Finding Boltzmann's Constant and Absolute Zero by Analyzing Johnson Noise

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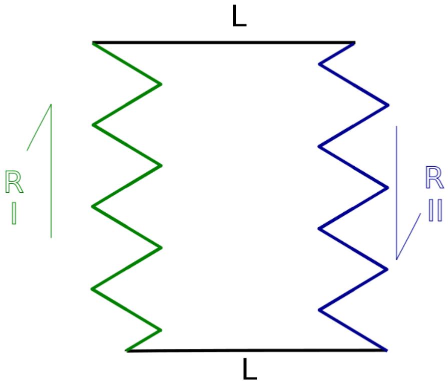
Experimental Goals

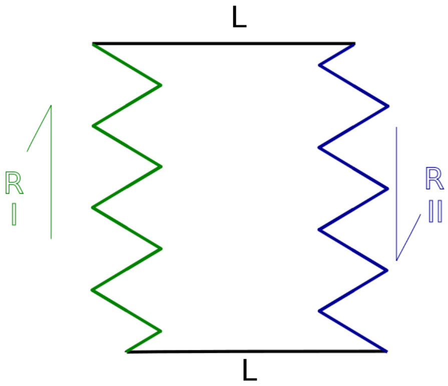
- Observe the Johnson (thermal) noise across a resistor
- Calculate Boltzmann's constant
- Calculate absolute zero in $^\circ$C

The Nyquist Theory of Johnson Noise



- Capacitance C, Inductance L
- Thermal Equilibrium $\Rightarrow$ $P_I = P_{II}$
- Each node = degree of freedom = kT energy
- $n + 1$ nodes $\Rightarrow$ natural frequency $\frac{n\nu}{2l}$

The Nyquist Theory of Johnson Noise



- Total energy over $d\nu$ is $\frac{2lkTd\nu}{\nu}$
- Average power dissipated by resistors: $kTd\nu$
- Relating power to voltage and analyzing circuit with gain function $g(\nu)$: $V^2 = 4R_\nu kTg^2(\nu)d\nu$
- $R_\nu = \frac{R}{1+(2\pi\nu CR)^2}$

The Nyquist Theory of Johnson Noise

In other words, the average thermal fluctuations in voltage obey

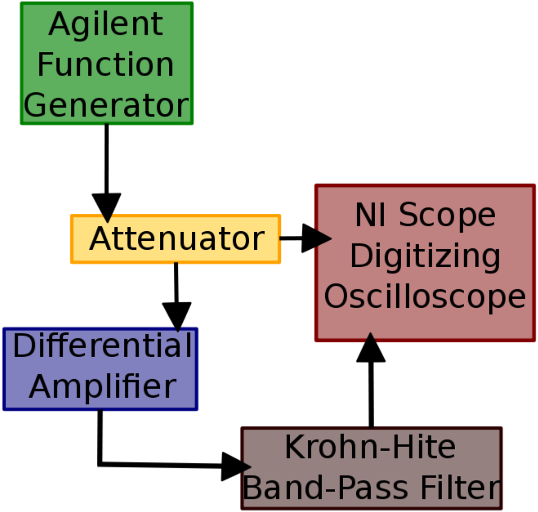
Johnson Noise Relation

$$\bar{V}^2 = 4RkTG(\nu) \tag{1}$$

where $G(\nu) = \int_0^\infty \frac{g^2(\nu)}{1+(2\pi\nu CR)^2} d\nu$

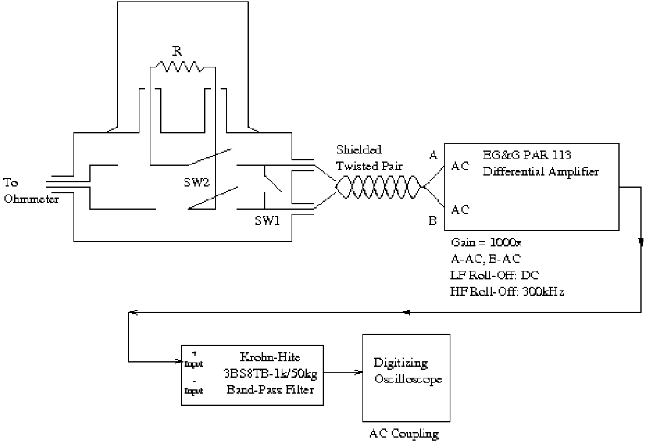
Experimental Setup

Calibration



Experimental Setup

Experiment



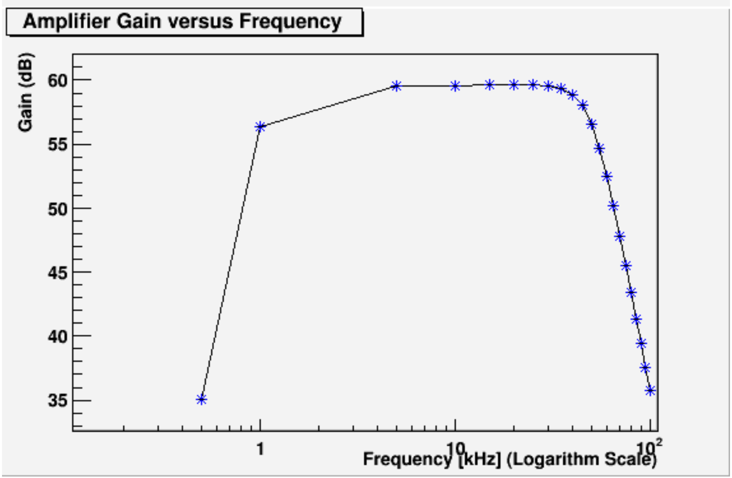
"Johnson Noise and Shot Noise: The Determination of the Boltzmann Constant, Absolute Zero Temperature and the Charge of the Electron", MIT Department of Physics, 7/11/08

Experimental Procedure

- Calibrate system amplification: Obtain gain curve
- Measure $V^2 = V^2_{resistor} - V^2_{shorted}$ and capacitance for:
  - varying temperatures
  - varying resistors

Results

Calibration Curve



Phadnis, Preeya. "Johnson and Shot Noise", MIT Department of Physics, 11/17/07

Results

Temperature Curve

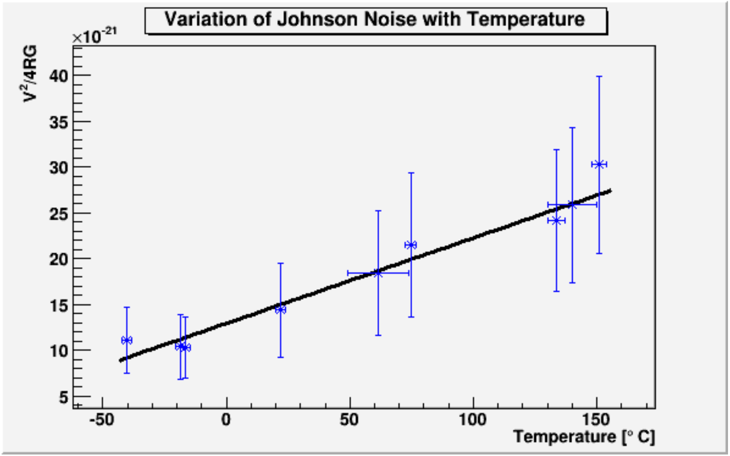


Figure: $\chi^2 = 0.635$, with probability $P(\chi^2)=0.9988$

- $k_b = 9.325 \times 10^{-23} \pm 2.952 \times 10^{-23}$ J/K
  - Generally accpted value: 1.381×10^{-23} J/K
  - Within 2.69σ
- 0K$= -138.97 \pm 49.79°$C
  - Generally accepted value: $-273.15°$C
  - Within 2.69σ

Error Analysis

Voltage Measurements

- Statistical error due to stochastic (thermodynamic) process
- Systematic error due to attenuation of signal in wires ($\approx 2\%$)

Error Analysis

Gain Measurements

- Propagate error from voltage measurements
- Systematic error from numerical integration
- Systematic error due to applying calibration curve from different data set to our temperature and voltage data

Error Analysis

Temperature, Capacitance, and Resistance Measurements

- Systematic error in temperature measurements due to temperature increasing or decreasing during measurements
- Random error in capacitance measurements due to significant fluctuations in capacitance while we were measuring it
- Small random error in resistance measurements due to minor fluctuations in resistance while we were measuring it

Error Analysis

Other sources

- Low bit rate on oscilloscope caused loss of data for higher frequency noises — could not correct for this due to the software licenses expiring
- High sensitivity to background events (such as people walking by)

Conclusions

- Temperature and $\frac{V^2}{4RG}$ appear to vary linearly
- Due to problems with the digitizing oscilloscope, problems arose that we could not correct for
- Obtained Boltzmann's constant and absolute zero to within 2.69σ
  - $k_b = 9.325 \times 10^{-23} \pm 2.952 \times 10^{-23}$ J/K
  - 0K$= -138.97 \pm 49.79°$C

Acknowledgements

I would like to thank my partner Anna Waldman-Brown for helping me collect and analyze data as well as the Junior Lab staff for helping me work out problems I encountered along the way. I would also like to thank Preeya Phadnis for furnishing me with the gain data used in this experiment.