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FREQUENCY AND TIME DOMAIN ANALYSIS OF VORTEX INDUCED VIBRATIONS FOR FREE SPAN PIPELINES

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ABSTRACT

The present paper will discuss various models for calculation of vortex induced vibrations (VIV) of free span pipelines, and present a new strategy for such analyses. Applications of traditional models are presented and their limitations discussed. The new approach is based on the combination of an empirical linear frequency domain model, and a non-linear time domain structural model. The first step is to carry out the VIV analysis according to linear response theory, and next introduce the calculated hydrodynamic forces to the non-linear structural model. The benefit from using the non-linear model is to describe stresses at the shoulders more accurately, which is important since fatigue damage in many cases will be largest in this area. The conclusion is that the interaction between pipe and seafloor is crucial for accurate stress prediction, and that a non-linear time domain model will give the most accurate result.

INTRODUCTION

Pipelines from offshore petroleum fields must frequently pass over areas with uneven seafloor. In such cases the pipeline may have free spans when crossing depressions. Hence, if dynamic loads occur, the free span may oscillate and time varying stresses may give unacceptable fatigue damage. A major source for dynamic stresses in deep water free span pipelines is vortex induced vibrations (VIV) caused by current.

A substantial research effort related to various aspects of VIV and free span pipelines has been seen during the last years. The hydrodynamics of a cylinder close to a wall was investigated by Bearman and Zdravkovich (1978), while Bryndum et.al. (1989) and Marchesani et.al. (1995) have reported results from laboratory tests on long slender beams. Full scale measurements were carried out by Bruschi et.al. (1989). Such research activities gave the background for a new set of guidelines issued by Det Norske Veritas (1998), also

referred to as DNV-G14. Important implications of these guidelines were published by Mørk et.al. (1998). Some work has also been published on direct calculation of VIV on free span pipelines, cf. Tura and Vitali (1990). Halse and Larsen (1998) used the combination of two-dimensional (2D) solutions of Navier-Stokes in combination with a 3D beam model.

It is interesting to observe that the offshore industry has applied guidelines such as DNV G-14 for fatigue analysis of free span pipelines, while direct calculations by use of empirical models have been carried out for vertical risers. The main differences are probably that the current is uniform for the pipeline while sheared profiles must be considered for risers, and that the pipeline will respond at low modes only, which is in contrast to a riser where higher order modes often are seen. However, the boundary conditions are more complex for a pipeline than a riser. One purpose of the present work is therefore to illustrate the significance of these boundary conditions and describe how they can be taken into account in a direct analysis.

Most empirical models are based on frequency domain dynamic solutions and linear structural models (Larsen 2000). A free span pipeline has, however, important non-linearities that should be taken into consideration. Both tension variation and pipe-seafloor interaction will contribute to nonlinear behaviour, which means that most empirical models will have significant limitations when dealing with the free span case. The need for time domain methods is therefore obvious.

The purpose of the present paper is to discuss nonlinear effects related to VIV of free span pipelines and describe how a best possible linear model can be established. An improved strategy for analysis will also be outlined. This is based on combined use of a traditional linear VIV analysis and a subsequent nonlinear simulation where information from the linear case is utilized. All models will be described and the influences from non-linearities illustrated by case studies.

NOMENCLATURE

Scalars

- A Oscillation amplitude
- D Diameter
- f_{0i} Eigenfrequency in still water, mode i
- f_{osc} Oscillation frequency
- \hat{f} Non-dimensional frequency $\hat{f} = \frac{f_{osc}D}{U}$
- U Current velocity
- U_R Reduced velocity, $U_{Ri}(z) = \frac{U(z)}{f_{0i}D}$

Matrices and vectors

- C_S Structural damping matrix
- C_B Damping matrix for pipe/seafloor interaction
- K Stiffness matrix
- M_S Structural mass matrix
- M_H Hydrodynamic mass matrix
- R Load vector, time domain
- r Displacement vector, time domain
- \dot{r}, \ddot{r} Velocity and acceleration vectors
- X Complex load vector, frequency domain
- x Complex displacement vector, freq. domain

- Describe the interaction between in-line and cross-flow VIV
- Describe the interaction between the pipe and seafloor in terms of non-linear stiffness and damping including friction for in-line oscillations of a pipe with seafloor contact.
- Take into account the influence from tension variation on stiffness and hence also on the actual eigenfrequency.
- Be able to analyse adjacent spans and the dynamic interaction between them.

Most empirical models for VIV analysis have been developed for the analysis of marine risers, Larsen (2000). They are therefore not able to meet all demands related to the free span pipeline case. Some aspects related to this problem will be discussed in the following section.

OUTLINE OF STANDARD ANALYSIS PROCEDURE

The computer programs VIVANA, Larsen et.al. (2000) and RIFLEX, Fylling et.al. (1998) have been used in the present study. The VIV analysis program VIVANA applies RIFLEX modules for system modeling and static analysis, and RIFLEX has been used for all time domain simulations.

A complete VIV analysis of a free span pipeline must include steps as follows:

1. Static analysis to define tension and shape of the span. Geometry of the seafloor and a model for pipe/seafloor interaction are essential features of this analysis.
2. Eigenvalue analysis for the actual static condition.
3. VIV analysis to identify response frequency and amplitudes for a given current condition.

Some details on these steps will be given in the following.

THE IDEAL VIV MODEL FOR FREE SPAN PIPELINES

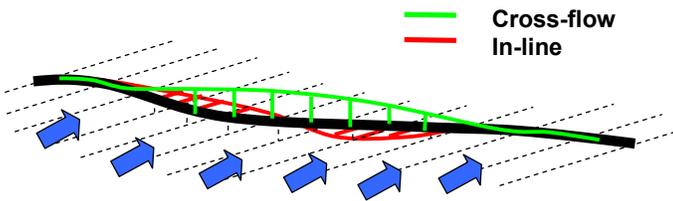


Figure 1 Static shape and VIV of free span pipeline

An ideal model for analysis of VIV for free span pipelines should be able to take into account the following effects:

- Start with a correct static condition found from a non-linear static analysis that gives a correct tension, seafloor contact and 3D shape of the pipe. Such an analysis must know the tension during installation and consider later changes in pressure, density of contents and temperature. This condition will define the in-line and cross-flow eigenfrequencies for the pipe, see Figure 1.
- Describe the necessary hydrodynamic coefficients (added mass, lift, drag and damping) for a pipe that oscillates in current close to a wall.
- Describe the local current profile including boundary layer effects close to the seafloor.
- Handle any current direction relative to the pipeline and also the combination of current and wave induced forces.
- Predict the correct dominating modes for in-line and cross-flow response for a given current condition.

Static analysis

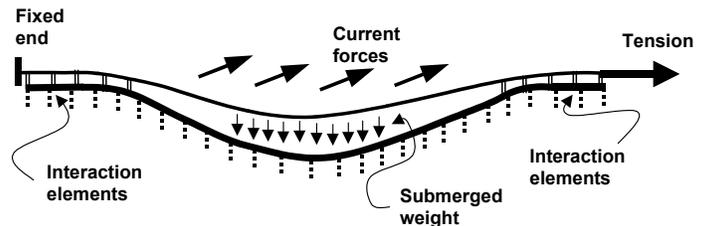


Figure 2. Model for static analysis of free span pipeline.

A 3D beam finite element model of the pipe is applied. The analysis will start from a stress free (horizontal straight lined) configuration without any seafloor contact. One end is position controlled for all degrees of freedom. The other end has all degrees of freedom position controlled except for the local axial direction where an external load identical to the applied laying tension is introduced.

A sequence of loads will be applied in such a way that the final condition will represent the real pipeline as accurately as possible. Non-linear bottom springs and friction elements take care of the pipe/seafloor interaction. A large number of load increments may be needed in order to maintain a stable solution during all intermediate conditions. The final load sequence will introduce current forces perpendicular to the plane defined by the pipe under influence from gravity and buoyancy loads. Hence, a correct 3D shape of the pipeline span is obtained, see Figure 2.

Eigenvalue analysis

An eigenvalue analysis can be carried out on the basis of the static solution. This is straightforward for the still water case since the added mass coefficient is well known. The effect of seafloor proximity may be taken into account, cfr. Jensen et.al. (1993), but this effect has been neglected in the present study. Eigenvalue analyses are also applied in order to identify the response frequency for a given current condition. The response frequency will in general appear as a compromise between the still water eigenfrequency and the vortex shedding frequency for the fixed cylinder, Larsen et.al. (2001). This frequency is found from an iteration where the added mass coefficient is given as a function of the non-dimensional frequency, see Nomenclature. Convergence is obtained when consistency is obtained between the added mass and eigenfrequency.

Excitation and damping models

Most empirical models for risers in sheared current will define an excitation bandwidth in terms of reduced velocity U_R or non-dimensional frequency \hat{f} . This bandwidth will identify an excitation zone along the riser for a given eigenfrequency. Damping will take place outside this zone. There is hence an energy balance between the excitation and damping zones as illustrated on Figure 3.

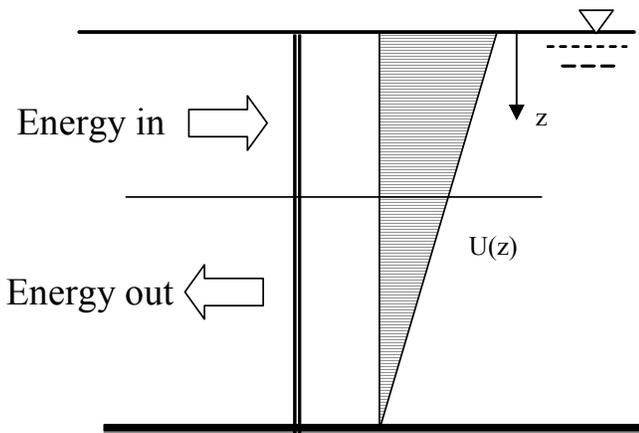


Figure 3. Energy balance for a riser in sheared current

This situation is different for the free span pipeline since current speed normally will be constant along the pipe. The damping mechanism is now linked to the oscillation amplitude and to the interaction between the pipe and the seafloor., see Figure 4.

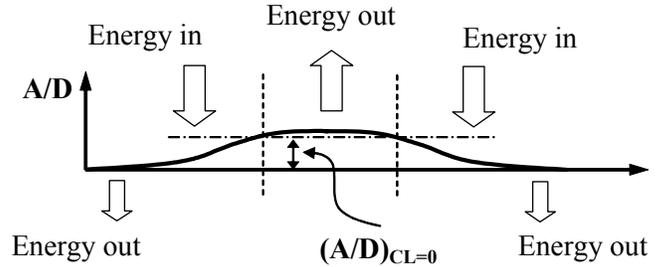


Figure 4. Energy balance for free span pipeline

Lock-in vibrations of a spring supported short cylinder is known to have a self-limiting amplitude even if no mechanical damping exists. This feature can be illustrated by the lift coefficient curve on Figure 5. If $C_L = 0$ there is no net transport of energy between the fluid and the cylinder, meaning that the oscillation is stationary. $(A/D)_0$ on the figure represents hence the lock-in amplitude. Larger amplitudes will give negative lift coefficient, meaning that we will have hydrodynamic damping, while smaller amplitudes will give excitation needed to drive an oscillation with some mechanical damping.

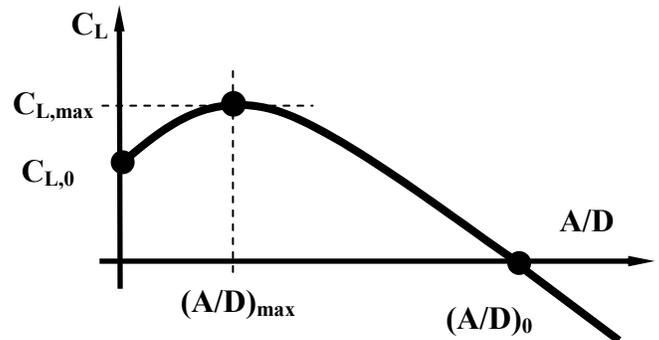


Figure 5. Lift coefficient curve as defined in VIVANA

If the seafloor damping is low, excitation will take place at moderate amplitudes, while we will have some damping at the largest amplitudes and at the shoulders (Figure 4). Increasing soil damping will reduce the response and the total span length may enter the excitation zone.

This way of modeling implies that the lift coefficient curve will define both excitation and damping. VIVANA applies a set of lift coefficient curves where each curve is defined by three points as seen from Figure 5. The points are given as functions of the non-dimensional frequency as seen from Figure 6. These curves are based on data from Gopalkrishnan (1993) and Vikestad (1998).

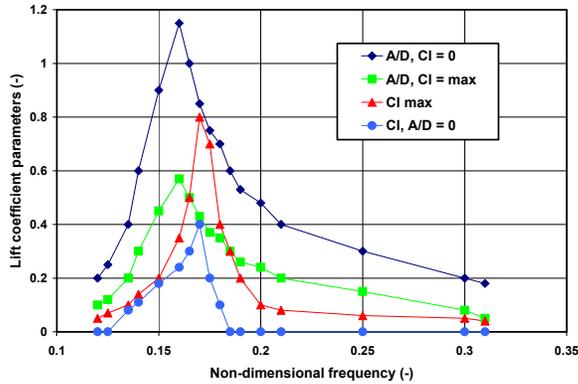


Figure 6. Lift coefficient parameters in VIVANA

Method for VIV response calculation

A free span pipeline is assumed to respond at one discrete frequency identified as an eigenfrequency with added mass valid for the given flow condition. The response may be calculated by using finite elements and the frequency response method. The equation of dynamic equilibrium may be written:

$$\mathbf{M}\ddot{\mathbf{r}} + \mathbf{C}\dot{\mathbf{r}} + \mathbf{K}\mathbf{r} = \mathbf{R} \quad (1)$$

The external loads will in this case be harmonic, but loads at all degrees of freedom are not necessarily in phase. It is convenient to describe this type of load pattern by a complex load vector with harmonic time variation:

$$\mathbf{R} = \mathbf{X} e^{i\omega t} \quad (2)$$

The response vector will also be given by a complex vector and a harmonic time variation. Hence we have:

$$\mathbf{r} = \mathbf{x} e^{i\omega t} \quad (3)$$

By introducing the hydrodynamic mass and damping matrices dynamic equilibrium can now be expressed as:

$$-\omega^2 (\mathbf{M}_S + \mathbf{M}_H) \mathbf{x} + i\omega (\mathbf{C}_S + \mathbf{C}_B) \mathbf{x} + \mathbf{K} \mathbf{x} = \mathbf{X} \quad (4)$$

The damping matrix \mathbf{C}_S represents structural damping and will normally be proportional to the stiffness matrix. \mathbf{C}_B contains damping terms from pipe/seafloor interaction. A simple matrix with elements on the main diagonal for vertical displacement has been applied in the present study.

Elements in the excitation vector \mathbf{X} are always in phase with the local response velocity, but a negative lift coefficient will imply a 180 degrees phase shift and hence turn excitation into damping. Since the magnitude of the lift coefficient depends on the response amplitude (cfr. Figure 5), an iteration is needed to solve the equation. Note that the response frequency is fixed during this iteration.

The iteration will identify a response shape and amplitude that gives consistency between the response level, lift

coefficients and the local flow condition. The mode shape corresponding to the selected response frequency is used as an initial estimate for the response vector only.

OUTLINE OF COMBINED APPROACH

The free span pipeline case involves several types of non-linearities, while the standard VIV model as described in the previous section is linear with respect to structural response. A non-linear time domain analysis procedure is hence wanted. The main reason for this is that maximum stress variation and hence also fatigue damage, will often occur at the shoulders where the influence from non-linear pipe/soil interaction is large.

However, empirical data for hydrodynamic coefficients are almost exclusively given as functions of amplitude and frequency, meaning that the response is assumed to be strongly dominated by one harmonic component. A time domain model must hence identify the frequency and amplitude in order to find the proper coefficients during a simulation.

System identification techniques have been used, cfr. Finn et.al. (1999), but the present work follows a completely different approach. The idea is to combine frequency and time domain analyses as follows, see also Figure 7:

1. Carry out an initial frequency domain analysis according to the procedure found in VIVANA. Response frequency and distribution of added mass, damping and lift forces along the pipe will then be known, but the structural response is found from a linear model.
2. Accept all hydrodynamic results from the initial analysis and use them as input to a time domain simulation. The only difference between the frequency and time domain analyses is the model for structural response including interaction with the seafloor.

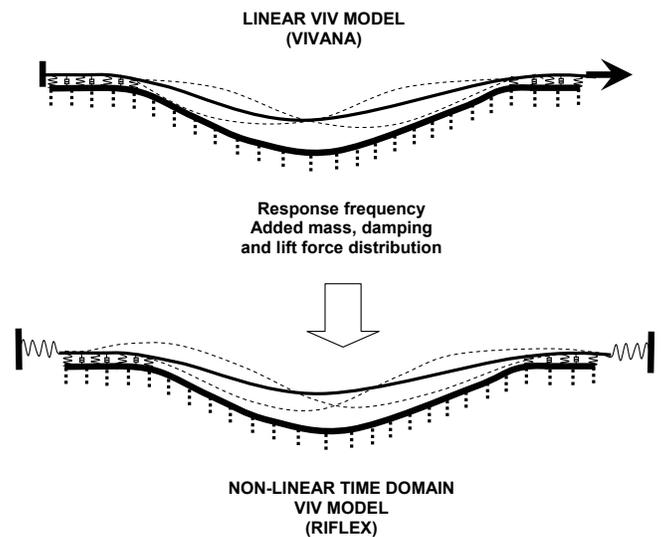


Figure 7. Frequency and time domain models

This approach is valid as long as the overall response amplitudes from the two analyses have minor discrepancies only. This limitation follows from the fact that some coefficients are amplitude dependent. If the difference is found to be significant, some kind of iteration will be needed. Structural damping might then be used to obtain a good agreement so that frequency domain results can be applied in the time domain. Note that the boundary conditions may be different in the two models as indicated in Figure 7.

Work to implement the approach outline above is at present in progress, but not completed. Results from such analyses can therefore not be presented herein, but some results from time domain simulations will be shown in order to illustrate important aspects of free span pipeline modeling.

CASE STUDIES

The purpose of the case study is to compare results from the various models and thereby illustrate shortcomings and modeling possibilities. Key data for the pipe are given in Table 1, while the profile of the pipe relative to the seafloor is seen on Figure 8. The tension of the pipe was 250 kN for all VIVANA analyses.

Table 1. Key data for the pipe

Outer diameter	0.556	m
Inner diameter	0.508	m
Wall thickness	0.024	m
Modulus of elasticity	2.06E+11	N/m ²
Density of pipe material	7850	kg/m ³
Density of content	148.07	kg/m ³
Submerged weight	942	N/m

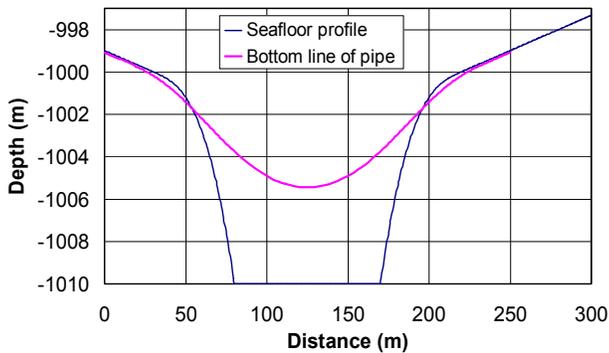


Figure 8. Geometry of free span pipeline

Use of standard model

Figure 9 and 10 present key results from VIV analyses according to the standard VIVANA procedure. Current speed is varied from a very low value up to 1.1 m/s. Initial cross-flow VIV is seen at a speed of 0.21 m/s. This corresponds to a reduced velocity of 3.1, which is in good agreement with DNV Guidelines (1998).

The first mode is seen to dominate up to a current speed of 0.6 m/s, corresponding to a reduced velocity of 8.8. At this point the second mode takes over. Reduced velocity for the corresponding eigenfrequency is 4.5. Similarly, the shift between the second and third mode takes place at a current speed between 1.0 and 1.1 m/s, where the reduced velocity for the second mode is between 8 and 9, and approximately 4.5 for the third. Non-dimensional response amplitudes are seen to reach a maximum value around 1.1, and experience some reduction with increasing speed before the next mode takes over.

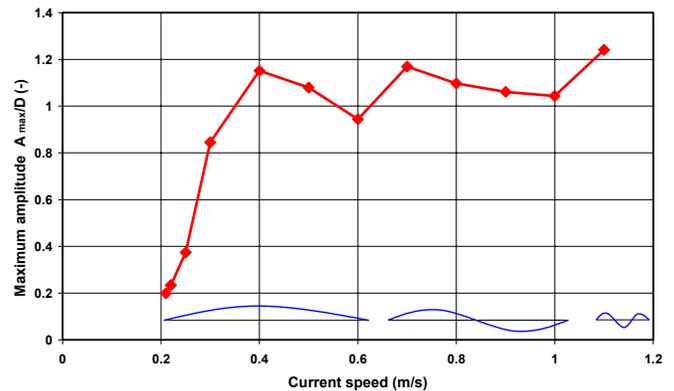


Figure 9. Response amplitudes and dominating modes as function of current speed

Eigenfrequencies and response frequencies as function of current speed are shown on Figure 10. Eigenfrequencies found by use of still water added mass will experience some influence from current speed since current forces will influence static shape and contact forces. This effect is not significant in the present study.

The trend for the response frequency is however, more interesting. At low current speed the response frequency is seen to be lower than the still water value. This is caused by the very high added mass coefficient for this condition. Increased speed leads to reduced added mass and hence also to increased response frequency.

The shift of dominating mode from first to second and from second to third is seen to appear without any dramatic change of response frequency, which is a consequence of the added mass modeling. This trend needs verification from experiments with flexible beams since the added mass curve is based on tests with short stiff cylinders.

It is seen that a new dominating mode takes over at a response frequency slightly lower than the corresponding still water frequency, and that the response frequency is close to linear with respect to current speed. One may interpret this trend as a fixed Strouhal number effect, valid for an oscillating beam. If so, one will arrive at a Strouhal number of 0.19. Note that this value is not an input value to VIVANA but a result of the analysis. VIVANA applies a Strouhal number valid for a fixed cylinder determined from Reynolds number, and the response frequency follows from an added mass/eigenfrequency iteration where the mass ratio will be an important parameter.

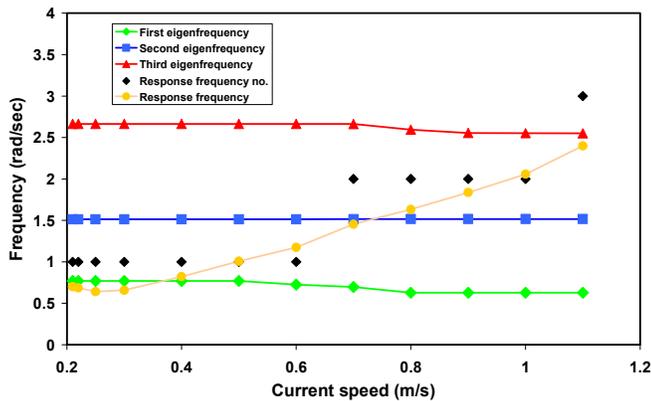


Figure 10. Eigenfrequencies for still water added mass and response frequencies as functions of current speed. Response frequency order indicated

Influence from linear soil damping

The same pipeline model was used to illustrate the influence from alternative damping assumptions. 3 cases were analysed:

1. No damping except for negative lift forces for large amplitudes
2. 10% damping modeled as a damping matrix proportional to the stiffness matrix. This model is often referred to as global damping or Rayleigh damping
3. Linear damping elements at all nodes with contact between the pipe and the seafloor. The magnitude of these dampers was tuned to obtain the same maximum response amplitude as for the global model. This model will be referred to as the local damping model.

Figure 11 shows the lift coefficient along the pipe for all cases. Case 1 is seen to have negative lift in the mid section caused by the large amplitudes found here. Energy is hence transported from the end sections to the mid section as illustrated on Figure 4. The lift coefficient distributions for the other two cases are seen to be nearly identical, which is a consequence of the intended similarity of response amplitude.

Response amplitudes are compared on Figure 12. 10% damping is seen to give approximately 40% reduction of the response. Amplitudes for the two damped cases are seen to be nearly identical, but minor differences are still seen. Amplitudes at the shoulders for the local model is found to be higher than for the global model. This follows from the fact that the only way energy can dissipate is by activating the dampers at the shoulders. Hence, amplitudes must be large enough to provide this dissipation.

Figure 13 shows the bending stresses along the pipe for all models. The global damping case is seen to have a uniform reduction of the moment as compared to the undamped case of the same magnitude as the oscillation amplitude. The local damping case has, however, a much smaller reduction at the shoulders than at the mid span. The consequence is that maximum dynamic stress has moved from the mid span to the shoulders, and is significantly higher than for the global model. The conclusion from this very simple study is that a global damping model may lead to under-prediction of bending stresses even if the response amplitude is correctly described. By introducing a local damping model, the response shape is altered and a simple modal approach will fail.

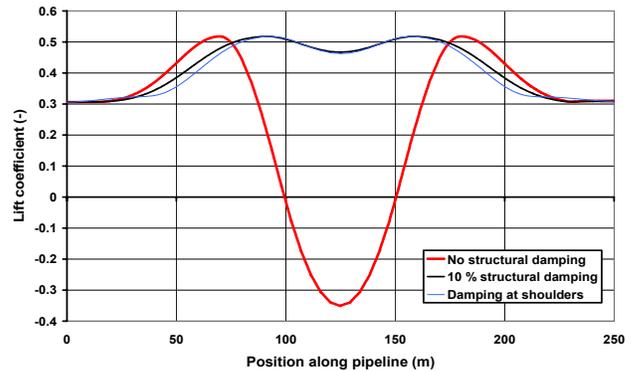


Figure 11. Distribution of lift coefficient along the pipe

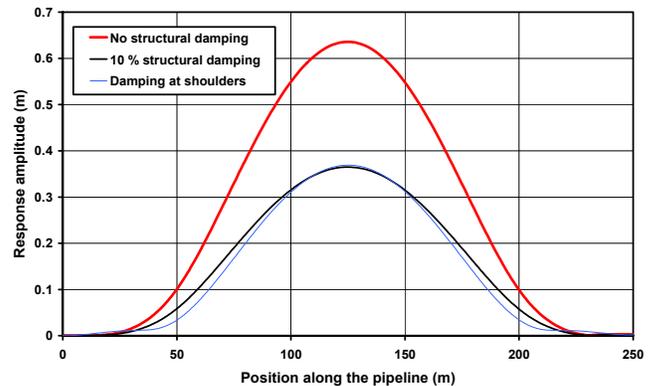


Figure 12. Response amplitudes

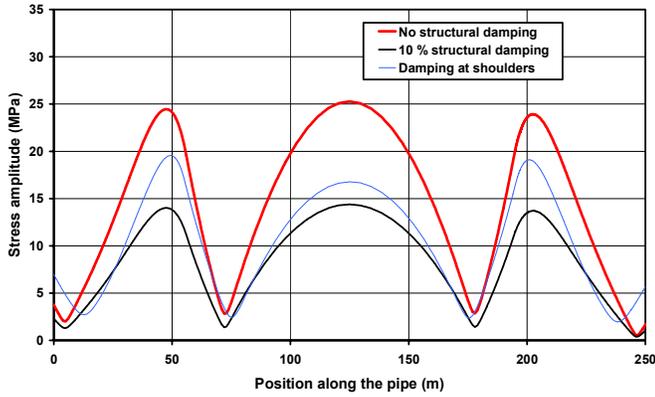


Figure 13. Bending stresses

Influence from non-linear boundary conditions

A simple case study using time domain models has been carried out. The purpose is to illustrate some aspects of modeling, and firm conclusions on how to establish a good time domain model for VIV analysis of free span pipelines can not be drawn.

The pipe and bottom profile for this study is different from the previous. The bending stiffness is reduced and an attempt has been made to have a smooth touch-down zone that will allow the touch-down point to have some movements during dynamic response. This was done in order to simulate the effect from local settlements that will take place underneath the pipe after installation. The distribution of contact forces will strongly depend on the local geometry of the seafloor in this area. This effect is easily observed from analyses, but no attempt has been made to study this in detail.

Pipe/seafloor interaction is modeled by discrete springs in the linear model. These springs are found in vertical, axial and lateral directions. The pipe is fixed at one end and has a constant tension at the other end. The non-linear model applies the same springs to represent the initial stiffness, but a dry friction model is introduced for axial and lateral directions. This will allow the pipe to slide under constant restoring force. The vertical springs are non-linear allowing the pipe to leave the bottom if tension occurs in the spring. A node without bottom contact at static equilibrium will initially not have springs, but these will be introduced if contact is obtained during the dynamic simulation.

The total model is more than 400 meters long, but the free span section at the mid part is slightly less than 100 meters. The bottom profile is symmetric.

Figures 14 to 17 show results from the linear and non-linear analyses. The loads are identical for the two cases, but no attempts were made to tune the models to give similar results. Figure 14 gives envelope curves for vertical displacements for the mid section of the pipe. The linear results are seen to be symmetric as expected, while the non-linear vertical springs have resulted in an asymmetric response for the non-linear

model. Figure 15 illustrates the uplifting and restrained downwards displacements as compared to linear response by zooming in on the touch-down zone. Note that these curves are envelopes and do not represent actual displacement shapes (snapshots). The non-linear response is also seen to be significantly higher than the linear. The reason for this will be commented later.

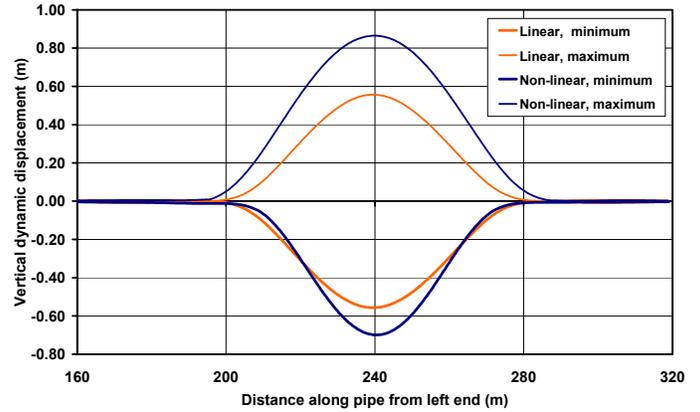


Figure 14. Vertical displacement envelope curves from linear and non-linear analyses

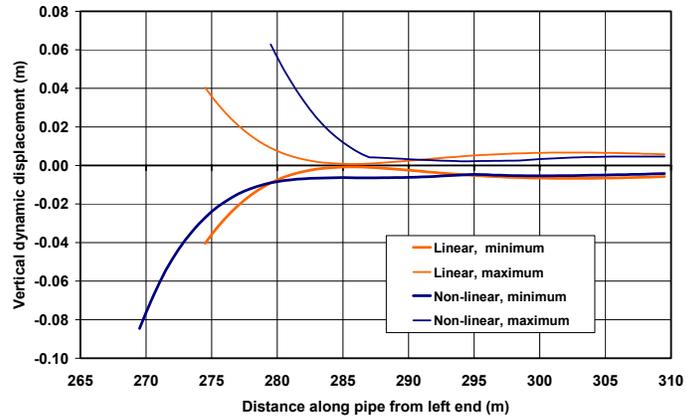


Figure 15. Displacements at the touch-down point

Bending moments and stress ranges are presented on Figure 16 and 17 respectively. Moments are shown in terms of envelope curves, meaning maximum recorded values during one load cycle and not simultaneous moment values.

The stress range curves shows that the non-linear stresses at the mid span is highest for the non-linear case, which is an obvious consequence of the response amplitudes at this part of the pipe. At the shoulders, however, the two models have given almost the same stress range. The linear case has almost the

same result for the mid span as for the shoulders, while the non-linear response is relatively lower at the shoulders. The reason for this is that the non-linear contact formulation will represent a more soft boundary condition than linear springs. The linear model will not allow the pipe to leave the bottom, but allow penetration at nodes without static seafloor contact.

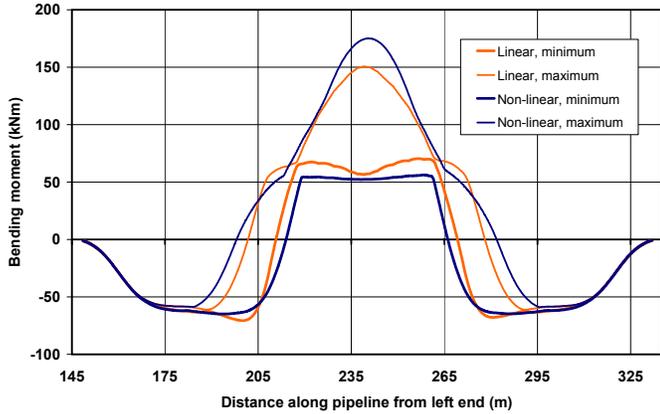


Figure 16. Bending moment from linear and non-linear analyses

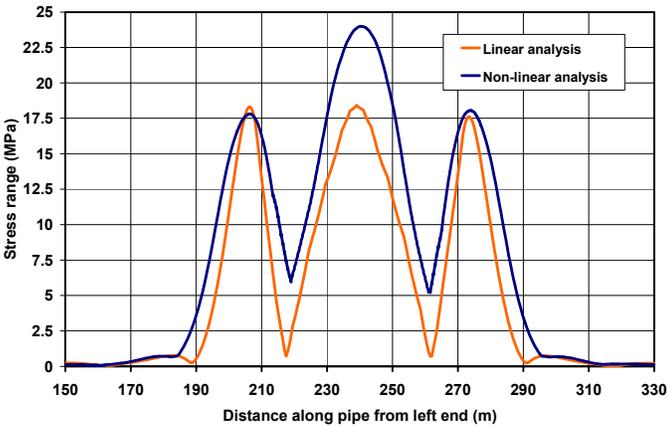


Figure 17. Stress range from linear and non-linear analyses

The large difference between linear and non-linear displacements at the mid span needs an explanation. The reason is not the difference in vertical contact formulation, but how interaction is modeled in axial direction. The springs in the linear model will act as a restriction for the sagged pipeline and give small axial displacements. Axial forces will, however, build up. This effect is prevented in the non-linear model where the friction formulation will limit axial contact forces and allow sliding and hence give larger response amplitudes.

A linear model may be tuned to give the same effect by adjusting the stiffness of the axial contact springs. This is illustrated on Figures 18 and 19, where results from models with different stiffness are presented. The “stiff spring” model is identical to the original, and the “soft spring” model applies springs with 10% stiffness as compared to the originals. The response is seen to increase considerably by reducing the stiffness of axial springs. It is easy to find linear springs that will give the same displacements as the more sophisticated friction model, but one should keep in mind that the effect is non-linear and hence amplitude dependent. It is also important to note that the local stresses at the shoulders will be over-predicted by a linear model even if the response amplitude at the mid span is correctly predicted.

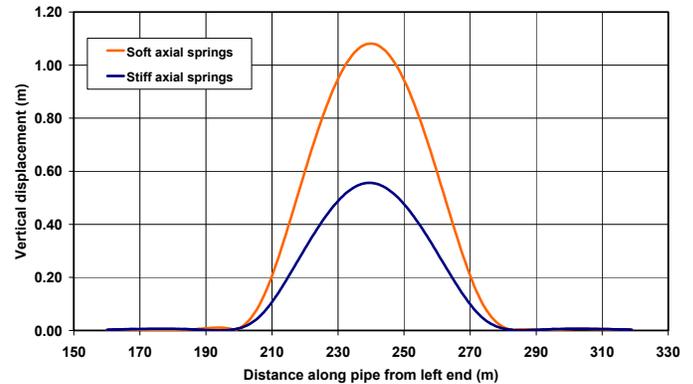


Figure 18. Vertical displacements from soft and stiff axial springs

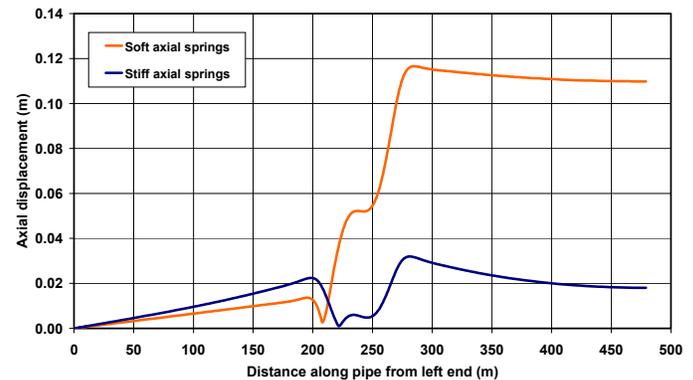


Figure 19. Axial displacements from soft and stiff axial springs

CONCLUSIONS AND RECOMMENDATIONS

An empirical model for prediction of VIV for marine risers have been applied on the free span pipeline case. Calculation of the static condition is important since this will be the basis for calculation of eigenfrequencies for the pipe.

The model predicts on-set of cross-flow VIV and the transition from one dominating mode to another reasonably good. Verification from comparisons with experiments are, however, needed.

Damping from pipe/seafloor interaction at the span shoulders should not be introduced as a global (modal) damping parameter since the effect of this damping mechanism is not a general response reduction.

The stresses at the shoulders will depend strongly on the local geometry of the bottom profile underneath the pipe. The only way of modeling this interaction in a dynamic analysis is to apply a non-linear formulation.

Axial interaction plays an equally important role. Tuning of linear models may lead to correct prediction of response amplitudes, but over-prediction of bending stresses at the shoulders.

The presented model does not represent an ideal solution. The model for added mass and hence also response frequency prediction needs verification. Further work should also be done on interaction between in-line and cross-flow response. Hydrodynamic coefficients for oscillating pipes close to the seafloor are also needed.

Further study of all aspects of pipe/seafloor interaction in order to identify parameters for interaction elements in a finite element model is recommended.

One way of combining frequency and time domain models have been outlined. This model needs to be implemented in the existing VIVANA/RIFLEX program system before one can conclude on its capability for reliable VIV prediction.

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