Superconducting Metals: Finding Critical Temperatures and Observing Phenomena

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(Dated: April 29, 2010)

Many metals display special properties below a certain critical temperature when they become superconducting. In this paper, we present findings for the critical temperatures of vanadium ($T_C = 5.36 \pm 0.13$ K), lead ($T_C = 7.30 \pm 0.05$ K), and niobium ($T_C = 9.7 \pm 0.6$ K) as well as observations of the suppression of the critical temperature of vanadium in the presence of a magnetic field and the $T = 0$ K critical field ($H_0 = 1480 \pm 105$ Oe). We also measured the Josephson effect in niobium and used our observations to determine the superconducting energy gap in niobium ($\Delta(0) = 4.23 \pm 0.18 \times 10^{-22}$ J) and to calculate the fundamental flux constant ($\Phi_0 = 2.11 \pm 0.07 \times 10^{-15}$ Wb). Lastly, we observed a persistent current in superconducting lead that held an initial internal zero flux.

1. INTRODUCTION

Superconductors are metals or alloys that have been cooled to a temperature below some critical value. When a metal capable of becoming a superconductor does so, it loses all electrical resistance and demonstrates several other properties, many of which will be discussed later in this paper.

2. BCS THEORY

BCS theory is the theory proposed by Bardeen, Cooper, and Schrieffer to describe the microscopic effects that cause a material to become superconducting.

Since electrons are fermions, they obey the Pauli exclusion principle. This means that only two electrons with opposite spin may occupy a given energy state at a time; when a metal is at its lowest energy level, there are only two electrons in the ground state, and the rest of the electrons fill successively higher energy levels, creating the Fermi sea. The highest energy level of the Fermi sea when the metal is in its ground state is known as the metal's Fermi energy.

In a normal metal, the lowest energy state of the metal is related to the Fermi energy. The two dominating forces that one must consider in a metal are the Coulombic repulsion between the electrons and the phononic interactions between the electrons and the nuclei. As the electrons moves through the surface of the metal, the attractive force between the electrons and nuclei cause slight distortions in the nuclear lattice. These waves are called phonons and tend to attract more electrons. Under ordinary conditions, these phonon interactions are insignificant—the thermal energy of the atoms is enough to break up any structure that the phonons may form, so they have little effect on the electrons.

As the temperature of the metal decreases, the thermal agitations become smaller and smaller until they can no longer break up the phonon waves. At this point, the attractive force of the phonons may become stronger than the repulsive Coulombic force. Two electrons with opposite spin may move towards each other and reach an entangled state known as a Cooper pair. Since the electrons have opposite spin, the Cooper pair has a total spin of 0 and is a boson, no longer constrained by the Pauli exclusion principle. The superconductor consists of several bosons, all of which may settle to the ground state at the same time. This means that the Cooper pairs will all occupy a total energy state lower than was achievable by the individual electrons. Additionally, since bosons may all occupy the same state, the metal will not have any resistance. This lack of resistance is one of the key qualities that define a superconductor.

At heart, BCS theory is more general than this. BCS theory predicts that any net attractive potential will cause electrons to form Cooper pairs. However, the mechanism described here is the one seen in most ordinary superconductors.

3. CRITICAL TEMPERATURE MEASUREMENTS

3.1. Experimental Setup

Figure 1 diagrams the probe used in this experiment. The probe was inserted into a dewar of liquid helium and lowered until the sample was shortly above the liquid helium, so that it would be cooled by the vapor. A

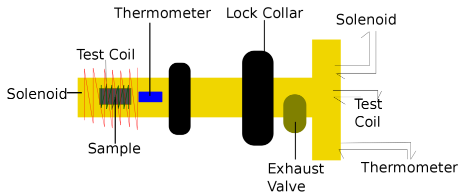

FIG. 1: A diagram of the probe used for measuring the critical temperature of vanadium and lead

pump was attached to the probe through the exhaust

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valve and was used for fine adjustments to the flow rate of the helium cooling the sample, allowing us to control the temperature of the sample.

An AC current was applied to the solenoid at a high frequency so that when the sample is not superconducting, it picks up the magnetic field generated by the solenoid. As this magnetic field changes, it induces a current in the test coil which was hooked up to an oscilloscope. Measuring this induced current shows a sine wave of a given height. When the sample is superconducting, the Meissner effect predicts that, beyond a certain skin thickness, the sample will not hold a magnetic field. This has the effect that less current is induced in the test coil since there is much less magnetic field inside of it, and so the sine wave on the oscilloscope decreases in amplitude.

Temperatures were measured using a diode placed a couple centimeters above the sample. Since the behavior of the diode materials at low temperature is known, the temperature of the diode can be determined by measuring the voltage across the diode. A conversion table from volts to Kelvin is provided in the lab guide[1]. In order to determine the temperature of voltages in between the ones provided by the table, we plotted a graph of the points provided, as seen in figure 2. As is clear in the figure, the curve appears to have one small region, followed by a discontinuity and then another smooth region. A sixth order polynomial was fit to each region and used to interpolate the temperatures corresponding to all of the measured voltages. For the region over the discontinuity, we extrapolated both fit functions and averaged their values together to determine the temperature of points in that region.

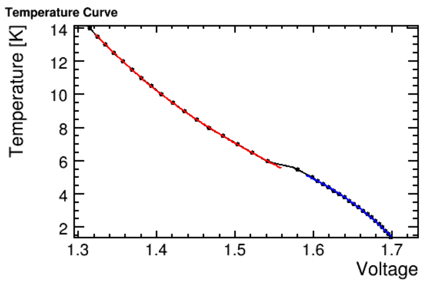

FIG. 2: The voltage-temperature calibration curve provided by the lab guide

Since there is a gap of about two centimeters between the sample and the thermometer, a temperature gradient will naturally be present and can add an additional systematic error to the data. To minimize the effects of this, we cooled the sample slowly to reduce the gradient.

3.2. T_C Measurements

To measure the critical temperatures of vanadium and lead, we cooled the probe for roughly two and a half hours to ensure that we would only have a small systematic error due to the temperature gradient.

As mentioned above, the sine wave reading on the oscilloscope should be smaller when the sample is in the superconducting phase than when it is a normal metal. In order to determine the critical temperatures, we found the temperature at which the sine wave's amplitude first began to decrease and continued to decrease the temperature until we found the point at which the sample just reached its minimum amplitude. By recording the temperatures at which the samples began and completed their transition to being totally superconducting several times and averaging these values together, we measured the critical temperatures of the samples. We found the critical temperature of vanadium to be 5.36 ± 0.13 K and the critical temperature of lead to be 7.30 ± 0.05 K.

The vanadium sample has a much larger random error because the measured temperature fell on the discontinuity in figure 2, where we had to extrapolate and average the values of the two fit functions.

Due to the aforementioned temperature gradient, there is a small unknown systematic error in the measurements of these temperatures; the actual values should be slightly lower than what we measured. For vanadium, this systematic error was small compared to the random error, but it is likely more significant for the lead sample which has less random error.

4. CRITICAL FIELDS

Exposing a metal to a magnetic field increases the energy gap between the superconducting and normal metal states by driving the Cooper pairs apart and weakening the attractive force. This suppresses the critical temperatures of the samples. For any given temperature less than the critical temperature, the magnetic field that will cause the sample to transition from being a superconductor to a normal metal is called the critical field.

Since the energy gap is proportional to the applied magnetic field, we have the relation ship $\frac{\Delta E(T)}{\Delta E(0)} = \frac{H_T}{H_0}$ where $\Delta E(T)$ is the energy gap at temperature T and H_T is the critical field at that temperature. Additionally, we know that $\frac{\Delta E(T)}{\Delta E(0)} = \cos\left(\frac{\pi T^2}{2T_C^2}\right)$[3]. Combining these two relations and Taylor expanding gives us the relation

$$H_T = H_0 \left(1 + \left(\frac{T}{T_C}\right)^2\right) \tag{1}$$

4.1. Measuring H_0

Using the same setup as we did to measure the critical temperatures, we can apply a DC current to the solenoid in addition the AC current. This offsets the zero of the AC oscillations, exposing the sample to a controllable magnetic field.

By recording the height of the sine wave on the oscilloscope as a function of temperature, as seen in figure 3 and fitting a logistic function[1], we were able to determine the critical temperature of the vanadium sample at $H_T = 0$ Oe and $H_T = 62.4$ Oe and solve for H_0 using equation 1. Doing so, we found $H_0 = 1480 \pm 105$ Oe.

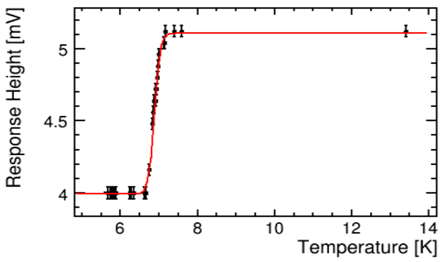

FIG. 3: Temperature versus test coil amplitude at $H_T = 62.4$ Oe. The logistic fit function fits with $\chi^2/NDF = 0.3$ and probability 99.9%

5. THE JOSEPHSON EFFECT

When two metals sandwich a thin insulator (known as a Josephson junction) and a voltage is applied to the metals, we expect an Ohmic (linear) response as the voltage between the metals varies linearly with the current through the insulator. However, when the two metals are lowered to below their critical temperatures, the probability of Cooper pairs quantum tunneling through the insulator becomes significant. This is known as the Josephson effect.

5.1. Measuring the Superconducting Energy Gap in Nb

We observed the Josephson effect using a niobium Josephson junction. The setup for this experiment was very similar to the previous one; the key differences are that the probe contains a Josephson junction in place of the sample and that we directly measure the temperature using a temperature sensor. We set the oscilloscope to plot the applied voltage across the junction versus the current between the plates.

When the niobium is non-superconducting, we expect to observe a linear relationship between the current and voltage as we observe Ohmic single electron tunneling across the junction. However, when the niobium is lowered to below its critical temperature, we observe what is shown in figure 4. Here, we see that at high voltages,

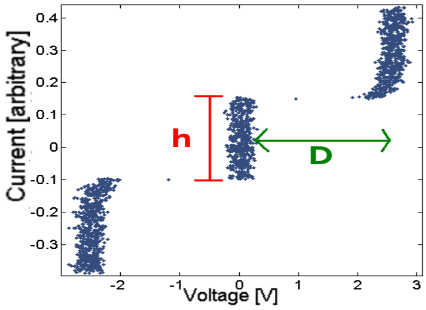

FIG. 4: V-I relation across a superconducting niobium Josephson junction

the energy of the electric field pulls apart Cooper pairs and we are left observing an Ohmic voltage-current relationship. However, when the voltage is too weak to do this, we find that the voltage drives tunneling of Cooper pairs back and forth so fast that there appears to be no current across the junction when the voltage is nonzero. When the voltage is zero, however, this is no longer the case and so the Cooper pairs may tunnel across the junction freely, resulting in the spike in the middle of figure 4.

By slowly adjusting the temperature of the junction, we can observe the temperature at which the Ohmic, non-superconducting line first begins to break and continue to decrease the temperature until it becomes steady in the form of figure 4. Doing so several times and averaging these temperatures together gives us a critical temperature for niobium of $T_C = 9.7 \pm 0.6$ K.

Since the current returns to being Ohmic when the voltage is strong enough to break the Cooper pairs, we can measure the superconducting energy gap by measuring this change in voltage, labeled D in figure 4. Since the Cooper pairs tunnel with a frequency $\nu = \frac{2e}{h}V$[4], we find that the energy gap $\Delta(T)$ corresponding to this transition is

$$\Delta(T) = \frac{De}{2} \tag{2}$$

Additionally, we expect the energy gap at a given temperature to be $\Delta(T) = 3.5 k_B T_c \sqrt{1 - \frac{T}{T_C}}$. Using these two equations and adjusting for the temperature at which the measurements were taken, we found $\Delta(0) = 4.23 \pm 0.18 \times 10^{-22}$ J.

5.2. Measuring the Flux Quantum

The flux quantum is the smallest possible "bit" of flux. This can be measured very precisely using our setup by applying a magnetic field to the Josephson junction. This can be done by running a current through the solenoid on the probe and observing the change in the amount Cooper pairs that tunnel through the junction (measured by h in figure 4). We expect h to follow the relationship

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[1] Physically, a logistic function makes sense here because we expect the electrons we base our observations on to follow a Fermi-Dirac distribution, which is of the same form

$h = A_0 \frac{\sin(\pi\omega B)}{\pi\omega B} + A_1$[4], where A_0, A_1 and ω are all fitting parameters. Plotting h versus magnetic field and fitting this function to the curve yields figure 5 where we find

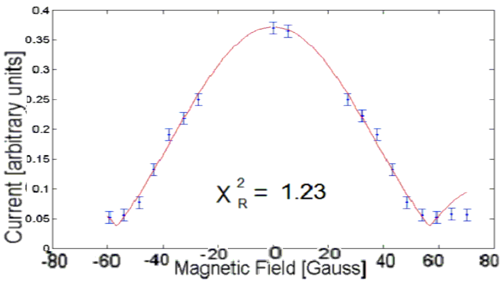

FIG. 5: Magnetic field versus h. The magnetic field axis was scaled from the current we applied to the solenoid using a factor of 540 Gauss/A[1]

$\omega = 213T^{-1}$

Noting that $\pi\omega B = \frac{\Phi}{\Phi_0}$, we find that $\Phi_0 = \frac{2\lambda + l)L}{\omega}$ where $\lambda = 46.6$ nm is the London penetration depth and $l = 1.75$ nm, $L = 5\mu$m describe the dimensions of the sample. We can therefore conclude that $\Phi_0 = 2.11 \pm 0.07 \times 10^{-15}$ Wb.

6. PERSISTENT CURRENT OBSERVATIONS

When a hollow cylinder of a metal is exposed to a magnetic field, the magnetic field is present in the cavity of the cylinder. If the metal then becomes superconducting, surface currents on the inside of the cylinder continue to generate the magnetic field in the cavity regardless of how the external field changes.

This effect can be observed experimentally using a probe similar to the one used to measure critical temperatures. The key difference between the probes is that this probe contains a hall sensor with a hollow lead cylinder inside instead of a sample.

We observed this effect by lowering a lead cylinder to below its critical temperature and then supplying a current to the solenoid around the cylinder, generating a magnetic field. As can be seen in figure 6, there continues to be no magnetic field inside the cylinder after the magnetic field has been turned on at A. As we increased the temperature of the sample, we began to see the external magnetic field appear when we got near the critical temperature of lead. This change signified the transition of the lead from a superconducting to normal metal state.

7. ERROR ANALYSIS AND CONCLUSIONS

Table I summarizes the results found by this experiment

Random errors dominated the errors in this experiment, with fitting errors comprising roughly 80% of the errors for most quantities. Temperature variations and digitization error from the oscilloscope contributed roughly equal amounts to the error, accounting for nearly 10% each.

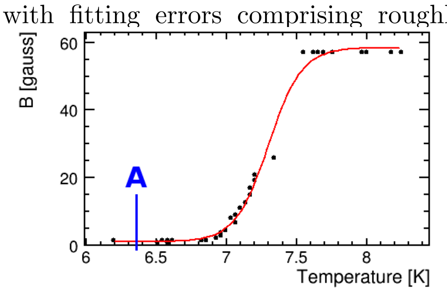

FIG. 6: Shows the persistent current effect in lead. The logistic fit function was used to scale the x-axis to the known T_C of Pb

	Value	±	Known[2]	Δ
$T_C(V)$	5.36 K	0.13	5.40 K	0.3σ
$T_C(Pb)$	7.30 K	0.05	7.20 K	2σ
$T_C(Nb)$	9.7 K	0.6	9.2 K	0.8σ
$H_0(V)$	1480 Oe	105	1408 Oe	0.7σ
$\Delta(0)_{Nb}$	4.23×10^{-22} J	0.18×10^{-22}	4.44×10^{-22} J	1.2σ
Φ_0	2.11×10^{-15} Wb	0.07×10^{-15}	2.07×10^{-15} Wb	$.6\sigma$

TABLE I: Quantities found in this experiment compared to their accepted values

There was also a small systematic error due to the temperature gradients between the samples and the thermometers. For most measurements this was small, although we believe it accounts for most of the deviation in the critical temperature of lead from its accepted value. For measuring quantities other than critical temperatures, we were able to adjust for this systematic error by normalizing our temperatures to our accurately measured temperatures.

[1] "Superconductivity: The Meissner Effect, Persistent Currents and the Josephson Effects", MIT Department of Physics, 1/29/09
[2] L.I. Berger and B.W. Roberts, "Properties of Superconductors", CRC Handbook of Chemistry and Physics, 90th ed (2010).
[3] T.P. Sheahen, "Rules for the Energy Gap and Critical Field of Superconductors", Physical Review 149,1 (1966).
[4] Y. Bruynseraede, et. al, "Giaever and Josephson Tunneling", NATO ASI Series, F59, Superconducting Electronics, Springer-Verlag (1989)