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## The Prediction of Lockin Vibration on Flexible Cylinders in a Sheared Flow

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### Abstract

A method is proposed for the prediction of the flow induced vibration response of flexible cylinders such as cables, pipes, and risers, in a sheared flow. The significance of material and hydrodynamic sources of damping is discussed. The reduced damping or response parameter plays a key role in response prediction. However, the dependence of the response parameter and therefore the response amplitude on the ratio of cylinder mass per unit length to the displaced fluid mass per unit length is shown to be widely misunderstood. Under lockin conditions, damping is important in determining response amplitude, but cylinder mass per unit length is not.

### Introduction

Flexible cylinders, such as cables, drill pipe, and marine risers, often exhibit an harmonic flow induced vibration response known as lockin. Under uniform flow conditions, lockin has been extensively studied and empirical response prediction techniques are often adequate. However, real ocean applications often require response prediction under non-uniform (sheared) flow conditions. Very long cylinders with closely spaced natural frequencies rarely exhibit lockin behavior and frequently behave as infinite strings (1). For shorter cylinders, with well separated natural frequencies, lockin with one mode is possible, even in the presence of shear. However, in such cases, response amplitude is very difficult to predict and it is often difficult to determine which mode, if any, will dominate the response. In this paper, a method for predicting lockin in a sheared flow is proposed. The method makes extensive use of the concept of the response parameter or reduced damping, as it is sometimes called.

A very common misconception regarding the response parameter is pointed out. The response parameter is shown to be primarily a function of damping and is specifically not a function of the cylinder mass per unit length.

References and figures at end of paper.

### Normal Mode Model of Lockin Vibrations

A pipe or cable under tension has, from an analytical view, an infinity of natural modes. When the cylinder is deployed with its longitudinal axis normal to an incident uniform flow, vibration is caused by the shedding of vortices in the wake of the cylinder. The vortex shedding process generates both fluctuating lift and drag forces on the cylinder. Under the correct circumstances, described extensively in the literature, (2,3) a phenomena known as lockin may occur. Lockin is characterized by the synchronization of the wake with either the cross-flow (lift direction) oscillations or with the in-line (drag direction) vibrations. This paper focuses on cross-flow lockin only, in which one cross flow mode dominates the response. At lockin in a uniform flow the lift forces are coherent over the entire length of the cylinder. A normal mode solution to the partial differential equation of motion may be obtained, and is briefly reviewed below.

Consider a beam or string under tension with fixed ends as defined in Figure 1. Let the vortex-induced cross-flow displacement be given by

$$y(x,t) = \sum_i q_i(t) \psi_i(x) \quad (1)$$

where the  $\psi_i(x)$  are the mode shapes and the  $q_i(t)$  are the modal amplitudes. Using the method of normal mode superposition, and assuming insignificant damping related intermodal coupling, a set of independent equations of motion are obtained, one for each mode. These equations are of the form:

$$M_i \ddot{q}_i + R_i \dot{q}_i + K_i q_i = N_i(t) \quad (2)$$

This equation is simply that of a linear, single degree of freedom mass-spring-dashpot system excited by a force  $N_i(t)$ , known as the modal exciting force for mode  $i$ . There exists one such

equivalent oscillator for each mode of interest.  $M_i$ ,  $R_i$ , and  $K_i$  are known respectively as the modal mass, damping and stiffness. The ratio of  $K_i$  to  $M_i$  yields the undamped natural frequency for the mode.

$$\omega_i = \sqrt{K_i/M_i} \quad (3)$$

$M_i$  and  $R_i$  are given by the following equations:

$$M_i = \int_0^L m(x) \psi_i^2(x) dx \quad (4)$$

$$R_i = \int_0^L r(x) \psi_i^2(x) dx \quad (5)$$

where  $m(x)$  and  $r(x)$  are the mass per unit length and equivalent linear damping coefficient per unit length.  $m(x)$  includes the added mass of fluid and  $r(x)$  has units of force per unit velocity per unit length.

The damping ratio for mode  $i$  is given by

$$\zeta_i = \frac{R_i}{2\omega_i M_i} \quad (6)$$

If one specifies an harmonic input and assumes an harmonic output of the following forms

$$N_i(t) = |N_i| e^{i\omega t} \quad (7)$$

$$q_i(t) = |q_i| e^{i(\omega t - \phi)} \quad (8)$$

then a solution for the magnitude of the response per unit input force and the phase between the force and the response may be directly obtained.

$$\frac{|q_i|}{|N_i|} = |H_i(\omega)| = \frac{1/K_i}{\left[ \left(1 - \frac{\omega^2}{\omega_i^2}\right)^2 + (2\zeta_i \frac{\omega}{\omega_i})^2 \right]^{1/2}} \quad (9)$$

$$\phi = \tan^{-1} \left[ \frac{2\zeta_i \omega / \omega_i}{1 - \frac{\omega^2}{\omega_i^2}} \right] \quad (10)$$

$H_i(\omega)$  is known as the frequency function or the response amplitude operator (RAO).

At resonance, the frequency of the external excitation is equal to one of the natural frequencies of the system, indicated here as  $\omega_i$ . If the corresponding modal damping ratio is small then the response of this mode will dominate the response of all other non-resonant modes. This is the case under cross-flow lockin conditions in a uniform flow. Therefore, it is appropriate to model the cross-flow, resonant lockin response in terms of the normal mode equivalent single degree of freedom system reviewed above.

At resonance, the magnitude and phase of the response reduce to

$$\phi_i = \pi/2 \quad (11)$$

$$|H_i(\omega_i)| = \frac{1}{2\zeta_i K_i} \quad (12)$$

Therefore, the response magnitude is

$$|q_i| = \frac{|N_i|}{K_i} \cdot \frac{1}{2\zeta_i} \quad (13)$$

The term  $N_i/K_i$  is the static deflection of the oscillator in response to a constant force  $N_i$ , and the term  $1/2\zeta_i$  is the dynamic amplification factor due to the resonance. Invoking the definition of the damping ratio,  $\zeta_i$ , from Equation 6, this response expression can be rewritten as:

$$|q_i| = \frac{|N_i|}{\omega_i R_i} \quad (14)$$

This expression will be of considerable use in the next section, on the interpretation of the response parameter. Henceforth, all discussion will pertain to the response of a single mode.

Understanding the Response Parameter,  $S_G = \zeta_s/\mu$

Due to a natural evolution in the understanding of the factors which determine lockin response behavior, over the years this critical parameter has been expressed in many forms, reviewed below.

Response parameter:

$$S_G = \zeta_s/\mu = 2\pi S_t^2 k_s \quad (15)$$

Structural damping ratio:

$$\zeta_s = \frac{\delta_s}{2\pi} = \frac{R_i}{2\omega_i M_i} \quad (16)$$

Mass ratio:

$$\mu = \frac{\rho D^2}{8\pi^2 S_t^2 m} \quad (17)$$

Reduced damping:

$$k_s = \frac{2m\delta_s}{\rho D^2} = \frac{4\pi m\zeta_s}{\rho D^2} \quad (18)$$

Response parameter

$$S_G = \frac{8\pi^2 S_t^2 m\zeta_s}{\rho D^2} \quad (19)$$

$\zeta_s$  is the damping ratio due to structural dissipation of energy only, and does not include hydrodynamic sources of damping.  $\delta_s$  is the associated logarithmic decrement.  $\mu$  is proportional to the ratio of the displaced fluid mass per unit length  $\pi\rho D^2/4$  to the mass (including added mass) per unit length of the cylinder,  $m$ .

For cylinders that do not have a constant mass per unit length, the  $m$  in these equations is replaced with an equivalent uniform mass per unit length  $m_e$ .  $m_e$  is the equivalent constant mass per unit length which would yield the same modal mass from Equation 4 as the actual variable mass per unit length  $m(x)$ . Therefore

$$m_e = \frac{\int_0^L m(x) \psi_i^2(x) dx}{\int_0^L \psi_i^2(x) dx} \quad (20)$$

For the remainder of this paper, a constant mass per unit length  $m$  shall be assumed, to simplify the analysis.

$D$  is the cylinder diameter, assumed constant, and  $S_t$  is the Strouhal number given by

$$S_t = \frac{f_s D}{U} \quad (21)$$

where  $f_s$  is the vortex shedding frequency and  $U$  is the free stream fluid velocity. At lockin the natural frequency and the vortex shedding frequency are assumed to be equal.

$$2\pi f_s = \omega_s = \omega_1 = 2\pi S_t U/D \quad (22)$$

Over many years the variety of these evolved forms has led to confusion and misinterpretation of the significance of the various terms which form the response parameter  $S_G$ .

The most serious misinterpretation is the implication that lockin response amplitude depends on the mass ratio,  $\mu$ . It has been generally believed that very dense cylinders respond with lower amplitudes than low density ones. This is not true. It is in fact dependent on fluid exciting forces and structural damping (not damping ratio). The mass per unit length of the cylinder is only important in determining the natural frequency. The validity of these statements can be demonstrated by simply drawing upon definitions, as shown below.

From Equations 18 and 6

$$k_s = \frac{4\pi m_e \zeta_s}{\rho D^2} = \frac{4\pi m R_i}{\rho D^2 2\omega_1 M_i} \quad (23)$$

Using the definitions of modal mass, and effective mass per unit length from Equations 4 and 20 yields,

$$k_s = \frac{2\pi R_i}{\rho D^2 \omega_1 \int_0^L \psi_i^2(x) dx} \quad (24)$$

For the case of constant damping constant per unit length,  $r(x)=r$

$$k_s = \frac{2\pi r}{\rho D^2 \omega_1} \quad (25)$$

If  $k_s$  is not a function of  $m(x)$  then from Equation 15 neither is  $S_G$ .

$$S_G = 2\pi S_t^2 k_s \quad (15)$$

$$S_G = \frac{4\pi^2 S_t^2 R_i}{\rho D^2 \omega_1 \int_0^L \psi_i^2(x) dx} \quad (26)$$

$R_i$  is the equivalent, linear, structural modal damping. The actual source of damping may not in fact be linear. For most interesting vibration cases the damping is low and for any specific steady state response amplitude an equivalent linear damping is an acceptable approximation.

There is experimental confirmation that  $S_G$  and hence the predicted response do not depend specifically on the mass ratio but on the ratio  $\zeta_s/\mu$ . As shown, this is because in taking this ratio the dependence on mass per unit length cancels out. Griffin in reference (7) presents a plot of response amplitude,  $2\bar{Y}/D$ , versus reduced velocity  $V_r = U/f_s D$  where  $f_s$  is the natural frequency. This figure is reproduced in figure 4.

Two different cases are shown, one in air and one in water. For both the ratio  $\zeta_s/\mu$  is approximately constant. However, the damping ratios and therefore the mass ratios are different by an order of magnitude. Botelho has also observed this apparent lack of specific dependence on  $\mu$  (8).

Both Griffin and Botelho have pointed out another interesting fact, which can be seen in Figure 4. The in water case has a larger damping ratio, by a factor of 10, and therefore it has a much broader bandwidth, than the in air case with lower damping. The halfpower bandwidth for a linear oscillator is equal to  $2\zeta_1 \omega_1$ . Thus one would expect to see a wider region of large amplitude response in a figure such as 4, for those cylinders with larger damping ratios. This author is of the opinion that the consequence of a higher damping ratio is to make lockin vibration of the cylinder less sensitive to local variations in flow velocity (hence reduced velocity) and therefore more tolerant of shear. In other words, two geometrically similar cables with the same reduced damping but different damping ratios will respond differently to a shear. The one with the higher damping ratio will likely experience lockin over a greater portion of its length.

For most engineers  $S_G$  has little physical meaning. In the next section, an attempt is made to clarify it.

#### An Interpretation of $S_G$ , The Response Parameter

No one denies its importance but a common sense interpretation is needed for  $S_G$ . To develop one requires a statement of the equation of motion for the normal mode excited at resonance during lockin. At lockin the lift force per unit length in phase with the cross-flow velocity of the cylinder can be expressed as

$$f(x,t) = 1/2 \rho U^2 DC_L(x) \omega_1 t \quad (27)$$

The modal exciting force is given by

$$N_i(t) = \int_0^L f(x,t) \psi_i(x) dx \quad (28)$$

$$= 1/2 \rho U^2 D \int_0^L C_L(x) \psi_i(x) dx e^{i\omega_i t} \quad (29)$$

Using the expression in Equation 29 for the modal exciting force, the non-linear feedback mechanisms which control response amplitude have been replaced with an equivalent linear exciting force in phase with the velocity of the cylinder. Implicit in this expression are the following assumptions:

- i. The lift coefficient  $C_L(x)$  must be chosen to yield the response amplitude which would be observed in an experiment and can be estimated from compiled data of  $S_G$  versus response amplitude, Figure 2.
- ii. Lockin exists over the entire cylinder length.
- iii. The modal damping on the left hand side arises from non-hydrodynamic sources only. This is because  $C_L(x)$  is a lift coefficient which reflects the net fluid dynamic force on the cylinder. It is in fact the difference between lift force in phase with the velocity of the cylinder due to circulation, and fluid resistive forces due to pressure drag and friction drag opposing the cross-flow velocity of the cylinder. In an experimental sense the net lift force is the only measurable quantity and is therefore used here. Under shear conditions, lockin over a portion of the length is likely. Outside of the lockin region fluid drag forces will have to be estimated and used to modify the estimate of  $R_i$ . This will be addressed in the section on response prediction in sheared flow.
- iv. Fluid forces in phase with the displacement and acceleration also exist. They are assumed to affect only the fluid added mass of the cylinder and are included in the expression for the modal mass,  $M_i$ .

Let the integral shown below be defined as  $P_u$ , where the  $u$  refers to the uniform flow case,

$$P_u = \int_0^L C_L(x) \psi_i(x) dx \quad (30)$$

Recalling Equation 14, an expression for the modal response amplitude at resonant lockin can be found

$$|q_i| = \frac{|N_i|}{\omega_i R_i} = \frac{1}{2} \frac{\rho U^2}{\omega_i R_i} P_u \quad (31)$$

From Equation 1 the response magnitude of the entire cylinder to the one resonant mode is

$$y(x) = |q_i| \psi_i(x) \quad (32)$$

which, when expressed as a double amplitude in diameters peak to peak, can be written as

$$\frac{2y(x)}{D} = \frac{2|q_i| \psi_i(x)}{D} \quad (33)$$

Substitution for  $|q_i|$  from Equation 31 leads to

$$\frac{2y(x)}{D} = \frac{2|N_i| \psi_i(x)}{D \omega_i R_i} \quad (34)$$

$$= \frac{\rho U^2 \psi_i(x) P_u}{\omega_i R_i} \quad (35)$$

Recalling that

$$U = \frac{\omega_i D}{2\pi S_t} \quad (23)$$

and the expression for  $S_G$  in Equation 26, leads to:

$$\frac{2y(x)}{D} = \frac{P_u \psi_i(x)}{S_G \int_0^L \psi_i^2(x) dx} \quad (36)$$

The maximum response occurs at the maximum value of the mode shape and therefore

$$\frac{2y_{\max}}{D} = \frac{P_u \psi_{i,\max}}{S_G \int_0^L \psi_i^2(x) dx} \quad (37)$$

Therefore,  $S_G$  is a dimensionless group which is an integral part of the expression one finds for a prediction of response amplitude, and therefore an experimentally observed dependence of response on  $S_G$  should not be surprising.

Griffin (2,4) has compiled and published data relating  $S_G$  to observed response. These data are given in Figure 2 and represent the results of many different types of experiments, including cantilevers, spring mounted cylinders, pivoted cylinders and cables. The horizontal axis is  $S_G$  and the vertical axis is

$$\frac{2y_{\max}}{D} \frac{I_i^2}{\psi_{i,\max}} \quad (38)$$

where 
$$I_i = \frac{\int_0^L \psi_i^2(x) dx}{\int_0^L \psi_i^2(x) dx} \quad (39)$$

For example, a string or a beam with pinned ends and constant tension have mode shapes which are given by

$$\psi_1(x) = \sin\left(\frac{i\pi x}{L}\right) \quad (40)$$

$$\text{and } I_1 = 3/4 \quad (41)$$

Other values for  $I_i$  corresponding to different mode shapes are given in Reference 4, as is a table identifying the source of the data used in Figure 2.

The factor  $I_i^{1/2}/\psi_{i\max}$  was used in an attempt to reduce the scatter in plotting response data for many different types of structures versus  $S_G$ . That this was the appropriate factor to use to accommodate various mode shapes was based on the assumption that the wake oscillator model correctly predicts response. Implicit in the wake oscillator model are particular assumptions regarding the spatial variation of  $C_L(x)$ . This author is of the opinion that such models are only approximations and that much of the scatter in the data is due to the fact that the correction factor has substantial error for some types of mode shapes.

It should also be noted that only very little of the data shown in Figure 2 is derived from cables and beams under tension such as risers and casing strings, which have essentially sinusoidal mode shapes. In the last few years a large amount of experimental data have been accumulated on such cylinders, and should be compiled in a separate plot of  $2y_{\max}/D$  versus  $S_G$  without correction factors such as  $I_i^{1/2}/\psi_{i\max}$ .

A Proposed Equivalent Response Parameter for Sheared Flow:  $S_{GE}$

Under sheared flow conditions lockin may occur over a limited portion of the cylinder defined by the range  $X_1$  to  $X_2$ . For sections of the cylinder outside of this range lockin does not occur and energy is lost due to hydrodynamic damping. In the analysis to follow it is assumed that only one mode has significant response, and even though exciting forces do exist outside of the lockin region they are not at the natural frequency and cause insignificant response. The method proposed is intended to be used to evaluate several possible vibration modes, one at a time, to determine which if any is likely to dominate the response.

A substantial database exists, which tabulates observed response versus the response parameter,  $S_G$ , but for uniform flows only. The approach proposed here takes advantage of this existing database by providing an estimate of the response parameter of an equivalent cylinder in a uniform flow, which would behave the same as the cylinder in the sheared flow. In order to be equivalent, both the cylinder in the sheared flow and the equivalent cylinder in the uniform flow must have the following characteristics.

- The modal response amplitude for each must be the same and therefore from Equation 14

$$\frac{N_{ie}}{\omega_i^{R_{ie}}} = \frac{N_{is}}{\omega_i^{R_{is}}} \quad (42)$$

where the subscripts e and s refer to the equivalent and sheared cases respectively.

- The exciting force over the region  $x_1$  to  $x_2$  must be the same for both cases. Outside of this region the forces contributing to lockin are assumed to be zero for the sheared case, and appropriate to that of a fully locked in cylinder in the equivalent case. The equivalent cylinder experiences lockin over its entire length and therefore additional power is fed into the resonant mode outside of the region  $x_1$  to  $x_2$ . In order for the response amplitude to stay constant the modal damping in the equivalent cylinder must be increased, so as to dissipate the greater injected power.

Solving for the equivalent damping

$$R_{ie} = R_{is} \frac{N_{ie}}{N_{is}} \quad (43)$$

The equivalent response parameter is obtained directly from Equation 26.

$$S_{GE} = \frac{4\pi^2 S_t^2 R_{ie}}{\rho D^2 \omega_i^2 \int_0^L \psi_i^2(x) dx} \quad (44)$$

$$= \frac{8\pi^2 S_t^2 m_e \tau_{ie}}{\rho D^2} \quad (45)$$

where

$$\tau_{ie} = \frac{R_{ie}}{2\omega_i M_i} \quad (46)$$

and  $m_e$  is defined in Equation 20. It remains to obtain a detailed expression of  $R_{ie}$  in terms of  $R_{is}$ , and  $N_{ie}/N_{is}$ . From Equation 29 and item (ii) above,

$$\frac{N_{ie}}{N_{is}} = \frac{\int_0^L C_L(x) \psi_i(x) dx}{\int_{X_1}^{X_2} C_L(x) \psi_i(x) dx} = \frac{P_u}{P_s} \quad (47)$$

and from Equation 5

$$R_{is} = \int_0^L (r_s(x) + r_h(x)) \psi_i^2(x) dx \quad (48)$$

where  $r_s(x)$  and  $r_h(x)$  are the structural and hydrodynamic damping constants per unit length, respectively. For the sake of example, let  $r_s(x)$  and  $r_h(x)$  be constant everywhere except in the region  $x_1$  to  $x_2$  where  $r_h(x)$  the hydrodynamic damping is required to be zero. This leads to

$$R_{is} = (r_s + r_h) \int_0^L \psi_i^2(x) dx - \int_{x_1}^{x_2} r_h \psi_i^2(x) dx \quad (49)$$

Perhaps the most convenient form in which to express  $S_{GE}$  is in terms of the  $S_G$  for the actual cylinder in a uniform flow.

Therefore from Equations 26, 43, and 44

$$S_{GE} = S_{GU} \frac{R_{ie}}{R_{is}} = S_{GU} \frac{P_u}{P_s} \frac{R_{is}}{R_{iu}} \quad (50)$$

where the subscript u has been added to clarify which quantities come from a uniform flow case and which are due to the sheared conditions. This expression reduces to

$$S_{GE} = \frac{P_u}{P_s} \left[ 1 + \frac{r_h}{r_s} - \frac{r_h}{r_s} \frac{\int_{x_1}^{x_2} \psi_i^2(x) dx}{\int_0^L \psi_i^2(x) dx} \right] S_{GU} \quad (51)$$

Both the quantity in brackets and the ratio  $P_u/P_s$  must always be greater than or equal to one. Therefore  $S_{GE}$  is always greater than or equal to  $S_{GU}$ . Note that in the limit as the sheared flow becomes uniform,  $S_{GE}$  equals  $S_{GU}$ , as expected.

To proceed farther requires knowledge of  $C_L(x)$ , the lift coefficient. As an instructive example, but admittedly without experimental justifications, let  $C_L(x)$  be proportional to the mode shape  $\psi_1(x)$ .

$$C_L(x) = C_L \psi_1(x) \quad (52)$$

Then the expression for  $P_u/P_s$  in Equation 47 simplifies greatly and

$$S_{GE} = \left[ \left( 1 + \frac{r_h}{r_s} \right) \frac{\int_0^L \psi_i^2(x) dx}{\int_{x_1}^{x_2} \psi_i^2(x) dx} - \frac{r_h}{r_s} \right] S_{GU} \quad (53)$$

Continuing the example, assume that the second mode of a cable with the mode shape

$$\psi_2(x) = \sin \frac{2\pi x}{L} \quad (54)$$

is excited over one fourth of its length;  $x_1=0$  to  $x_2=L/4$ , as shown in Figure 3. In that case,

$$S_{GE} = \left( 4 + \frac{3r_h}{r_s} \right) S_{GU} \quad (55)$$

If the distributed damping per unit length has equal hydrodynamic and material components, then

$$S_{GE} = 7S_{GU} \quad (56)$$

In this particular example, the cylinder in a sheared flow, with the top quarter of its length

experiencing lockin in the second mode, would respond at the same amplitude as the same cylinder in a uniform flow but with a response parameter seven times as great.

### Conclusions and Recommendations

If only one mode has a natural frequency excitable by a sheared flow, then a worst case prediction is given by the method described above. However, if two or more modes are potential candidates for resonant lockin excitation then the equivalent response parameter for each should be computed. The mode with the lowest  $S_{GE}$  is the most likely lockin candidate. If two or more modes have low  $S_{GE}$  values, multimodal non-lockin response or beating between modes may be observed.

The accurate response prediction of flexible cylinders in sheared flows requires much more experimental data. Areas of particular weakness are: i. the current state of knowledge of the hydrodynamic damping on the non-locked in regions of a cylinder; ii. the form of  $C_L(x)$  for both uniform and sheared cases; iii. the means of estimating the extent of locked-in regions in sheared flows; iv. the dependence of the locked-in region on damping ratio and bandwidth..

One model for predicting the locked-in region has been offered in the literature (5,6). Experimental observation is needed.

### Nomenclature

$C_L(x)$	lift coefficient
$D$	cylinder diameter
$f$	vortex shedding frequency (Hz)
$H_1^s(\omega)$	frequency response function or RAO
$I_{i1}$	mode shape correction factor
$K_{i1}$	modal stiffness
$k_{i1}$	reduced damping
$L^s$	length of flexible cylinder
$M_{i1}$	modal mass
$m, m(x)$	constant and variable mass per unit length
$m_e$	constant $m$ equivalent to a variable $m(x)$
$N_{i1}$	modal force
$N_{ie}$	equivalent system modal force
$N_{is}$	modal force for sheared case
$P_u, P_s$	integrals in uniform flow and sheared cases
$q_i(t)$	modal amplitude
$R_{i1}$	modal damping constant
$R_{ie}, R_{is}$	$R_i$ in equivalent and sheared cases
$S_G^e, S_G^s$	response parameter
$S_{GU}, S_{GE}$	$S_G$ for uniform flow and sheared flow equivalent
$S_t$	Strouhal number
$U$	free stream velocity
$V$	reduced velocity
$x^r$	longitudinal coordinate
$x_1, x_2$	range of lockin
$y(x, t), y(x)$	cross flow response amplitude
$y_{max}$	maximum value of $y(x)$
$\delta_s$	logarithmic decrement for structural damping
$\zeta_{i1}$	modal damping ratio
$\zeta_s$	structural modal damping ratio
$\zeta_{ie}$	equivalent $\zeta_i$
$\omega$	frequency (radians/sec)
$\omega_{i1}$	natural frequency

$\omega_s$	shedding frequency
$\rho$	density of water
$\mu$	mass ratio
$\phi_1$	phase angle
$\psi_1(x)$	mode shape
$\psi_{1max}$	maximum value of the mode shape

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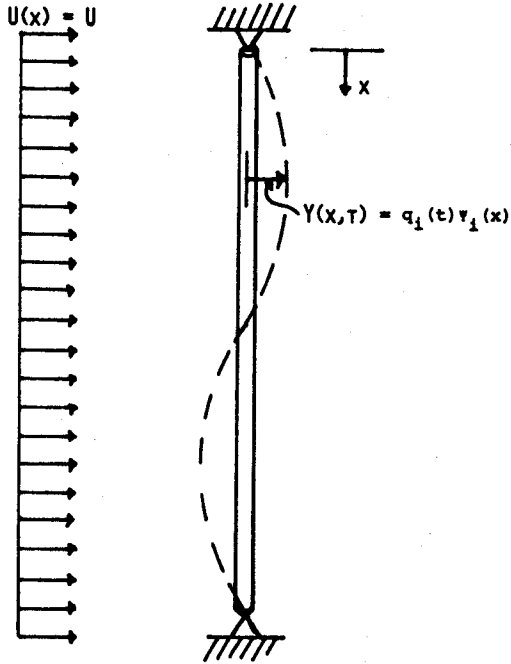


Fig. 1—Flexible cylinder in a uniform flow.

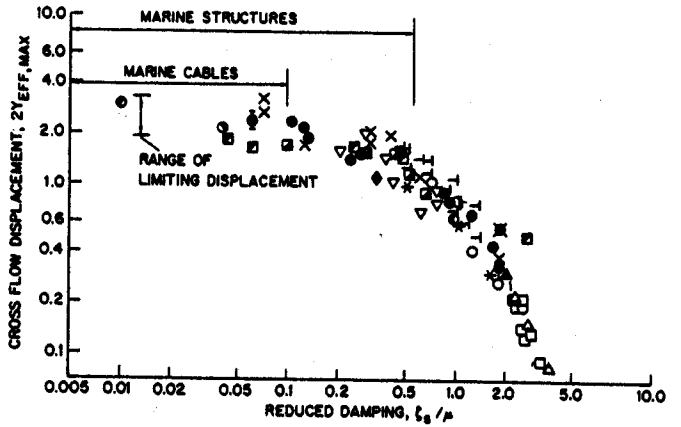


Fig. 2—Response parameter,  $S_D$ , vs.  $2Y_{eff,max}/D$ , from Griffin, 2,4,5,7

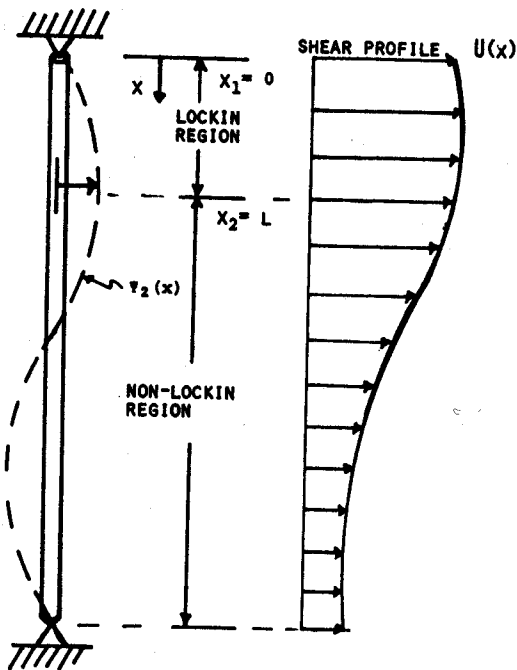


Fig. 3—Second mode partial lockin in a sheared flow.

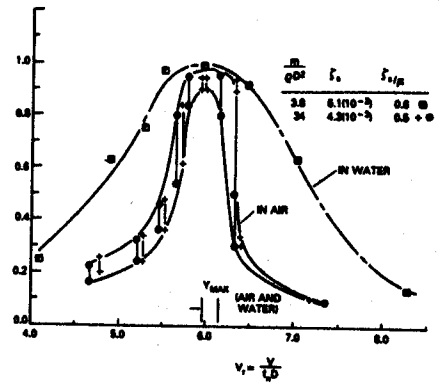


Fig. 4—Reduced velocity vs.  $2Y_{max}/D$ , from Griffin, 7