We generalize Narasimhan’s (1988) model of retail promotion to include multiple products and general demand functions. Doing so allows us to further characterize optimal promotion strategies. We find that firms prefer to offer deeper promotions on products for which switching customers have stronger demand than loyal customers and/or for which the price sensitivity of demand is high for both switching and loyal customers. We further show that firms will offer deeper promotions on products which enjoy complementary relationships with other products that they sell rather than on products for which the firm sells a substitute.

(Retail Promotions; Pricing; Mixed Strategies)

1. Introduction

The interpretation of temporary price promotions as mixed strategy equilibria was proposed by Varian (1980) and Narasimhan (1988). We begin by generalizing their models to include multiple products and general demand functions. Rather than choosing from a distribution of individual prices, multi-product firms choose from a distribution of efficient price vectors; where each vector represents the most profitable strategy for offering switching customers a fixed level of surplus. The inclusion of multiple products and general demand functions yields a richer understanding of the role that customer demand characteristics play in determining firms’ relative willingness to promote different products.

Other researchers have studied multiproduct pricing strategies. In particular, Lal and Matutes (1989) identify two pure strategy equilibria. In the first, competing firms sell the same product to price sensitive (poor) customers at marginal cost while another product is sold only to the less price sensitive (rich) customers. In the second equilibrium, the poor customers buy one product from each firm, paying a price equal to their reservation values. The authors note that both equilibria are symmetric so that neither equilibrium predicts which product each firm prefers to promote. Lal and Villas-Boas (1994) consider a market incorporating competition at both the retail and the manufacturer level in order to investigate the impact of channel intermediaries and brand competition on the depth and frequency of promotions. In doing so they extend the standard Varian model to include two competing brands and a rich specification of customer heterogeneity. In a related paper, Agrawal (1996) also incorporates two competing brands to investigate how the size and strength of customer loyalty influence manufacturers’ advertising and trade promotion mixes. These mixes are determined in part by the anticipated response of a monopolist retailer. The author provides empirical support for his conclusions using data from a variety of product categories.

The current study focuses on identifying which product categories rather than which competing brands firms prefer to promote. For this reason, we increase the breadth rather than the depth of the customers’ product selection so that in equilibrium, all customers purchase both products. Moreover, these earlier papers all assume unit demand functions, in which each customer demands a single unit of a product whenever the product’s price is less than the customer’s reservation value for that product. This restricts the marginal (opportunity) cost of offering additional surplus to switching
customers to be constant across products and price levels. As a result, the efficient price vectors are not unique because firms and customers are generally indifferent as to which combination of prices is used to offer a fixed level of surplus. In contrast, we contemplate markets in which customers derive decreasing marginal utility from increased purchase quantities and show that firms' preferences for promoting each product depend upon the customers' demand characteristics.

2. Existence of a Mixed Strategy Equilibrium

Consider a market with two competing firms, \( f \in \{0, 1\} \), which both sell two products \( i \in \{a, b\} \). Each firm has a homogenous segment of loyal customers of mass \( Y \). These customers always purchase both products from the firm to which they are loyal (according to their demand functions, which follow). In addition, there exists a homogenous segment of switching customers of mass \( X \), who purchase both products from the firm offering the greatest joint surplus (from purchases of both products). The switching customers first observe the prices charged by each firm and then decide which firm to purchase from.

The assumption that customers buy both products from the same firm is an important one. Our findings apply only to those purchase occasions in which the transaction cost incurred when purchasing from multiple firms is sufficiently high that very few customers do so. For example: for some telecommunication services it is technically infeasible to purchase different services from different firms, few diners are willing to travel between restaurants to eat appetizers and entrees at different locations, while many customers visit just one supermarket to purchase all of their weekly groceries (although they may visit different stores in alternate weeks). In markets in which customers purchase from multiple firms, competition will focus on individual product prices rather than the prices charged for bundles of goods.

Consumer surplus is equal to the utility that a customer derives from consuming both products less the joint cost:

\[
S_j(q_j, p) = U_j(q_j) - q_j'p
\]

(1)

where \( q_j \) and \( p \) are quantity and price vectors (respectively); \( U_j(q_j) \) is the utility that a customer (of type \( j \)) derives from consuming the quantity vector \( q_j \) and \( j \in \{x, y\} \) denotes a customer's type.

Assume that the hessian of \( S_j \) is negative definite and that \( S_j \) is increasing and at least twice continuously differentiable in each \( q_j \). The expected profit earned by firm \( f \) is defined as:

\[
\Pi_f(p) = \alpha Xq^*_f(p)'(p - c) + Yq^*_f(p)'(p - c)
\]

(2)

where \( c \) is a cost vector; \( \alpha \) is equal to the probability that the switching customers purchase from the firm; and

\[
q^*_f(p) = \arg\max_{q_j} S_j(q_j, p).
\]

(3)

To ensure that the expected profit function is strictly concave and continuously differentiable, we restrict attention to utility functions yielding derived demand functions which are concave in price and have negative own-price derivatives. In accordance with our focus on demand characteristics, we simplify by setting marginal costs equal to zero. If a firm does not expect to attract any switching customers it will maximize the profits that it earns from its loyal customers:

\[
\hat{\rho} = \arg\max_{\rho} q^*_f(p)'p.
\]

(4)

Let \( R \) represent the profit that a firm earns under these conditions and let \( S^f \) represent the resulting surplus offered to the switching customers:

\[
R = Yq^*_f(\hat{\rho})'\hat{\rho},
\]

(5)

\[
S^f = S^*_f(\hat{\rho}), \quad \text{where}
\]

(6)

\[
S^{f*}_j(p) = S_j(q^*_f(p), p).
\]

(7)

Because either firm can earn profit \( R \) from selling just to its loyal customers, neither will offer a price bundle that yields profits lower than \( R \) when selling to both the switching customers and the firm's loyal customers. In particular, the maximum surplus (\( S^{f*} \)) that a firm will
offer switching customers in order to guarantee that they come, can be found by identifying the price vector that maximizes the switching customers' surplus when firm profits (from both the firm's loyal customers and the switching customers) are equal to \( R \):

\[
S^H = S^*_f(p^*), \quad \text{where} \quad p^* = \arg\max_p S^*_f(p) \tag{8}
\]

\[
\text{s.t. } Xq^*_f(p)'p + Yq^*_{s}(p)'p = R. \tag{9}
\]

For the reasons described by Varian (1980), we know that in any mixed strategy equilibrium, there are no point masses in the equilibrium pricing strategies (Varian, Proposition 3). Unfortunately several of Varian's other results depend upon an assumption that customers have unit demand for the single product in his model. The introduction of multiple products and a more general derived demand function compels further justification for these results. Proofs for each of the lemmas and propositions that follow (together with a proof of existence) are presented in a separate Technical Appendix available from the author.

**Lemma 1.** In any mixed strategy equilibrium, \( S^*_f(p) \in [S^L, S^H] \).

In Varian's model, \( S^L \) equals zero, so any \( S^*_f < S^L \) would violate the switching customers' participation constraints.

**Lemma 2.** In any mixed strategy equilibrium, \( \tilde{p} \) and \( p^* \) must receive positive support.

In any mixed strategy equilibrium, it is clear that the probability of attracting the switching customers (\( \alpha \)) is determined in part by the price vector that a firm charges. Moreover, when the demand characteristics of the customer segments differ, the price vector that a monopolist would charge depends upon \( \alpha \). For any \( \alpha \), how does the monopoly price vector compare with the price vector at which the probability of attracting the switching customers is equal to \( \alpha \)?

**Lemma 3.** In any mixed strategy equilibrium, let \( p \) be any price vector receiving positive support and define \( \alpha(p) \) as the probability that a firm charging \( p \) will attract the switching customers. Further define the monopoly price vector \( M(p) \) as the price vector that maximizes expected firm profits for a given \( \alpha = \alpha(p) \). In any mixed strategy equilibrium, \( S^*_f(M(p)) = S^*_f(p) \) when \( \alpha \) equals zero, but \( S^*_f(M(p)) < S^*_f(p) \) otherwise.

This important result highlights the role of competition in the market. Each firm would offer less surplus to the switching customers if doing so did not reduce the chances that these customers would purchase from the firm. We use this result to prove that there are no gaps in the range of surplus offerings receiving positive support.

**Lemma 4.** In any mixed strategy equilibrium, for every \( S \in [S^L, S^H] \) there exists a price vector, \( p \), receiving positive support for which \( S^*_f(p) = S \).

Together Lemmas 1 and 4 imply that, in any mixed strategy equilibrium, the surplus offered to the switching customers must fall in the range \([S^L, S^H]\) but may occur anywhere in that range with positive probability. Define \( p^*(s, \alpha) \) as an efficient price vector, where:

\[
p^*(s, \alpha) = \arg\max_p \alpha Xq^*_f(p)'p + Yq^*_{s}(p)'p, \quad \text{s.t. } S^*_f(p) = s \tag{10}
\]

where \( \alpha \in [0, 1] \) is equal to the probability that the switching customers purchase from the firm.

We further define the set \( P = \{ p \in \mathbb{R}^2: p = p^*(s, \alpha), \text{ where } s \in [S^L, S^H] \text{ and } \alpha \in [0, 1] \} \) as the set of efficient price vectors.

**Proposition 1.** No mixed strategy equilibrium can give positive support to any price vector \( p \notin P \).

We restrict attention to utility functions that yield sufficient concavity in the derived demand functions to ensure that there exists a unique interior solution to Eq. 10. This excludes the unit demand function, under which competing firms and customers are generally indifferent as to what combination of price promotions are used to offer a fixed level of surplus.

**Proposition 2.** It is an equilibrium for both firms to adopt the following mixed strategy:

\[
f(p) = \begin{cases} 
    f(s) & \text{when } p = p^*(s, \alpha) \text{ and } \alpha = 1 - F(s), \\
    0 & \text{otherwise}, 
\end{cases}
\tag{11}
\]

where \( F(s) \) is defined (implicitly) as:

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Application of the implicit function theorem confirms that \( F(s) \) is an increasing function of \( X \) and a decreasing function of \( Y \).\(^2\) Firms offer a deeper range of price promotions when there are many switching customers and/or few loyal customers present in the market. For this reason, we would expect to see firms which have recently entered the market offering more generous promotions than firms which have had time to establish a sizeable following of loyal consumers. It is also possible to conjecture about the influence of substitute or complement relationships between the products. Complement relationships increase the surplus that customers enjoy from purchasing larger quantities of each product, reducing the rate at which marginal utility falls as the quantity purchased increases. If this is true, the introduction of complement relationships will yield greater increases in demand and firm profit when prices are lower, mitigating the cost of promotions and increasing the depth of the average promotion. The opposite may be expected to occur in the presence of substitute relationships.

3. Characterizing the Depth of Promotions

Recall that each efficient price vector represents the least costly strategy for offering switching customers a fixed level of surplus (Eq. 10). The Kuhn Tucker conditions from the Lagrangian yield the following expression:

\[
\frac{d \Pi_i(p, \alpha)}{dp^i} = \frac{d \Pi_i(p, \alpha)}{d S_i(p)} - \frac{d S_i(p)}{dp^i} = \frac{\partial q_i^*(p)}{\partial p^i} = \frac{q_i^*(p)}{X \frac{dq_i^*(p)}{dp^i} + \alpha X \frac{dq_i^*(p)}{dp^i} + \alpha X q_i^*(p)} \]

Equation (13) implies that in equilibrium, the (opportunity) cost of a marginal increase in the surplus offered to switching customers is the same for each product.\(^3\) Expanding this equation through differentiation (and use of the envelope theorem) will help us to further characterize the behavior of the efficient price vectors:

\[
\frac{d \Pi_i(p, \alpha)}{dp^i} - \frac{d S_i(p)}{dp^i} = \left[ X \frac{dq_i^*(p)}{dp^i} + \alpha X \frac{dq_i^*(p)}{dp^i} + \alpha X q_i^*(p) \right] \frac{1}{q_i^*(p)}
\]

where \( \frac{dq_i^*(p)}{dp^i} \) = the stacked vector of the derivatives of \( q_i \) with respect to \( p^i \).

This expression, \( Z'(p, \alpha) \), is analogous to the marginal rate of substitution for product \( i \) and represents the marginal cost of offering switching customers additional surplus through product \( i \). Our concavity assumptions require that \( Z' \) decreases as \( p^i \) is increased. It should be clear that if \( Z^a > Z^b \), a firm can increase its profits by raising \( p^a \) and reducing \( p^b \). It is this relationship that we exploit to predict how customers’ demand characteristics influence each firm’s preference for promoting each product. Define \( OP \) as the own-price sensitivity of demand for product \( a \) and \( CP \), as the cross-price sensitivity of demand for product \( a \):

\[
OP_a = \frac{dq^a}{dp^a}, \quad CP_a = \frac{dq^a}{dp^b}.
\]

In Table 1 we characterize the demand of each customer type in terms of the relative strength of customer

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Impact of an Increase in Each Variable on the Efficient Price Vector ((p^a, p^b))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable</td>
<td>( p^a )</td>
</tr>
<tr>
<td>( OP_a )</td>
<td>increase</td>
</tr>
<tr>
<td>( OP_b )</td>
<td>increase</td>
</tr>
<tr>
<td>( CP_a )</td>
<td>increase</td>
</tr>
<tr>
<td>( CP_b )</td>
<td>increase</td>
</tr>
<tr>
<td>( q_i^a )</td>
<td>decrease</td>
</tr>
<tr>
<td>( q_i^b )</td>
<td>increase</td>
</tr>
</tbody>
</table>

\(^2\) Similar results were formally derived by Narasimhan (1988).

\(^3\) Except in the case of boundary solutions. Note that this relationship continues to hold when the number of products sold by each firm is increased.
demand \((q^*_j)\)^4 and the sensitivity of demand to price changes \((OP_j \text{ and } CP_j)\). While an improvement in the strength of switching customers' demand, \(q^*_j\), increases the marginal cost of a price reduction, this is more than offset by the increase in the marginal rate at which surplus is offered to the switching customers. Therefore, as \(q^*_j\) increases, the marginal cost of offering switching customers additional surplus though product \(a\) is decreased. This implies a reduction in the price of product \(a\) and an increase in the price of product \(b\). In contrast, when the demand of loyal customers increases, the rate at which surplus is offered to the switching customers does not change. As a result, increases in \(q^*_j\) cause \(Z^*_j\) to rise, which increases the firms' preference for discounting product \(b\) (relative to product \(a\)).

Note that we earlier assumed that \(OP_j\) is negative \((\forall j)\) so an increase in \(OP_j\) implies that product \(a\)'s demand is less sensitive to its (own) price. As a result, the marginal cost of offering surplus to the switching customers by reducing \(p^a\) is increased. Now consider \(CP_j\). If the sign of \(CP_j\) is positive, an increase in this variable should be interpreted as a strengthening of the substitute relationship between the two products. Alternatively, when \(CP_j\) is negative, an increase in \(CP_j\) implies the weakening of a complementary relationship between the products. In either case, increases in \(CP_j\) increase the marginal cost of offering switching customers additional surplus through product \(a\). As a result, firms would prefer to offer deeper promotions on product \(b\) and more shallow promotions on product \(a\). In general firms would rather offer deeper promotions on products whose prices have strong complementary effects on the demand for other products sold by the firm.

4. Conclusions
We have demonstrated the existence of a mixed strategy equilibrium in which multi-product firms randomize over a distribution of efficient price vectors. In equilibrium, each efficient price vector is located at a point at which the opportunity cost of a marginal increase in the surplus offered to switching customers is the same for each product. We use this finding to interpret how customers' demand characteristics influence relative promotion depths for each product. We find that when the demand for a product is more sensitive to its own price, marginal reductions in the price of this product are relatively less costly. As a result, firms will prefer to attract switching customers by discounting products which exhibit this characteristic. For analogous reasons, firms would also rather discount products whose prices have strong complementary effects on the demand for other products sold by the firm. Discounting products whose prices have a substitution effect on the demand for other products reduces the profits earned on both products. The analysis further reveals that it is more efficient to discount products for which switching customers have strong demand and loyal customers have weak demand. This is consistent with the principle of target marketing: discounting to attract switching customers is least costly and most effective when switching and loyal customers prefer to purchase different products.

Note that in the interest of brevity, these results focus solely on relative promotion depths. They do not take into account the impact of demand characteristics on the overall depth and/or frequency of promotions. Extending the analysis to investigate these phenomenon requires the straightforward application of the implicit function theorem to Eq. 12. It is similarly straightforward to extend the analysis to include more than two competing firms or a broader range of products. Interestingly, the introduction of additional competitors may not result in lower average prices (Rosenthal 1980). As the number of competitors increases, any single firm is less likely to capture the switching customers, so firms are more reluctant to offer deep promotions.

These results offer a variety of readily testable hypotheses once access to sufficient cross-category data becomes available. However, while the results may provide the subject for future empirical tests, they also introduce an additional complexity to such tests by demonstrating that the impact of promotions on customer demand helps determine which products are promoted. We are unaware of any studies that have identified or addressed this endogeneity issue.5

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4 We simplify notation by denoting \(q^*_j\) using \(q^*_j\).

5 This paper has benefitted from comments by Mary Sullivan and Birger Wernerfelt.
References


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