Neutrino Mass Problem: Masses and Oscillations

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In this paper I will discuss the phenomenon of neutrino masses and how the mixing of flavor and mass eigenstates leads to oscillations. A theoretical treatment of neutrino mass eigenstates will first be brought up to differentiate between the possibilities of neutrinos being Majorana and Dirac particles. This paper will explain the theory behind neutrino oscillations and will motivate discussion with the solar neutrino problem.

1. INTRODUCTION

In the standard model of particle physics, there are three different generations of leptons: electronic, muonic, and tauonic leptons. Each of these generations consists of two flavors: a particle (electron, muon, or tauon) and a corresponding neutrino. Unlike their counterparts, the neutrinos all carry zero electric charge and have very little mass, only noticeably interacting with matter through the weak nuclear force.

In addition to there being six flavors of neutrinos, each flavor has an antiparticle. In the case of neutrinos, which do not have charge, the only property that differentiates between neutrinos and antineutrinos is their helicity. [1]. It is from this helicity that we observe that only left-handed neutrinos and right-handed antineutrinos interact with the weak force. This fact, as I will explain shortly, makes the question of determining the mass of the neutrinos interesting and will lead to a discussion of neutrino mass oscillations.

The question of the neutrino's mass is an important one in much of modern astrophysics and particle physics, as knowing its mass will reveal fundamental information about the nature of the neutrino and will let us discover new things about the state of the early universe.

2. MASSLESS NEUTRINOS

Since left-handed fermions couple with the left-handed components of particles, while right-handed fermions couple with right-handed components, we find that electron neutrinos interact with the left-handed components of electrons, forming a doublet, while electron antineutrinos interact with the right-handed components of positrons, also forming a doublet. We therefore expect helicity to differentiate the behavior of neutrinos and antineutrinos in weak interactions. However, we consider a neutrino traveling at subluminal speeds and an observer traveling alongside of it. If the observer is traveling slower than the neutrino, it will observe a momentum of $\vec{p}$, a spin of $\vec{S}$ and a helicity of h. However, when the observer increases its speed to faster than that of the neutrino, the momentum will appear to switch direction, while the spin will stay the same. With a momentum now of $-\vec{p}$ and the same spin, the helicity will switch to h, rendering the neutrino an antineutrino from the observer's reference frame. However, since the neutrino and the antineutrino interact differently, the observer would expect to see different interactions in both reference frames. This appears to form a contradiction, as the laws of physics should be the same in all reference frames.

The original response to this was to require that neutrinos be massless. If neutrinos have no mass, then they may travel at the speed of light, preventing this apparent contradiction from occurring.

However, recent observations such as neutrino oscillations provide evidence that neutrinos must have masses, while several beta decay experiments have shown that if neutrinos do have masses, they must be very small in comparison to other standard model particles.

3. MASSIVE NEUTRINOS

3.1. Majorana vs. Dirac Particles

To discuss the nonzero masses of neutrinos, it is first necessary to differentiate between two different types of particles. The first type is known as a Majorana particle and describes a particle that is the same as its antiparticle (this includes photons, for example). The second type is known as a Dirac particle, and describes the more familiar case in which particles and antiparticles are distinct, such as for electrons. Since it is not currently known whether neutrinos are Majorana or Dirac particles, it is necessary to discuss both cases, as they lead to slightly different results.

Considering a two-component spinor ρ (that is, a vector whose components represent two different particles), we can find the Lagrangian density for a free spin-$\frac{1}{2}$ spinor by considering the Klein-Gordon equation acting on spinors of two spin-$\frac{1}{2}$ Lorentz invariant particles to be:

$$\mathcal{L} = -i\rho^\dagger \sigma_\mu \partial_\mu \rho - \frac{m}{2}\rho^T \sigma_2 \rho + \text{Hermitian Conjugate} \quad (1)$$

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[1] The helicity of a particle, given by $h = \hat{S} \cdot \hat{p}$, is the projection of the particle's spin onto its momentum

Where ρ is the spinor whose components are a neutrino and its antineutrino, m is the mass of the particle, μ is one of $1, 2, 3, 4$, and the σ_μ are the Pauli matrices plus $-i1$—that is:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \sigma_4 = \begin{pmatrix} -i & 0 \\ 0 & -i \end{pmatrix}$$

Applying the Euler-Lagrange equation to equation 1 gives us the equation of motion,

$$-i\sigma_\mu \partial_\mu \rho = m\sigma_2 \rho^* \tag{2}$$

We can then write general solutions to 1 as,

$$\psi = \begin{pmatrix} \chi \\ \sigma_2 \phi^* \end{pmatrix} \tag{3}$$

We can explore the differences between Majorana particles and Dirac particles in this context by transforming these solutions under charge conjugation formalism. Charge conjugating a charged particle returns an antiparticle with the opposite charge of the original particle. Since neutrinos are neutrally charged, the result of applying charge conjugation is the neutrinos' antiparticle. In this form, the charge conjugation matrix C is commonly given as,

$$C = \begin{pmatrix} -\sigma_2 & 0 \\ 0 & -\sigma_2 \end{pmatrix} \tag{4}$$

Here we arrive at the result that $C\psi = \psi'$,

$$\psi' = \begin{pmatrix} \phi \\ \sigma_2 \chi^* \end{pmatrix} \tag{5}$$

If the neutrino is a Majorana particle, we therefore have that $\chi = \phi$. This is not necessarily true in the Dirac case. Note that χ represents the left-handed component of ψ and therefore the neutrino, while ϕ represents the right-handed component and therefore the antineutrino.

To consider the Dirac case in more depth, we can look at how equation 1 acts on our our solutions in equation 3. Considering the sum over the Lagrangian density for the two components of ρ (ρ_1 and ρ_2), we find that we can relate χ and ϕ to ρ_1 and ρ_2 by,

$$\chi = \frac{1}{\sqrt{2}}(\rho_2 + i\rho_1) \tag{6}$$
$$\phi = \frac{1}{\sqrt{2}}(\rho_2 - i\rho_1) \tag{7}$$

Equations 6,7 relate the states of Majorana fermions, ρ_i to Dirac fermions χ, ϕ. This relation shows that a Dirac particle is equivalent to two Majorana particles of equal mass.

From here it becomes clear that since Majorana and Dirac fermions have different wave functions, we can expect them to behave differently in physical situations. There are currently several experiments underway that are designed to test whether neutrinos are Majorana or Dirac particles.

3.2. Finding the Mass

One important difference between the two types of neutrinos is that they may decay differently. In particular, materials such as germanium that undergo double beta decay may typically emit two neutrinos in the process. However, if neutrinos are Majorana particles, there should be a small probability that the reaction will occur without releasing any neutrinos. What happens here is that the neutrino released from one beta decay may be absorbed into a nucleon and induce a second beta decay. Typically this reaction would require the nucleon to absorb an antineutrino, and therefore is not possible if neutrinos are Dirac particles, but if neutrinos are Majorana particles, there should be a small chance that this will occur.

Since the nature of the neutrino determines its decay physics, we can theoretically use measure the decay of nuclei to determine the mass of the neutrino. Regardless of which type of particle neutrinos are, such experiments should yield similar results, since they should both be very small. However, if neutrinos are Majorana particles, the mass should be slightly greater, due to a positive correction term in the matrix element describing this reaction.

While the exact masses of neutrinos have not yet been determined, there are limits on the masses of each of the neutrinos, as shown in table I This table shows the upper bounds on these neutrinos' masses. Most of the bounds were found from studying the electron emission spectra from beta-decaying nuclei. The values presented in table I are the average masses of flavor eigenstates of neutrinos. As I will discuss shortly, the flavor eigenstates and mass eigenstates of neutrinos are not the same; flavor eigenstates are superpositions of the three different mass eigenstates and vice versa, meaning that if a neutrino is in a definite flavor state, as they are in the experiments that placed the limits, the masses will vary according to how the mass eigenstates are mixed. This process of neutrino masses changing over time is known as neutrino oscillations.

	$\langle m \rangle$
ν_e	225 eV
ν_μ	0.19 MeV
ν_τ	18.2 MeV

TABLE I: Upper bounds on neutrino flavor mass expectation values, from the Particle Data Group

4. NEUTRINO OSCILLATIONS

4.1. Flavor and Mass Eigenstates

There are currently three known flavors of neutrino, the electron, μ, and τ, corresponding to the three generations of leptons. Experiments have also observed three different mass eigenstates that neutrinos may assume, denoted by ν_1, ν_2, and ν_3. The three flavor eigenstates correspond to the different reactions that produce those neutrinos, and differently flavored neutrinos all display different properties, making them distinguishable. In particular, weak interactions that produce electrons, may only produce electron neutrinos or electron antineutrinos.

We know that neutrinos exist largely through their roll in conserving energy—reactions such as beta decay involve the nucleus of an atom reducing its energy by an amount greater than is seen in the emitted electron, strongly hinting that a neutrino must be emitted to carry away the rest of the energy. Additionally, we can determine the flavor of the emitted neutrino by trying to conserve lepton flavor number, where particles of a particular generation carry a flavor number of $+1$ while their antiparticles carry a flavor number of -1. In the example of beta decay above, we know that the neutrino emitted is an electron antineutrino, since the antineutrino's lepton flavor number must balance with the number of the emitted electron.

As we have seen through experiments that lead to conundra such as the solar neutrino problem—which we will discuss later—the mass eigenstates are eigenstates of the Hamiltonian describing a free neutrino, but the flavor eigenstates—eigenstates of the weak interactions—are not. However, when a reaction occurs that produces a neutrino, we can know for sure what their flavor is, but not what their mass is. Since the emitted neutrinos are not at a definite mass eigenstate, they are free to oscillate between all of the possible states, as per the laws of quantum mechanics.

4.2. Mixing of the Two-State System

Since the tauon and ν_τ are both relatively massive[2] (the tauon is massive compared to other leptons, while ν_τ is massive compared to the other neutrinos), both particles are rarely seen in nature under ordinary circumstances, as they quickly decay into lower energy states. For this reason, it is instructive to first neglect the tau generation and begin our discussion of neutrino oscillations with a study of the simpler two-state system, neglecting the effects of ν_τ. Similarly, since ν_3 is so much more massive than the other mass eigenstates and is most commonly seen in ν_τ, we can neglect ν_3 in our early evaluations as well.

[2] It technically is not correct to call ν_τ massive, since it is a weak eigenstate of the neutrino rather than a mass eigenstate, so it does not actually make sense to talk about its mass. By referring to the mass of ν_τ, I am actually referring to its mass expectation value, coming from a combination of the mass eigenstates and the probability of ν_τ occupying each of those states

4.3. The Mixing Matrix

Making these simplifications, we encounter a system of two flavor eigenstates, ν_e and ν_μ along with their energy eigenstates ν_1 and ν_2. Since the flavor eigenstates are all orthonormal and span the basis of our model system (and the same is true for the mass eigenstates), we can write each flavor eigenstate as a superposition of mass eigenstates.

$$|\nu_e\rangle = U_{11}|\nu_1\rangle + U_{12}|\nu_2\rangle \tag{8}$$

$$|\nu_\mu\rangle = U_{21}|\nu_1\rangle + U_{22}|\nu_2\rangle \tag{9}$$

where $|\langle\nu_1|\nu_e\rangle|^2 = |U_{11}|^2$ represents the probability of the electron neutrino being observed with a mass of ν_1, and so on. Since the $|\nu_2\rangle$ state is of the same energy for both the $|\nu_e\rangle$ state and the $|\nu_\mu\rangle$ state, both flavor eigenstates are equally likely to be in that state. For this reason, we have the condition that $|U_{12}|^2 = |U_{21}|^2$. By conservation of probability, we are also left with the condition that $|U_{11}|^2 = |U_{22}|^2$.

As the notation suggests, a convenient notation for this system relies on the matrix representation and is commonly written as,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} U_{11} & U_{12} \\ U_{21} & U_{22} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{10}$$

$$= \begin{pmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} \tag{11}$$

We may call this θ-dependent matrix R_θ for shorthand. The use of cos and sin was chosen for this representation to enforce normalization, while the angle θ_{12} is known as the neutrino mixing angle between those two mass eigenstates and represents how likely a given neutrino is to be found in a particular eigenstate. The signs of matrix elements were chosen to make the matrix have unit determinate, a condition that we require for rotation matrices such as this.

Alternatively, we can solve equation 11 for the mass eigenstates and find that,

$$\begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix} = R_\theta^T \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \tag{12}$$

When discussing the oscillations of neutrino flavors that follow from this, we find that the spacial variation of the flavor states can be described by,

$$i\frac{d}{dx}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{1}{2E}M^2 \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} \tag{13}$$

where E is the energy of the neutrino and the matrix M^2 is given by,

$$M^2 = \frac{1}{2} R_\theta \begin{pmatrix} -\Delta m^2 & 0 \\ 0 & \Delta m^2 \end{pmatrix} R_\theta^T \tag{14}$$

4.3.1. The MSW Effect

When neutrinos pass through materials, the presence electrons in the material may alter the observed mixing angles. Since any given material will contain a large number of electrons and nuclei which will cause the neutrinos to scatter. Electron neutrinos may scatter off of electrons via exchange of W bosons, while any type of neutrino may scatter off of electrons or nucleons via the exchange of Z bosons. This effect is the eponymous Mikheyev-Wolfenstein-Smirnov effect, generally known as the MSW effect, and must be accounted for when determining the weak mixing angles for neutrino oscillations. This effect is particular important in observing solar neutrinos.

More quantitatively, the neutrinos passing through a material with electron density N_e will experience an effective increase in potential of

$$V_{\nu_e} = \sqrt{2} G_F N_e \tag{15}$$

where $G_F = 1.166 \times 10^{-5}\text{GeV}^{-2}$ is the Fermi constant.

We can account for this change in potential by altering the matrix M^2 from equation 14 as,

$$M^2_{MSW} = M^2 + E \begin{pmatrix} V & 0 \\ 0 & -V \end{pmatrix} \tag{16}$$

By considering all of the materials that the neutrinos must pass through, we can therefore correct our mass difference and observed mixing angles to account for the neutrinos' actual values.

4.3.2. Mass Differences

Considering the relativistic energy of a neutrino traveling through free space, we have the equation,$E^2 = m^2 + p^2$ where E is the energy of the particle, m is its mass, and p is the magnitude of its momentum. Solving this equation for energy and assuming that the momentum is large compared to the rest energy, we arrive at the relation $E = p + \frac{m^2}{2p}$. Taking the change of energy as the neutrino oscillates from one flavor state to another, we have that

$$\Delta E = \frac{\Delta(m^2)}{2E} \tag{17}$$

On a different note, we may consider the probability of a neutrino oscillating as it travels through space. In this paper, we will follow an electron neutrino traveling through space and find the probability that it has turned into a muon neutrino after a given flight length.

Earlier in our discussion of neutrino oscillations, we found that a muon neutrino is in a superposition of mass eigenstates as described by equation 11. The muon state of a particle traveling at time t is therefore given by,

$$|\nu_\mu\rangle = \cos\theta_{12} |\nu_2\rangle - \sin\theta_{12} |\nu_1\rangle \tag{18}$$

We also know the mass eigenstates in terms of the flavor eigenstates, as given by equation 12. Putting these equations into Schrödinger's equation we find that for a particle that is initially an electron neutrino in the $|\nu_1\rangle$ state,

$$|\nu_1(t)\rangle = -\sin(\theta_{12}) e^{-iE_1 t} \tag{19}$$

$$|\nu_2(2)\rangle = \cos(\theta_{12}) e^{-iE_2 t} \tag{20}$$

Combining equations 18, 20, and 20, we can find that the probability of the electron neutrino oscillating into a muon neutrino after a time t has elapsed is,

$$P_{\nu_e \to \nu_\mu}(t) = |\langle \nu_\mu(t) | \nu_\mu(t) \rangle|^2 = \left[\sin(2\theta_{12}) \sin(\frac{E_2 - E1}{2} t) \right]^2 \tag{21}$$

Now we can substitute in equation 17 into equation 21 while noting that, for neutrinos traveling near the speed of light, the distance traveled $x = t$. This yields the equation

$$P_{\nu_e \to \nu_\mu}(t) = \left[\sin(2\theta_{12}) \sin(\frac{\Delta(m^2)}{4E} x) \right]^2 \tag{22}$$

This equation reveals the length scale for neutrino oscillations,

$$L = \frac{4E}{\Delta(m^2)} \tag{23}$$

the distance after which a given neutrino is most likely to be found in the muon state.

By measuring neutrinos from a source that is producing them at a known and constant flavor at a certain distance away, we can determine how many of the neutrinos had changed state and therefore solve for the quantity $\Delta(m^2)$.

4.4. Mixing of the Three-State System

Now that we have studied how neutrinos oscillate in a two-state system, we can perform a similar analysis to explore the three-state case.

4.4.1. The MNS Matrix

Again, we begin with a system of equations relating the flavor and mass eigenstates.

$$|\nu_e\rangle = U_{11}|\nu_1\rangle + U_{12}|\nu_2\rangle + U_{13}|\nu_3\rangle \quad (24)$$
$$|\nu_\mu\rangle = U_{21}|\nu_1\rangle + U_{22}|\nu_2\rangle + U_{23}|\nu_3\rangle \quad (25)$$
$$|\nu_\tau\rangle = U_{31}|\nu_1\rangle + U_{32}|\nu_2\rangle + U_{33}|\nu_3\rangle \quad (26)$$

We can write this equation in terms of a matrix similar to what we did in developing the oscillation matrix in equation 11. In this case, there are three relevant mixing angles, θ_{12}, θ_{23}, and θ_{13}, relating to the the probabilities of a neutrino of a given flavor being observed in a given mass eigenstate. With the addition of a phase factor δ, we can describe the three-state neutrino mixing phenomenon using the mixing matrix U given by considering two-state rotations from ν_1 to ν_2, ν_2 to ν_3, and ν_1 to ν_3. These three rotation matrices are given respectively by,

$$\begin{pmatrix} \cos(\theta_{12}) & \sin(\theta_{12}) & 0 \\ -\sin(\theta_{12}) & \cos(\theta_{12}) & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (27)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_{23}) & \sin(\theta_{23}) \\ 0 & -\sin(\theta_{23}) & \cos(\theta_{23}) \end{pmatrix} \quad (28)$$

$$\begin{pmatrix} \cos(\theta_{13}) & 0 & \sin_(\theta_{13})e^{-i\delta} \\ 0 & 1 & 0 \\ -\sin(\theta_{13})e^{i\delta} & 0 & \cos(\theta_13) \end{pmatrix} \quad (29)$$

Multiplying these three matrices together gives us the total mixing matrix,

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (30)$$

Here we use the standard notation where $s_{ij} = \sin(\theta_{ij})$ and $c_{ij} = \cos(\theta_{ij})$. This matrix was first written by Maki, Nakagawa, and Sakata, and so it is commonly known as the MNS matrix for short. Using this matrix, we commonly write the weak eigenstates as,

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad (31)$$

Additionally, since U is unitary, we can find the mass eigenstates in terms of the weak eigenstates simply by taking the conjugate transpose of U.

The phase factor δ is zero if neutrino oscillations are to obey CP symmetry[3]. However, a common explanation for the matter dominance of the universe today is that CP symmetry is broken through neutrino oscillations. For this reason we include the factor of δ to account for this possibility.

Additionally, as we discussed at the beginning of this paper, there is a slight variation between the case in which neutrinos are Majorana particles and the case in which they are Dirac particles. Since antineutrinos and neutrinos are equivalent in the Majorana case, we can say that a Majorana neutrino is in a superposition of of states between a neutrino and an antineutrino, and so we introduce the two phase factors α_1 and α_2 and multiply U by an additional factor of

$$\begin{pmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (32)$$

Since equation 32 affects U as an overall phase (it acts on all of U equally with a unit determinate), it does not end up affecting the oscillations themselves. However, α_1 and α_2 becomes significant when considering other Majorana neutrino phenomena such as neutrinoless double beta decay. $\alpha_1 = \alpha_2 = 0$ if neutrinos are Dirac particles.

From studying solar neutrinos, atmospheric neutrinos, and nuclear reactor-borne neutrinos, we can compare the observed number of neutrinos of each flavor to the number produced to obtain values for the weak mixing angles θ_{12} and θ_{23}. Additionally, we can compare these measurements to limits placed on observations of the cosmic microwave background to place a limit on θ_{13}. These experiments have provided us with the values given in table II.

θ_{12}	$33.9° \pm 2.4°$
θ_{23}	$45° \pm 7°$
θ_{13}	$< 10.3°$

TABLE II: Data from solar, atmospheric, and reactor neutrino oscillation experiments have provided us with rough values for the weak neutrino mixing angles, shown above

From these angles, we can determine the mixing amplitude of each mass eigenstate with each weak eigenstate by substituting these values of θ_{ij} into the matrix in equation 30. These probabilities can be visualized as in figure 1.

[3] CP symmetry is a combination of charge conjugation symmetry (C) and parity symmetry (P). C symmetry means that the particle and the antiparticle behave the same. P symmetry means that the particle behaves the same if you reverse all spacial coordinates. CP symmetry means that the particle behaves the same under both transformations simultaneously. Note that this does not necessarily mean that the particle behaves the same under each one individually. There is also a third type of symmetry that commonly follows these two called T symmetry, which is a symmetry of reversing the flow of time

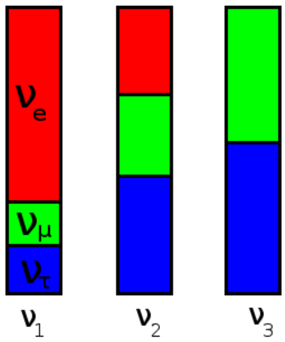

FIG. 1: The approximate flavor contents of the mass eigenstates. ν_3 is made up nearly entire of ν_μ and ν_τ eigenstates; the mixing with ν_e is too small to be shown in this figure

4.4.2. Mass Differences

By a similar analysis to what we did in our discussion of the two-state system, we can similarly come upon an equation relating the number of neutrinos that have oscillated from one state to another to the difference in the squares of the mass of the particles and the distance traveled by the neutrinos. By fitting our observations from the aforementioned oscillation experiments to our equations, we can find the mass square differences for the three mass eigenstates.

For the three neutrino mass eigenstates, there are two linearly independent differences to be calculated. These can be found from solar neutrino observations and from atmospheric neutrino observations and gives us values shown in table III.

$\Delta(m^2)_{12}$	$8 \times 10^{-5} \pm 0.6$ eV^2
$\Delta(m^2)_{23}$	$2.4 \times 10^{-3} \pm 0.6$ eV^2

TABLE III: The differences in the squares of the neutrino masses. $\Delta(m^2)_{12}$ comes from solar neutrino observations while $\Delta(m^2)_{23}$ comes from atmospheric neutrino observations

These values indicate that two of the masses are relatively close together, while the third is far away. Because these values are differences, they do not indicate which mass is greater. By convention, we call ν_1 the lower of the two masses that are close to each other and ν_2 the higher of the two close masses, leaving ν_3 to be the mass that is either far above or far below the mass of the other two neutrinos. As a result, we end up with two relative mass distributions, known as the normal and the inverted neutrino spectra, shown in figure 2

Since the electron and the muon have relatively similar masses while the tauon is significantly more massive, we intuitively expect the neutrinos to follow the same general pattern—that is to say, we naturally want to say that the mass that is far away from the other two is also the heaviest mass. For this reason, we call the spectrum on the left of figure 2 the "normal" spectrum and the one

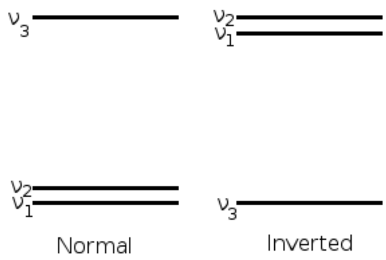

FIG. 2: The normal and inverted relative neutrino mass spectra

on the right the "inverted" spectrum. Of course, this is mostly just our intuition; with what is currently known, both distributions are equally likely to reflect reality.

5. THE SOLAR NEUTRINO PROBLEM

Now that we have discussed the theory of neutrino oscillations, we can understand what was going on in the experiment that initially motivated the study of neutrino oscillations. Before the solar neutrino problem was encountered, neutrinos were widely considered to be massless, as predicted by the standard model. However, this problem has revolutionized the way we think about neutrinos as will soon be apparent.

5.1. The pp Chain

To begin discussion, it is important to examine the source of neutrinos from the sun. The neutrinos created in the sun are all initially electron neutrinos. While stars in general may undergo a large number of neutrino-producing reaction, the reaction the dominates solar mass stars such as the sun is known as the pp chain.

The pp chain begins with two protons combining to form a deuteron, releasing an electron neutrino (and either releasing a positron or absorbing an electron in the process). The deuteron then combines with another proton to form a ^{3}He and a γ. From here, any of three things may happen. The ^{3}He may react with another proton, yielding an α particle, a positron, and another electron neutrino. Alternatively, the ^{3}He may interact with another ^{3}He, forming an α and two protons, or the ^{3}He may interact with an α, resulting in a ^{7}Be and a γ. The ^{7}Be may then interact with an electron, resulting in a ^{7}Li and an electron neutrino, emit a positron and an electron neutrino, or undergo a series of other reactions that do not produce neutrinos.

Since the energies for each of these reactions can be determined and the mass and temperature of the sun can easily be approximated, we can easily estimate the neutrino flux we expect to see coming from the sun.

5.2. The Davis Experiment and the Missing Neutrinos

In 1968, Ray Davis performed an experiment to test the theoretical predictions of the neutrino flux. For his experiment, he set up a large vat of liquid chlorine, deep in the Homestake mine in South Dakota to avoid background from cosmic rays. When a neutrino approaches a chlorine atom, there is a chance that it will induce a beta decay, turning the chlorine into an argon atom. Since the probability of this happening was well-known at the time, Davis set the tank up for several months and counted the number of argon atoms in the tank at the end of the experiment. From this number, he was able to work out how many neutrinos must have been emitted from the sun as a whole during that time period.

The number of neutrinos that Davis's experiment measured was only about a third of the theoretical neutrino flux. Many other similar experiments including the Kamiokande series in Japan as well as the Solar Neutrino Observatory (SNO) in Sudbury, Ontario, followed Davis's experiment, trying to measure the solar neutrino flux. All of these experiments failed to measure the expected amount; most of them also appeared to have missed about two thirds of the neutrinos.

Eventually SNO managed to solve this problem. Using D_2O instead of H_2O (using heavy water instead of water), SNO was able to observe three different possible reactions:

$$\nu_e + d \rightarrow e^- + p + p \tag{33}$$

$$\nu_x + d \rightarrow \nu_x + p + n \tag{34}$$

$$\nu_x + e^- \rightarrow \nu_x + e^- \tag{35}$$

where ν_x can be any flavor of neutrino. By monitoring all three different reactions, SNO measured the expected number of neutrinos. However, when it only looked at the results from reaction 33, it only measured one third of the expected number of neutrinos. This was the first clue that neutrinos from the sun must be oscillating into different flavors of neutrinos along their path to earth.

From these experiments, the modern theories of neutrino masses and neutrino oscillations were born.

6. CLOSING REMARKS

While the standard model predicts that neutrinos should not have mass, experiments such as SNO's measuring of solar neutrinos have shown neutrinos actually behave as if they have mass eigenstates between which they can oscillate. This discovery lead to the whole theory of neutrino oscillations, as both their mass and weak eigenstates oscillate alongside each other. Since we know that neutrinos must be massive in order for their mass eigenstates to oscillate, we also know that our standard model view of neutrinos is incorrect. This discrepancy signifies a need for beyond the standard model physics in order to fully understand neutrinos. Many theories arise, explaining how neutrinos may get their mass. These lead into several complex and beautiful theories, many of which—such as the Higgs mechanism—run deep into heart of modern particle physics.

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