A New Interpretation of the Response Parameter S_g

October 30, 2000

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Introduction: One of the key problems in VIV response prediction in sheared flow has been the prediction of single-mode dominance in sheared flows in spite of many competing modes. This challenge has stimulated a new look into the meaning of the dimensionless parameter, which is known variously as the "response parameter", the "reduced damping" or the Scruton number". It is known to be a rough predictor of response amplitude A/D. The various forms have small differences and there has been some debate over the years as to which is correct. Most of the controversy centers on the definitions of mass ratio and damping ratio(ζ). Our recent studies have shed some new light on this.

Analysis: Consider a simple 2D rigid cylinder of unit length, mounted on springs in a uniform flow. It is free to respond in the transverse direction. Assume that the cylinder is vibrating under lock-in conditions and that the lift force in phase with velocity may be characterized as a cosine function with a single frequency at the vortex shedding frequency. This ignores the higher harmonic terms in the forcing function and therefore only addresses the dominant response at the shedding frequency. The cylinder is a single degree of freedom oscillator with the following equation of motion, where we assume that the fluid force in phase with acceleration may be lumped into an added mass term, m_a .

$$\left(m_s + m_a\right)\ddot{x} + R_s\dot{x} + kx = \frac{1}{2}C_{L,v}\rho_w U^2 D\cos\left(\omega_s t\right) \qquad (1)$$

Under lock-in conditions, the vortex shedding frequency equals the natural frequency.

$$\omega_s = \omega_n = \sqrt{k/\left(m_s + m_a\right)} \qquad (2)$$

At resonance $x = A\sin\left(\omega_n t\right)$ is a solution of the equation of motion. Substituting in for x(t) shows that the inertial and stiffness terms cancel, leaving the far simpler relationship that follows:

$$R_s\dot{x} = R_s A\omega_n\cos(\omega_n t) = \frac{1}{2}C_{L,v}\rho_w U^2 D\cos\left(\omega_n t\right) \qquad (3)$$

Solving for A/D yields:

$$\frac{A}{D} = \frac{C_{L,v}}{2}\frac{\rho_w U^2}{R_s\omega_n} \qquad (4)$$

The last expression is a simple statement of dynamic equilibrium between lift force and damping forces. It tells us that at lock-in the response amplitude is insensitive to mass ratio. This expression contains a dimensionless group, which characterizes the lock-in response of a 2D cylinder. This parameter is identical to one usually called S_g, providing that the damping ratio and mass ratio are defined in one particular way. The commonly accepted definitions of S_g, mass ratio, damping ratio and Strouhal number are as follows:

$$S_G = 2\pi S_t^2 \left(\frac{m}{\rho D^2} \right) 4\pi\zeta, \qquad mass\,ratio = \frac{m}{\rho_w} D^2, \qquad \zeta = \frac{R_s}{2\omega_n m}, \; and \quad S_t = \frac{\omega_n}{2\pi} \frac{D}{U} \quad (5)$$

The usual definition of the mass per unit length, *m*, in the mass ratio expression is that it is structural mass only. However, to be consistent with the original equation of motion, the mass/length, as it is used in the definition of S_g above and in the expression for damping ratio, must include the added mass term. That is to say: $m = m_s + m_a$. Upon substituting for S_t *and* ζ in the above expression for S_g, t he mass/length terms cancel out, leading to an expression for S_g, which is not a function of the cylinder mass, as shown below:

$$S_g = \frac{R_s \omega_n}{\rho_w U^2} \quad (6)$$

This is the dimensionless group, which appeared in Equation (4). Restating equation (4) and expressing it in terms of S_g leads to:

$$\frac{A}{D} = \frac{C_{L,v}}{2} \frac{\rho_w U^2}{R_s \omega_n} = \frac{C_{L,v}}{2S_g} \qquad (7).$$

Solving for $C_{l,v}$ leads to the following equation of a hyperbola:

$$C_{l,v} = 2S_G \frac{A}{D} \qquad (8)$$

On a linear plot of Sg versus A/D lines of constant lift coefficient are hyperbolas, which form straight lines on a log-log scale. The last figure is such a log-log plot with the lines of constant $C_{l,v}$ shown. Also shown is a broad swath, which is the region where much experimental data exists, as compiled by Owen Griffin[1984]. Thus, a consistent use of the mass/length in the response calculation has lead to an explicit relationship between the parameter S_g and the lift coefficient in phase with velocity. Although developed for the spring mounted cylinder, this analysis may be adapted to the multi-mode, sheared flow case as shown in reference [Vandiver, 1985].

REFERENCES

O. Griffin, “Vibrations and Flow-Induced Forces Caused by Vortex Shedding”, Symposium on flow-Induced Vibration, Volume 1, ASME Winter Annual Meeting, Dec. 1984.

Vandiver, J. K., "Prediction of Lockin Vibration on Flexible Cylinders in Sheared Flow," Proceedings of the 1985 Offshore Technology Conference, Paper No. 5006, Houston, May 1985

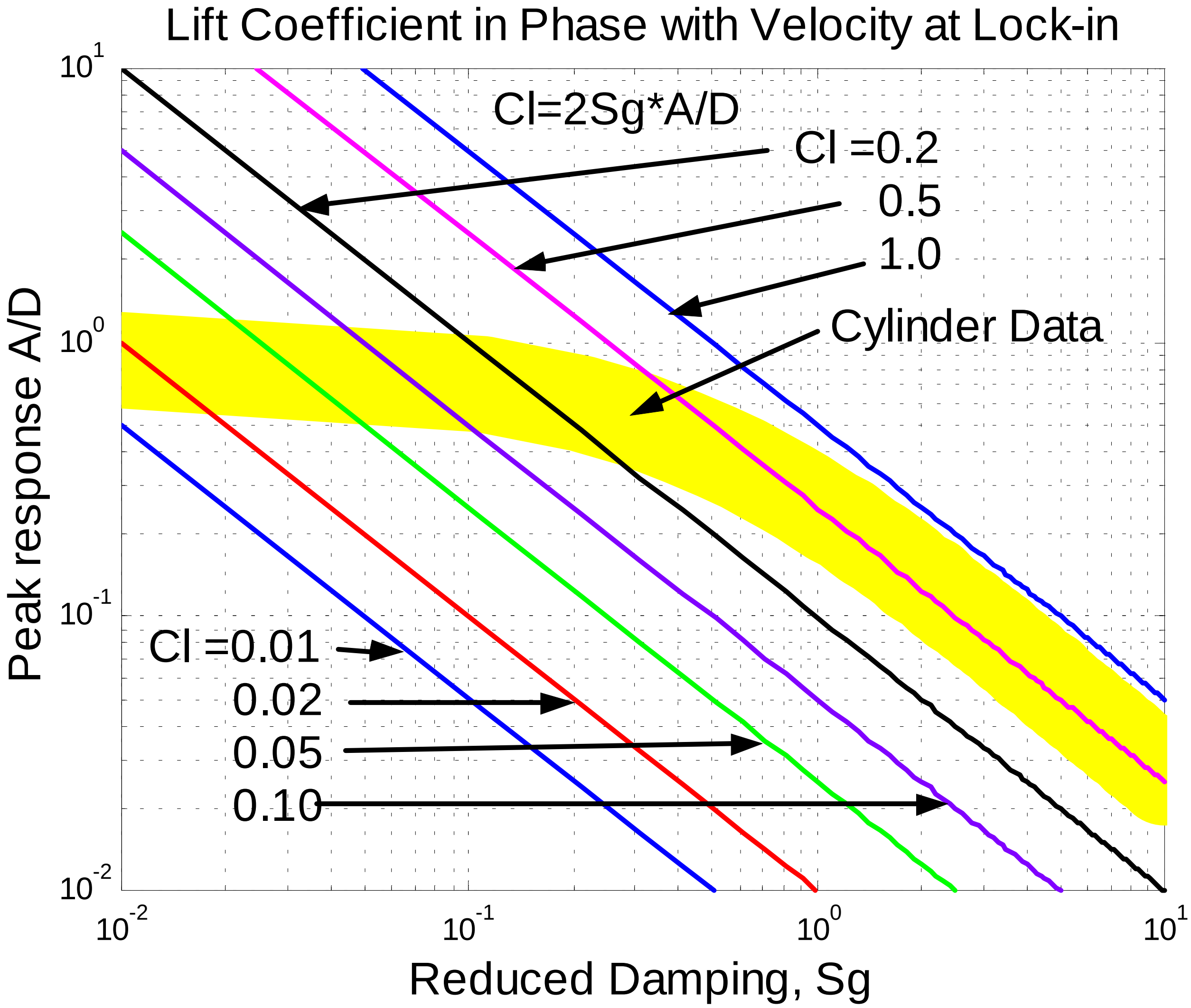

Figure 1. Lines of constant lift coefficient in phase with velocity for spring mounted cylinders, as a function of peak response and the response parameter, S_g