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A GAUSSIAN MODEL FOR PREDICTING THE EFFECT OF UNSTEADY WINDSPEED ON THE VORTEX-INDUCED VIBRATION RESPONSE OF STRUCTURAL MEMBERS

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ABSTRACT

In a previous study [1] the authors have shown that the expected duration of visit by the windspeed to the critical velocity interval of a structural member is an important time scale in determining the ultimate fatigue damage of the member due to vortex-induced vibration in naturally time varying winds. In this paper, a Gaussian windspeed assumption is introduced in which the expected duration of visit can be expressed explicitly in terms of simple wind statistics. This assumption is verified with high sampling rate maritime wind data. The wind statistics necessary for calculation of the expected duration of visit are extracted from the raw wind data.

INTRODUCTION

Vortex-induced vibrations of structural members have been the source of fatigue damage to offshore platforms during fabrication and transportation and to flarebooms during in-service conditions. To avoid failures, it is important for designers to be able to predict such vibrations as well as the resulting fatigue damage.

Current response prediction methods generally assume that when the mean wind speed is within the critical wind speed range for a given structural member, then it is adequate to compute the steady state response of the member and the associated fatigue damage rate. However, practical experience has revealed [2] that these methods *over-predict* the response, and, predict structural failures *too* frequently.

Fei & Vandiver [1] discovered from wind tunnel experiments that large-scale variations in the mean windspeed typically prevent vortex-excited vibrations of an elastic cylinder from reaching steady state amplitudes. In other words, if the duration of time that the windspeed stays within the critical velocity range for the member is less than the transient buildup time for the lightly damped vibration of the member, then the fatigue damage rate is reduced. When the wind is considered as a random process, the ratio between the two time scales is important in determining the ultimate fatigue damage rate. The two time scales are the duration of visit by the windspeed to a critical velocity interval and the rise time of the structural response.

Based on the results of wind tunnel experiments, Fei & Vandiver [1] proposed a probabilistic model for the prediction of the expected fatigue damage rate of a structural member excited by random winds. The model accounts for the effects of unsteady windspeeds and finite structural response rise times. The variability of the windspeed is characterized by the expected duration of a visit by the mean windspeed to the critical velocity interval of a particular structural member. For a stationary wind process, Fei [3] has shown that the expected duration of visit depends on the cumulative distribution functions and the mean upcrossing rates of the windspeed evaluated at the boundaries of the critical velocity interval.

In this paper, a Gaussian windspeed assumption is introduced, which allows the expected duration of visit to be expressed explicitly in terms of simple wind statistics. This assumption is tested by comparison to results computed directly using high sampling rate real maritime wind data. Wind statistics, extracted from the data, are used in the Gaussian model to predict the expected duration of visit.

These predictions are compared to the direct results.

EXPECTED DURATION OF A VISIT BY THE WINDSPEED IN A CRITICAL VELOCITY INTERVAL $[a, b]$

The duration of a visit by the wind speed to an interval $[a, b]$ is defined as the undisrupted length of time that the wind speed spends between levels a and b. The definition of the duration of a visit by the wind speed to an interval can be illustrated in Figure 1. $\mathcal{T}_{[a,b]}$, the duration of a visit to an interval, starts with either an upcrossing of the windspeed at level a or a downcrossing of windspeed at level b, and ends with either an upcrossing at b or a downcrossing at a. In the same figure, $\mathcal{T}_{]a,b[}$ is the period that the windspeed spends outside of $[a, b]$.

In the case of random wind, the duration of visit is a random variable that depends on the mean rates of crossings by the wind speed at levels a and b. The exact distribution of the duration of a visit by the wind speed to an interval is not known except for very few random processes [4]. However, it was shown in Fei [3] that the exact mean of the duration of visit can be calculated as follows, provided that the wind speed is a stationary random process:

To derive the mean value $E[\mathcal{T}_{[a,b]}]$, we need to apply a nonlinear transformation to the windspeed process $V(t)$. Let $X(t)$ be a random process that can be derived from $V(t)$ in the following way:

$$X(t) = y - (V(t) - a)(V(t) - b) \tag{1}$$

where y is an arbitrary positive real constant.

This transformation, as expressed in Equation 1, establishes a nonlinear mapping from $V(t)$ to $X(t)$. Specifically, the windspeed samples within the velocity interval $[a, b]$ are mapped to the samples of the process $X(t)$ which have the values greater or equal to y. The windspeed samples outside the interval $[a, b]$ are mapped to the samples of the process $X(t)$ which have the values less than y. Therefore, calculating the mean duration of a visit by the windspeed to the interval $[a, b]$ is equivalent to calculating the mean length of stay by the process $X(t)$ above the level y.

Figure 2 shows the time history of the process $X(t)$, which is derived from the windspeed process $V(t)$ shown in Figure 1, through the nonlinear transformation expressed in Equation 1. T_y and T'_y are, respectively, the successive times which $X(t)$ spends above and below the threshold y. Since $a \le V \le b$ corresponds to $X \ge y$, thus $T_y = \mathcal{T}_{[a,b]}$. In the following, we will derive $E[T_y]$ in terms of the statistics of $X(t)$, then express $E[\mathcal{T}_{[a,b]}]$ in terms of the statistics of $V(t)$.

$E[T_y]$, the mean value of T_y, can be expressed as below for a stationary random process $X(t)$ [5].

$$E[T_y] = \frac{1 - F_X(y)}{\mu_y^+} \tag{2}$$

where $F_X(y)$ is the CDF of the process $X(t)$ evaluated at y. μ_y^+ is the mean rate of crossing the level $X(t) = y$ at positive slopes.

Since $a \le V \le b$ corresponds to $X \ge y$, the probability of the windspeed within the critical velocity interval is equivalent to the probability that the derived process $X(t)$ exceeds the level y. Since each upcrossing at the level $X(t) = y$ corresponds to either a simultaneous upcrossing at the level $V(t) = a$ or a simultaneous downcrossing at the level $V(t) = b$, the frequency of crossing the level $X(t) = y$ at positive slopes is equivalent to the frequency of crossing the level $V(t) = a$ at positive slopes and crossing the level $V(t) = b$ at negative slopes. Therefore, $F_X(y)$ and μ_y^+ can be related to the statistics of $V(t)$ as follows.

$$1 - F_X(y) = F_V(b) - F_V(a) \tag{3}$$

$$\begin{aligned} \mu_y^+ &= \nu_a^+ + \nu_b^- \\ &= \nu_a^+ + \nu_b^+ \end{aligned} \tag{4}$$

where $F_V(b)$ and $F_V(a)$ are, respectively, the cumulative distribution functions (CDFs) of the windspeed process $V(t)$ evaluated at b and a. ν_a^+ is the mean rate of crossing the level $V(t) = a$ at positive slopes, ν_b^+ and ν_b^- are the mean rates of crossing the level $V(t) = b$ at positive and negative slopes respectively. Since every up-crossing is followed by a down-crossing, $\nu_b^+ = \nu_b^-$. The mean rate of crossing the level $V(t) = \theta$ can be expressed as follows [5].

$$\nu_\theta^+ = \nu_\theta^- = \frac{1}{2}\int_{-\infty}^{\infty} |\,\dot{v}\,|\, p_{V\dot{V}}(\theta, \dot{v})d\dot{v} \tag{5}$$

where $p_{V\dot{V}}(v, \dot{v})$ is the joint probability density function (PDF) of the random process $V(t)$ and its time derivative process $\dot{V}(t)$.

Substituting Equations 3 and 4 into Equation 2, we arrive at the equation for the mean duration of a visit by the windspeed to the critical interval $[a, b]$.

$$E[\mathcal{T}_{[a,b]}] = \frac{F_V(b) - F_V(a)}{\nu_a^+ + \nu_b^+} \tag{6}$$

Where $F_V(c)$ is the cumulative distribution function (CDF), which specifies the probability that the wind speed is less than or equal to c; ν_c^+ is the mean rate of crossing the level $V(t) = c$ with positive slopes, and:

$$F_V(c) = \int_0^c p_V(v)dv \quad (7)$$

$$\nu_c^+ = \frac{1}{2}\int_{-\infty}^{\infty} | \dot{v} | p_{V\dot{V}}(c,\dot{v})d\dot{v} \quad (8)$$

Where $p_V(v)$ is the PDF of the wind speed and $p_{V\dot{V}}(v,\dot{v})$ is the joint PDF of the wind speed and its time derivative. Usually lower case symbols are used as arguments of PDF's. Upper case symbols are used as real time dependent variables.

The windspeed interval $[a,b]$ is usually determined from the range of reduced velocities that will allow the vortex shedding frequency from the member to lockin or synchronize with the natural frequency of the member. The reduced velocity is determined as $V_r = \frac{V}{f_n \times D}$, where V is the windspeed. f_n is the member natural frequency and D is the diameter. The critical velocity of the member is for the purpose of this study taken to be that value of V which yields a reduced velocity of 6.0. The range, $[a,b]$, of the critical velocity interval is defined in terms of the lower and upper bound values of reduced velocity for the interval of windspeed which allow lockin. In this study those values are taken as 5.0 and 6.5. The use of a reduced velocity of 6.0 and the proposed critical reduced velocity range (from 5.0 to 6.5) are based on extensive wind tunnel experiments on a pinned-pinned beam [3]. These values may be applicable to most offshore structural members with both ends attached since they have mode shapes similar, if not identical, to that of a pinned-pinned beam.

It is clear from Equation 6 that the mean duration of an undisrupted visit to a critical velocity interval depends not only on the probability distribution of the wind speed $V(t)$, but also on the properties of its time derivative process, $\dot{V}(t)$, due to the dependence of Equation 6 on the mean rates of crossings. Mean upcrossing rates of random processes can be calculated from Equation 8 for known joint PDF of the windspeed and its time derivative. However Equation 8 is not the only way to calculate mean upcrossing rates of random processes. Grigoriu [6] showed that mean upcrossing rates of non-Gaussian random processes can be determined from related Gaussian processes, through a univariate, nonlinear transformation as follows.

Let $\tilde{Y}$ be a Gaussian random process of the same sampling rate and total length as $V(t)$, which consists of random variables $\tilde{Y}_i$ of zero mean and unit variance. Then there exists a real function $h(\tilde{Y}_i)$ such that

$$V_i = g(\tilde{Y}_i) = F_V^{-1}(\Phi(\tilde{Y}_i)) \quad (9)$$

where Φ is the CDF of $\tilde{Y}_i$ and $\Phi(\tilde{Y}_i) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\tilde{Y}_i} \exp(-0.5\tilde{y}^2)d\tilde{y}$. Since both F_V and Φ are monotones, g is guaranteed to possess one to one mapping. The mean upcrossing rate of level c of the random process $V(t)$, can be obtained from crossings of level $\tilde{y} = g^{-1}(c)$ of the stationary Gaussian process $\tilde{Y}(t)$, since the g-function increases monotonically and since $V(t)$ and $\tilde{Y}(t)$ upcross the levels c and $\tilde{y}$ respectively at the same instances. Thus

$$\nu_c^+ = \frac{\sigma_{\dot{\tilde{Y}}}}{\sqrt{2\pi}}\phi(g^{-1}(c)) \quad (10)$$

where $\sigma_{\dot{\tilde{Y}}}$ is the standard deviation of $\dot{\tilde{Y}}(t)$. It can be determined from $\sigma_{\dot{V}}$ and the CDF of $V(t)$ [6]; $\phi(\tilde{y}) = \frac{1}{\sqrt{2\pi}}\exp(-0.5\tilde{y}^2)$ is the PDF of the standard normal variable.

However, if the windspeed can be simplified as a Gaussian process, then both the CDF and the mean upcrossing rates can be expressed in the following standard analytical forms:

$$F_V(c) = \int_0^c \frac{1}{\sqrt{2\pi}\sigma_V}\exp(-\frac{(v-\bar{V})^2}{2\sigma_V^2})dv \quad (11)$$

$$\nu_c^+ = \frac{1}{2\pi}\frac{\sigma_{\dot{V}}}{\sigma_V}\exp(-\frac{(c-\bar{V})^2}{2\sigma_V^2}) \quad (12)$$

where $\bar{V}$ is the mean windspeed; σ_V is the standard deviation of the windspeed; $\sigma_{\dot{V}}$ is the standard deviation of the time derivative of the windspeed (or the wind acceleration).

Combining Equations 6, 11 and 12, $E[\mathcal{T}_{[a,b]}]$ can be expressed explicitly in terms of wind statistics as given below:

$$E[\mathcal{T}_{[a,b]}] = \frac{\int_a^b \frac{1}{\sqrt{2\pi}\sigma_V}\exp(-\frac{(v-\bar{V})^2}{2\sigma_V^2})dv}{\frac{1}{2\pi}\frac{\sigma_{\dot{V}}}{\sigma_V}\left\{\exp(-\frac{(a-\bar{V})^2}{2\sigma_V^2}) + \exp(-\frac{(b-\bar{V})^2}{2\sigma_V^2})\right\}} \quad (13)$$

INTRODUCTION OF A GAUSSIAN WINDSPEED APPROXIMATION

In the analysis of random winds, the Gaussian windspeed approximation is very attractive because the PDF of the Gaussian windspeed $p_V(v)$ only depends on two statistics, namely the mean windspeed $\bar{V}$ and the standard deviation of the instantaneous windspeed σ_V:

$$p_V(v) = \frac{1}{\sqrt{2\pi}\sigma_V}\exp(-\frac{(v-\bar{V})^2}{2\sigma_V^2})$$

Furthermore, the Gaussian windspeed approximation enables the expected duration of visit to be expressed in terms of only a few wind statistics. Before the Gaussian windspeed approximation can be employed with confidence, it must be verified using real maritime wind data. It is known that natural winds are not always Gaussian random processes. Nonetheless a Gaussian model can be shown to be useful if it produces conservative engineering predictions.

Raw wind data with varying mean windspeeds and turbulence levels were evaluated. These wind data were a product of a measurement program sponsored by the Statoil Joint Industry Project on Maritime Turbulent Wind Field Measurements and Models. Project members included: Amoco Norway Oil Company, Conoco Norway Inc., Elf Aquitaine Norge A/S, Exxon Production Research Company, A/S Norske Shell, Norsk Hydro, Statoil and Saga Petroleum A/S. The database consists of several hundred hours of high quality wind data, obtained at exposed sites on the western coast of Norway. The raw wind data analyzed in this paper consisted of five hundred raw windspeed records taken at 5 different elevations. Each raw windspeed record is 40 minutes long with a sampling frequency of 0.85 Hertz. The 40-minute mean windspeed varied between 13 [m·s^{-1}] and 31 [m·s^{-1}], and the turbulence level varied between 7% and 30%. A description of the wind measurement program and some wind data may be found in Odd Jan Andersen and Jorgen Lovseth [7].

The Gaussian windspeed approximation was tested in the following way. For a given raw windspeed record, the expected duration of visit, $E[\mathcal{T}_{[a,b]}]$, was evaluated by two methods. Method one was to evaluate $F_V(x)$ and ν_x^+ numerically from the raw record, and then calculate $E[\mathcal{T}_{[a,b]}]$ by definition in Equation 6. This value of the expected duration of visit was defined as the *Numerical Duration*. Method two was to estimate the sample wind statistics $\bar{V}$, σ_V and $\sigma_{\dot{V}}$ from the raw record, and then calculate the expected duration of visit by Equation 13. Since Equation 13 assumes that the underlying windspeed is a Gaussian process, the expected duration calculated using this method was denoted as the *Gaussian Duration*. The comparison between the *Numerical Duration* and the *Gaussian Duration* should indicate the adequacy of the Gaussian windspeed approximation.

Figures 3 and 4 show the variations of the *Numerical Duration* and the *Gaussian Duration* with different values of V_{crit} and therefore with different $[a, b]$. In Figure 3, twelve 40-minute windspeed records with similar values of wind statistics ($\bar{V}$ = 15 [m·s^{-1}] and $\frac{\sigma_V}{\bar{V}} = 0.17$) were selected, and the *Numerical Duration* corresponding to each value of V_{crit} was averaged across all 12 records. The *Gaussian Duration* was calculated from the average values of sample wind statistics from all 12 records. In Figure 4, eleven 40-minute windspeed records with similar values of wind statistics ($\bar{V}$ = 20 [m·s^{-1}] and $\frac{\sigma_V}{\bar{V}} = 0.084$) were selected, and both the *Numerical Duration* and the *Gaussian Duration* were calculated similarly.

Figures 3 and 4 are typical examples revealing that both the *Gaussian Duration* and the *Numerical Duration* follow similar trends, and that more importantly the *Gaussian Duration* appears to be a consistently conservative estimate compared to the *Numerical Duration*. These observations also hold for wind records with different statistics [3]. Since a conservative estimate in the expected duration leads to a conservative estimate in fatigue damage rate, the Gaussian windspeed model is a useful and conservative predictor of fatigue damage.

ANALYSIS OF WIND STATISTICS FROM MARITIME WIND DATA

To implement the proposed probabilistic prediction methodology requires the input of wind statistics and structural parameters. These wind statistics include the mean windspeed $\bar{V}$, the standard deviation of the instantaneous windspeed σ_V, and the standard deviation of the time derivative of the windspeed (wind acceleration) $\sigma_{\dot{V}}$. In this section, wind statistics σ_V and $\sigma_{\dot{V}}$ are analyzed from high-sampling rate real maritime wind data.

Figure 5 shows the observed values of σ_V as a function of $\bar{V}$ at three different elevations. Each pair, ($\bar{V}$, σ_V), was calculated numerically from a 40-minute maritime windspeed record with a sampling frequency of 0.85 Hertz.

The observed σ_V shows poor correlation with the mean windspeeds. The scatter of the data is caused by differences in the atmospheric stability. When the atmosphere is stable, the velocity fluctuation is generated only by the shear gradient in the velocity profile, resulting in a small value of σ_V. When the atmosphere is unstable, the velocity fluctuation is generated not only by the shear gradient in the velocity profile, but also by unstable atmospheric convection, resulting in a large value of σ_V. A strong atmospheric instability is characterized by a strong negative temperature gradient in the vertical direction (higher temperature close to ground).

To obtain the values of $\sigma_{\dot{V}}$, the windspeed sequence needs to be differentiated with respect to time using an accurate numerical differentiation scheme. Figure 6 shows the observed values of $\sigma_{\dot{V}}$ as a function of $\bar{V}$ at three different elevations. The time sequence of $\dot{V}$ was derived numerically from 40-minute raw windspeed records using the *central difference scheme*.

At a given elevation, Figure 6 shows that $\sigma_{\dot{V}}$ is highly correlated with $\bar{V}$. $\sigma_{\dot{V}}$ increases with $\bar{V}$, suggesting higher rates of turbulence production at higher windspeeds. At a constant value of $\bar{V}$, $\sigma_{\dot{V}}$ decreases as elevation increases. This is because the presence of the earth's boundary layer causes a larger shear gradient in the velocity profile near the surface, which in turn generates more turbulence and contributes to a larger $\sigma_{\dot{V}}$. It is worth pointing out that unlike σ_V, $\sigma_{\dot{V}}$ appears to be independent of atmospheric stability. The data in Figures 5 come from widely varying stability conditions.

In terms of the expected duration of visit, the ratio between σ_V and $\sigma_{\dot{V}}$ is more important than $\sigma_{\dot{V}}$, as shown in Equation 13. Mathematically $\frac{2\pi\sigma_V}{\sigma_{\dot{V}}}$ is the inverse of the mean upcrossing rate at the mean windspeed. Figure 7 shows the value of $\frac{\sigma_V}{\sigma_{\dot{V}}}$ as a function of $\bar{V}$. This figure is compiled from the data in Figures 5 and 6.

The scatter of the values of $\frac{\sigma_V}{\sigma_{\dot{V}}}$ against $\bar{V}$ is caused by the effect of stability on σ_V. Although stability is not indicated in this plot, the minimum value of $\frac{\sigma_V}{\sigma_{\dot{V}}}$ is 2.5 [s] independent of mean windspeed and occurs in stable atmosphere. It can be as large as 15 [s] 46 [m] above ground, when the atmosphere is strongly unstable. 10 [s] is an upper bound in all but 4 of the sample cases, and is suggested as a rough rule of thumb for estimates of $\frac{\sigma_V}{\sigma_{\dot{V}}}$ for maritime winds. Fei [3] shows that typical durations of visit of the wind to critical intervals defined by a reduced velocity range of 5 to 6.5 are from 10 to 20 seconds. Typical rise times of structural members are given by $t_r = \frac{1}{2\pi\zeta f_n}$, where ζ is the structural damping ratio of the structural member. t_r may be as large as 100 periods of vibration for lightly damped welded steel members. For a typical vibration frequency of 3 Hertz, this equates to a rise time of 33 seconds. Therefore the typical rise time of a member on an offshore platform is substantially longer than the expected duration of visit to the critical velocity interval. Steady state response is frequently not achieved and actual damage rates are less than steady state assumptions would predict.

CONCLUSIONS

The extension of these results to other geographical locations will require analysis of local wind statistics. These results do not include the effect of variations of wind direction. This is a topic for additional research.

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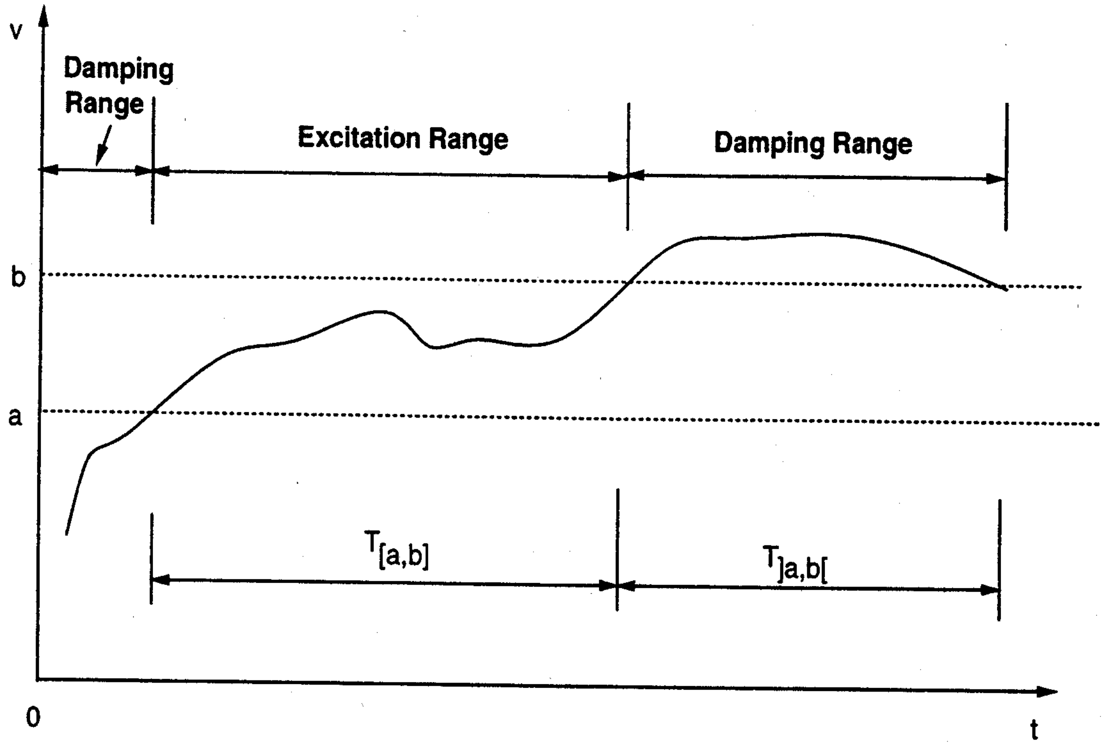

Figure 1: Duration of a visit by the wind speed to an interval $[a, b]$

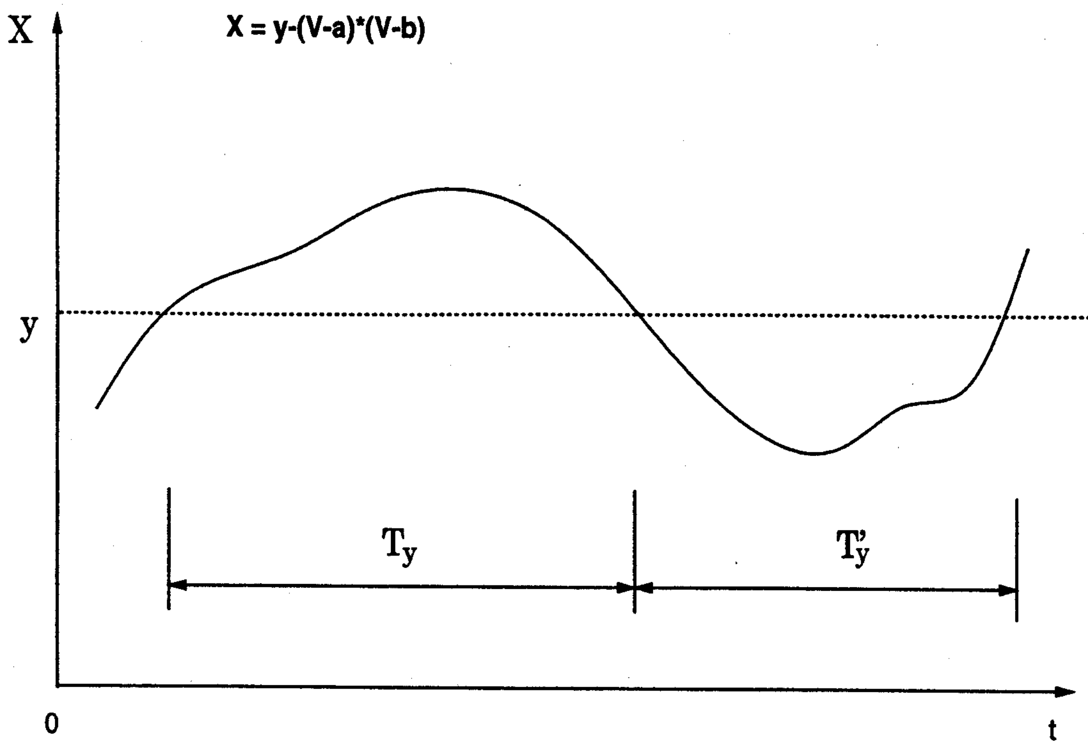

Figure 2: Durations of stay above and below a fixed threshold y

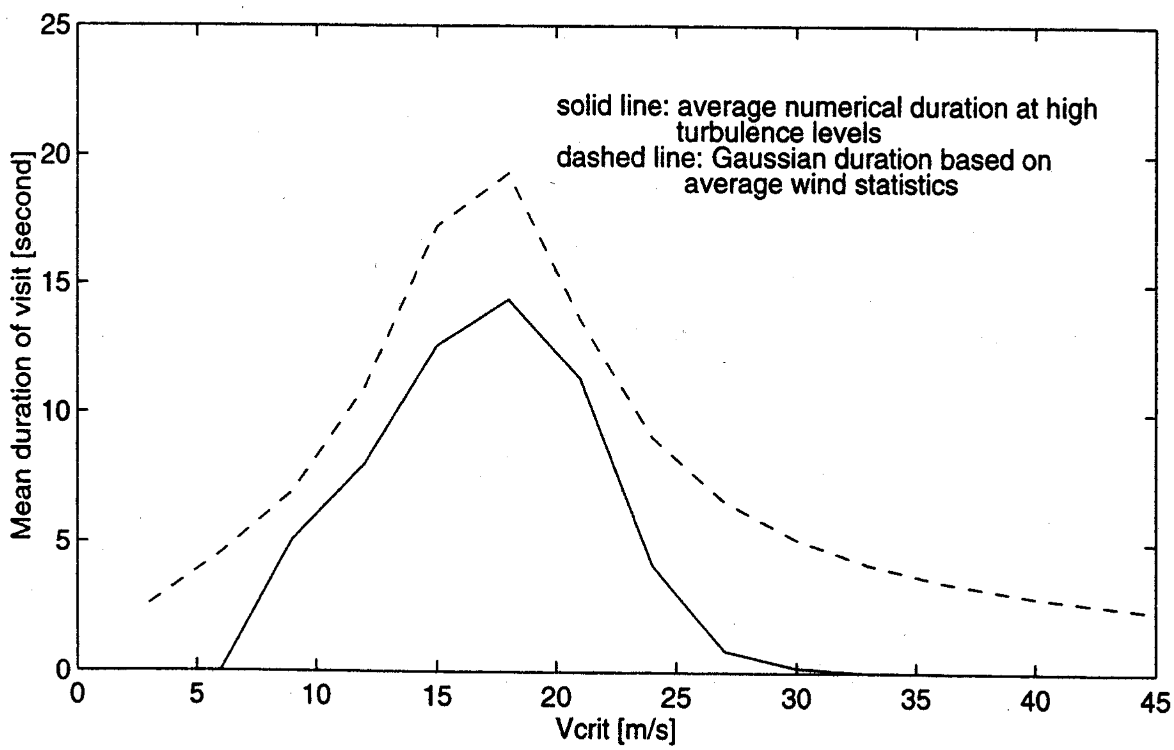

Figure 3: Variation of average numerical and Gaussian durations with critical velocity, from 12 windspeed records with $\bar{V} = 15$ [m·s^{-1}] and large velocity fluctuations ($\frac{\sigma_V}{\bar{V}} = 17\%$)

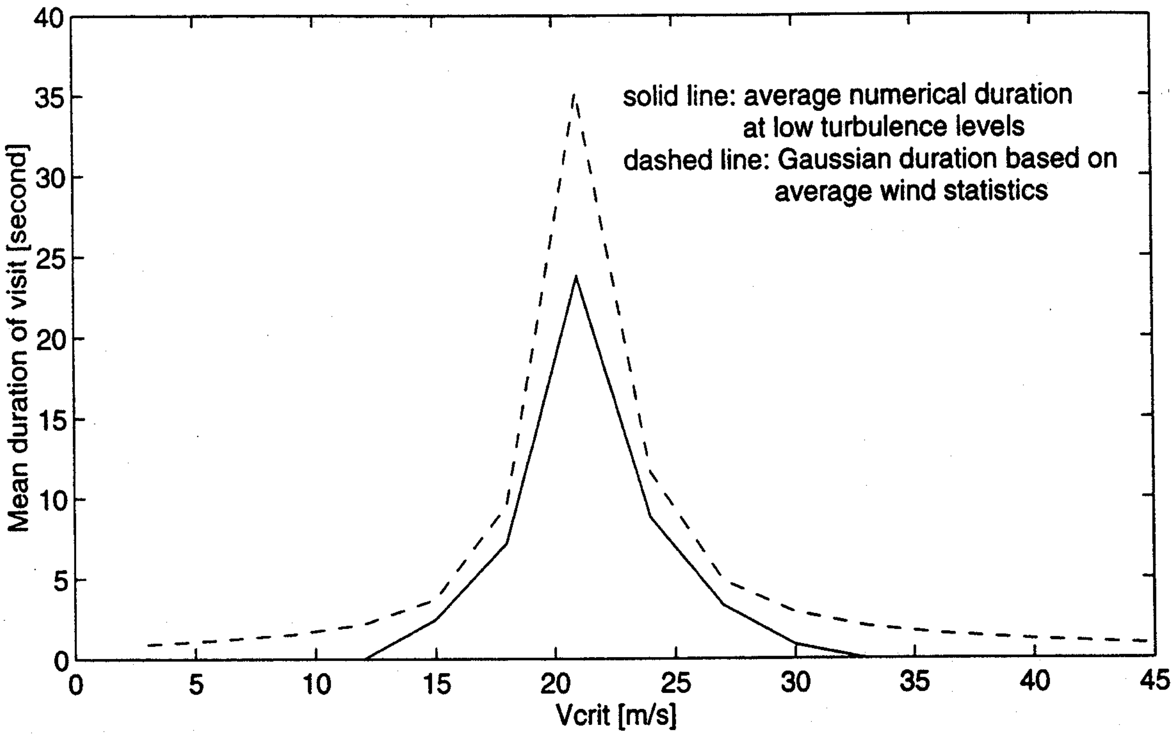

Figure 4: Variation of average numerical and Gaussian durations with critical velocity, from 11 windspeed records with $\bar{V} = 20$ [m·s^{-1}] and small velocity fluctuations ($\frac{\sigma_V}{\bar{V}} = 8\%$)

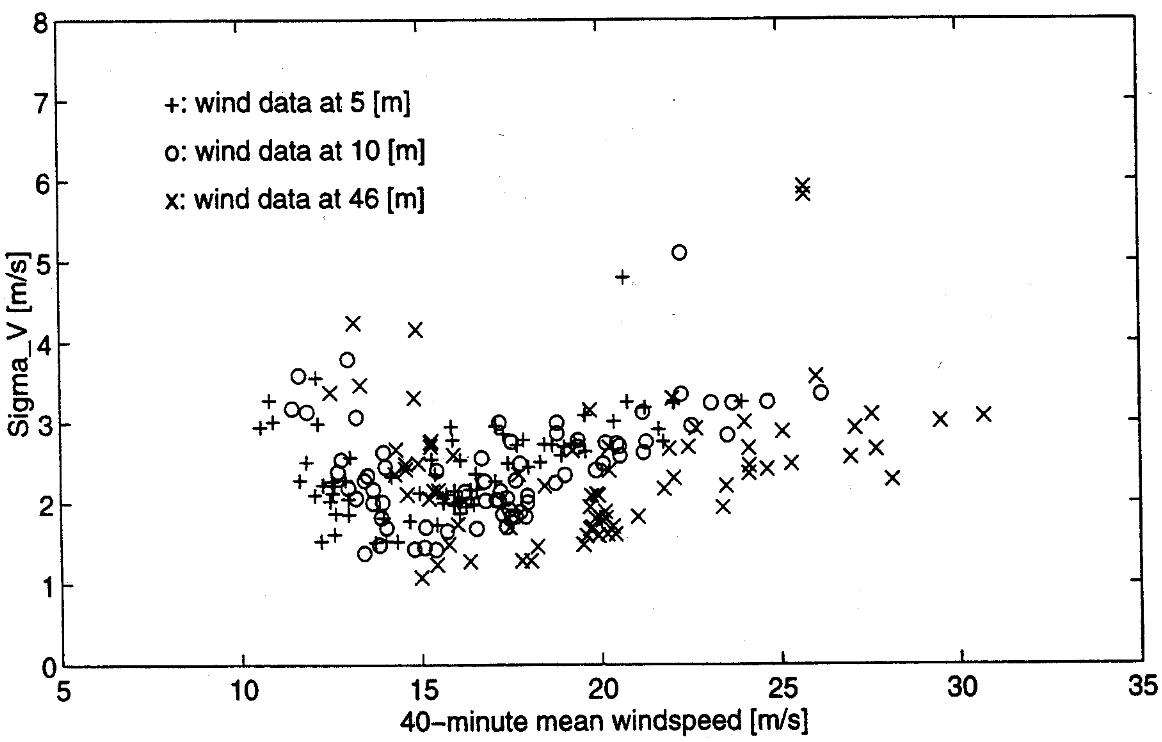

Figure 5: Variation of σ_V with $\bar{V}$ at three different elevations

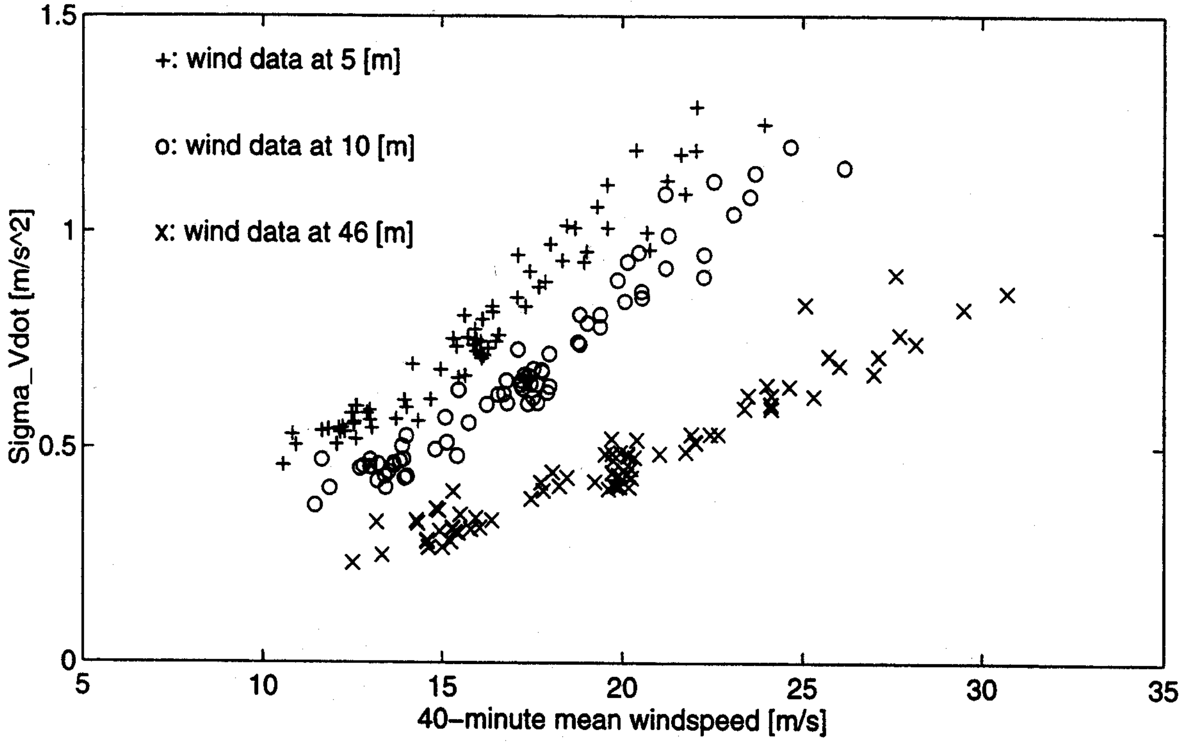

Figure 6: Variation of $\sigma_{\dot{V}}$ with $\bar{V}$ at three different elevations

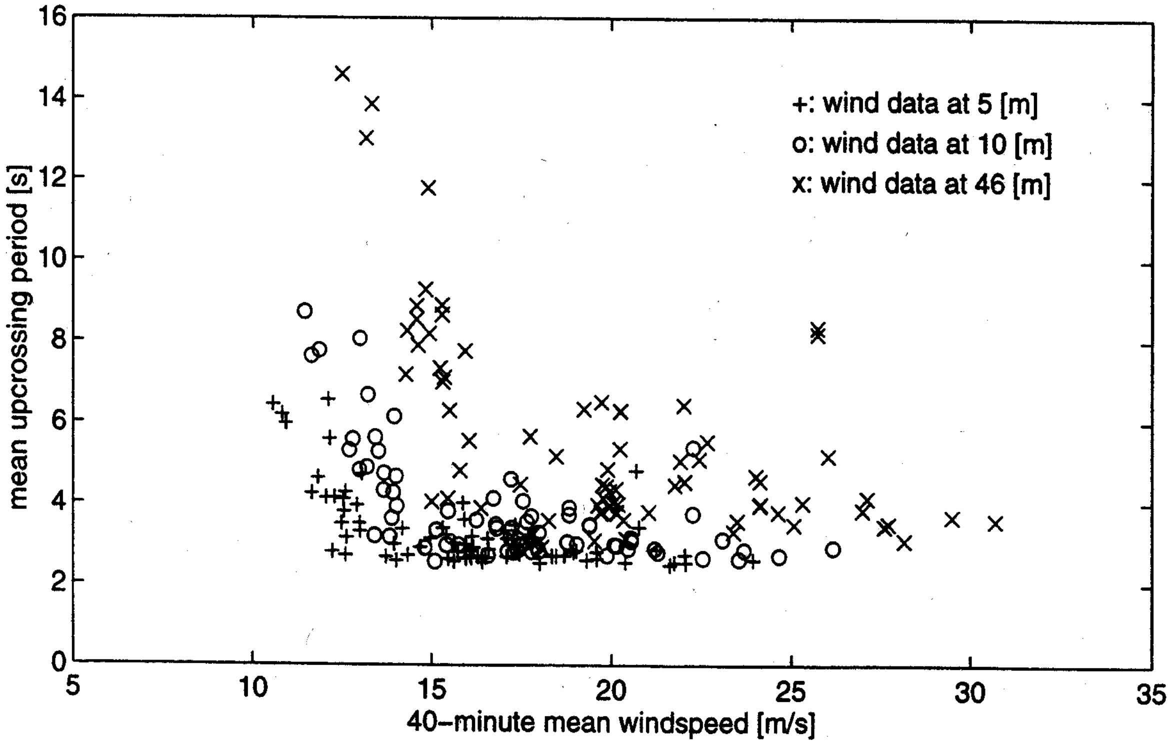

Figure 7: Variation of mean upcrossing periods $(=\frac{\sigma_V}{\sigma_{\dot{V}}})$ with $\bar{V}$ at three different elevations