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Optimum Investments to Mitigate Catastrophic Risk: Application to Food Industry Firms [1]

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Abstract

A simple analytical model is developed and calibrated, using probability measures for catastrophic risks from prior research, the "Risk Metrics Project," together with the results of a "Benchmarking and Assessment Survey" to estimate the level of optimum investments needed by firms in the food industry to mitigate exposure to catastrophic risk. The limitations on the availability of catastrophic risk insurance and their high level of deductibility, together with the one-time nature of the alternative risk-mitigating investments suggest that such investments should be undertaken whether or not catastrophic risk insurance is available, particularly since these investments have a large impact on risk financing. Such investments may protect long term reputations and brand ratings in addition to mitigating potential catastrophic losses.

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1 Introduction

The numerous difficulties that have thus far plagued the catastrophic risk insurance market and particularly those dealing with terrorism risk insurance (c.f., Kunreuther. et. al., 2005 and Auerswald, et. al. 2006) lead us to consider what firms and businesses can do by way of ex-anti investments in risk mitigating strategies. One of the principal reasons why catastrophic and especially terrorism insurance markets have not been adequate is the difficulty that exists in assessing low frequency high impact risks and the uncertainties that are associated with measuring such risks. In a previous line of research conducted by Mohtadi and Murshid (2004-2006) and broadly identified as "Risk Metrics Project," [3], we have attempted to take a first step at addressing this shortcoming by developing quantitative probability measures using a novel statistical methodology. While it is hoped that these new measures find their way to the insurance industry and are ultimately adopted, that task is beyond us at the present time. In the meantime, however, the probability measures developed under the "Risk Metrics Project," and based on primary data collected under this project (Mohtadi and Murshid, 2006a), may be used to find a "best practice" strategy. For the purposes of this research, a best practice strategy consists of finding the optimum level of investments that firms wishing to mitigate the impact of a catastrophic risks should undertake. This is the task of the present study. This work is made possible by developing an analytical model that uses not only the "Risk Metrics" probability measures but also the results from another project initiated by Closs and Kinsey (2005-2007) broadly identified as the "Benchmarking and Assessment Survey."[4] Some of the econometric results from that survey, which are found from a working paper by Agaiwal and Mohtadi (2007), are then used in the present study.

Our results suggest that whether the optimum level of investments to mitigate risk is larger or smaller than the cost of catastrophic risk insurance for equal coverage, depends on the *level* of risk. However, the limitations that exist on the availability of catastrophic

[3] This research was sponsored by the University of Minnesota's National Center for Food Protection and Defense (NCFPD) and the Department of Homeland Security (DHS) and has led to a number of publications and working papers several of which are cited at the end of this paper.

[4] This work is also sponsored by the University of Minnesota's National Center for Food Protection and Defense (NCFPD) and the Department of Homeland Security (DHS) and is still ongoing.

insurance and their high level of deductibility, together with the one-time nature of the alternative risk-mitigating investment strategies suggest that such investments should be undertaken whether or not catastrophic risk insurance is available, particularly as their impact on risk financing may not be negligible at all.

Following the development of the analytical model in section 2, the estimation method and the results are presented in section 3. Section 4 makes some concluding observations based on the comparison of our results with those of catastrophic insurance.

2 A Simple Analytical Model

Consider investing K in infrastructure/ business practice/ etc. ex-anti to improve responsiveness to a catastrophic event, should such an event occur. Let L be the nominal loss in the case of an event. Then an initial investment of K, could reduce the impact of the loss by a fraction $\lambda(K)$ such that $\lambda(K)L < L$ and $\lambda(K) < 1$. Higher values of initial investments reduce the impact of the loss L so that $\lambda'(K) < 0$. Thus, $\lambda(K)$ may be interpreted as the *"loss reduction coefficient."* However, this reduction in loss is bounded from below (by zero) as the law of diminishing marginal productivity would imply. Thus without loss of generality one can assume that $\lambda^{"}(K) > 0$ and that $\underset{\lim k\to\infty}{\lambda(K)} \longrightarrow 0$, as shown below:

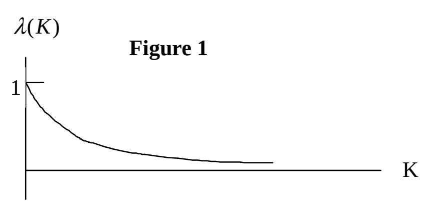


Figure 1

Many functional forms satisfy the above requirement. We shall return to this issue shortly.

If π_L is the probability of an event of magnitude L, then the expected loss in the absence of investment is

$$E(L|\pi_L)^{no\ invest} = \pi_L L \tag{1}$$

In the case of investing K the expected loss is:

$$E(L|\pi_L)^{invest} = \pi_L \lambda(K) L + K \tag{2}$$

Thus, while the event cannot be avoided, prior mitigating action can reduce the extent of loss from the event. Suppose the overall stream of profits from production is Π_o. Then expected profits from investing and non-investing in security measures are, respectively,

$$E(\Pi^{invest}) = \Pi_o - E(L|\pi_L)^{invest} = \Pi_o - \pi_L \lambda(K) L - K \tag{3}$$

$$E(\Pi^{no\ invest}) = \Pi_o - E(L|\pi_L)^{no\ invest} = \Pi_o - \pi_L L \tag{4}$$

The expected net gain from investing, given the probability π_L, is denoted by $G(\pi_L)$ and is found from

$$G(\pi_L) = E(L|\pi_L)^{\ invest} - E(L|\pi_L)^{no\ invest} = \pi_L[1 - \lambda(K)]L - K \tag{5}$$

Notice that $G(\pi_L)$ is $\gtreqless 0$ if $\pi_L \gtreqless \bar{\pi}$ where $\bar{\pi}$ is the threshold value of event probability and is given by $\bar{\pi} \equiv (K/L).(1/[1 - \lambda(K)]$. This implies that for events with very low probability, a firm could lose by investing K!

Catastrophic events, which are the focus of this work, are by nature low frequency high impact events. Developing accurate metrics of catastrophic risk is not easy. The challenge arises because of the need to extrapolate from observed levels in data to unobserved levels. Classical statistical methods are not well-suited for this task (Coles 2001, Embrechts et al. 1997). Instead, the appropriate approach is to estimate catastrophic risk using extreme value (EV) analysis. By exploiting limiting arguments extreme value models can provide an approximate description of the stochastic behavior of extremes. This is both discussed and measured in Mohtadi and Murshid (2006b, 2006c and 2006d). Thus, $\pi(L) \sim f(L)$,where $f(L)$ is an EV probability distribution function The expected gain from an event of that leads to a loss of size in the range L_o to L_1 is obtained from the EV probability distribution function such that,

$$G(\pi_{L_0<L<L_1}) = [1-\lambda(K)] \int_{L_o}^{L_1} f(L).L.dL - K \tag{6}$$

or,

Owing to the complexity of the term inside the integral, we will approximate G by replacing L inside the integral with its linear mean, $\bar{L} = (L_o + L_1)/2$.Thus, we have:

$$G(\pi_{L_0<L<L_1}) \cong [1-\lambda(K)]\bar{L} \int_{L_o}^{L_1} f(L).dL - K =$$
$$[1-\lambda(K)]\bar{L}[F(L|_{L>L_o}) - F(L|_{L>L_1})] - K \tag{7}$$

where $F(L)$ is the cumulative density function of L.

2.1 Optimum Investments

Maximizing the net gains from security investments, $\underset{(K)}{Max} G(\pi_{L_0<L<L_1})(K)$, leads to a value K^* that satisfies the first order condition below[5]:

$$\lambda'(K^*)\bar{L}[F(L|_{L>L_o}) - F(L|_{L>L_1})] = -1 \tag{8}$$

Deriving an explicit expression for optimum capital expenditures, K^*, depends on the functional forms used. But such functions must all satisfy the criteria discussed earlier. If we use the following form,

$$\lambda(K) = \frac{1}{1+\theta K^{\alpha}} \quad \theta > 0,\ 0 < \alpha < 1 \tag{9}$$

[5] The second order condition is satisfied since $\lambda" > \mathbf{0}$

then the requirement that $\lambda' < 0$, $\lambda'' > 0$, $\lambda(0) = 1$, and $\lambda(K)_{\lim k \to \infty} \longrightarrow 0$, are all satisfied. For this specification, the first order condition yields:

$$(1 + \theta K^{*\alpha})^2 = [F(L|_{L>L_o}) - F(L|_{L>L_1})]\bar{L}\theta\alpha K^{*\alpha-1} \tag{10}$$

We can use equation 10 along with 9 to both simplify the expression for K^* and also to substitute for the value of θ in terms of λ. The reason for the latter is that, as we shall see below, the empirical results from the "Benchmarking Survey" are closely associated with the values of λ rather than θ. Substituting from 9 into 10 we find:

$$\frac{1}{\lambda^2} = [F(L|_{L>L_o}) - F(L|_{L>L_1})]\bar{L}\theta\alpha K^{*\alpha-1} \tag{11}$$

In turn, we find from 9 that $\theta K^\alpha = (1 - \lambda)/\lambda$. Substituting this value into 11 we find the optimum level of investments K^* to be:

$$K^* = \alpha\bar{L}[F(L|_{L>L_o}) - F(L|_{L>L_1})]\lambda(1 - \lambda) \tag{12}$$

3 Estimation and Results

The cumulative probability values of $F(L|_{L>L_o})$ are based on data complied by Mohtadi and Murshid (2006a). This work, sponsored under a grant from Department of Homeland Security (DHS) compiles 448 incidents of Chemical Biological and Radionuclear (CBRN) attacks from 1960s to 2005. Mohtadi and Murshid (2006b and 2006d) then calculate the probability of any such attack worldwide of a given magnitude or larger (in terms of the number of injuries) for different time horizons. These probability values are presented in the third column of table 1 below. The fourth and the fifth column then convert this cumulative probability to the probability values $F(L|_{L_o<L<L_1})$ compatible with equation 7. Although any attack on the food sector will be of a CBRN nature, not every CBRN attack involves food. In fact, only about 60 of these incidents revolved around food (For a full chronology see Mohtadi and Murshid (2006a). Assuming a uniform distribution of CBRN attacks on food and non-food as our first prior, the above probabilities are adjusted in column five by the factor 60/448 or 0.134.

Table 1
Probabilities of a CBRN Attack of Various Magnitudes (based on Extreme Value Analysis) and Extrapolated Probabilities of Attacks on Food Sources

Number of casualties	Time Horizon	Cum. Probability of CBRN at various casualty levels	Probability of a CBRN with # of casualties between two levels		Adjusted probability for attacks on food
1000	Current risk	0.310	1000-5000	0.143	0.019
5000		0.167	5000-10000	0.034	0.005
10000		0.133	10000-15000	0.055	0.007
15000		0.078			
1000	5-year forecast	0.546	1000-5000	0.251	0.034
5000		0.295	5000-10000	0.071	0.009
10000		0.225	10000-15000	0.095	0.013
15000		0.130			
1000	10-year forecast	0.732	1000-5000	0.211	0.028
5000		0.520	5000-10000	0.110	0.015
10000		0.410	10000-15000	0.119	0.016
15000		0.291			
1000	20-year forecast	0.863	1000-5000	0.095	0.013
5000		0.768	5000-10000	0.057	0.008
10000		0.712	10000-15000	0.077	0.010
15000		0.634			

Source for column 3: Mohtadi and Murshid (2006b) and additional extrapolations

Next, the "levels of injury" must be translated into dollar loss. To do this, we rely on a simple conversion factor from the forensic literature, where the value of \$1 million is used for the loss of life. But since the above table includes injuries and mortalities, we will adjust the conversion factor to \$0.5 million for the case of injuries which by far dominate the data. The assumption is that the an injury or death will generate a liability for which the firm is ultimately responsible. We recognize that this is an overly simplified approach. For example, the economic impact on the firm might include such things as reputation, loss of future markets, etc., and this impact might be vastly greater than the liability loss associated with injury or death. But in defense of this approach, it may be pointed out that those effects are indirect effects whereas the focus at this point are direct effects. In any case, the result will be that the size of the expected gain (expected loss avoidance) from risk mitigating investments by firms may be significantly larger than the values obtained here and that our estimates will be only a conservative lower bound for what is actually needed.

Values of λ are estimated from the results of a survey based on another Nantional Center for Food Protection and Defense project (NCFPD), titled "Benchmarking Survey" led by Closs and Kinsey (2005-2007). This is done as follows: Consider the following survey question with a response rating from 1 (significantly reduced) to 5 (significantly increase) and 6 (N/A) (to be inserted in the blanks). From the survey we have the following sample of questions:

"Our firm's security investment has resulted in——security incidents."

Response to this question indicated a significant reduction in reported incidents by firms which have made security investments in the first place. While there is a possibility that an action taken by a firm could lead to the *perception* of success by a respondent, whether or not the actual number of incidents has declined, this issue cannot be settled in this paper. This is the subject of an ongoing work by Agiwal and Mohtadi (2007) that is based on Latent Trait Analysis, a method common in psychology, to decipher underlying traits in surveys. For now, however, we will assume that the results of the survey do in fact capture objectively the relation between security investments and the outcome of the investments, in terms of reduced risk. With this caveat, the results from Agiwal and Mohtadi (2007), when they are broken down into food manufacturers and food retailers yield the following means and variances as well as the associated value of λ. These value are entered based on the discussion that follows this table:

	mean	variance	associated λ
food manufacturing	1.72	.47	0.67
food retail	1.73	.46	0.67

It would be reasonable to assume that reductions in security incidents are linearly proportional to reductions in expected losses from such incidents. This is given by the expression $[1 - \lambda]L$. However, the mapping from the values in the survey to the actual values of λ are somewhat arbitrary. For the lack of a more rigorous criterion, we will assume that an answer of "1" to the survey question, i.e., significant reduction in the number of security incidents, implies a reduction in incidents by 2/3 ($\lambda = 2/3$) and that an answer of 2 implies a reduction by 1/2 ($\lambda = 1/2$). Interpolating, the mean value of 1.7 from the above table would imply that $\lambda \simeq .67$. This is entered in the last column of the table.

We now have sufficient information to estimate the optimum value of security investments a firm would need as self-insurance. For $\lambda = 0.67$, and for the various levels of injury and time horizons given and the associated probabilities in table 1 the optimum level of risk mitigating investments can be calculated from equation 12 (using a values of 0.5). The results are reported in the table below:

Table 2

Estimated Values of secuity investment for different loss levels and Time Horizons

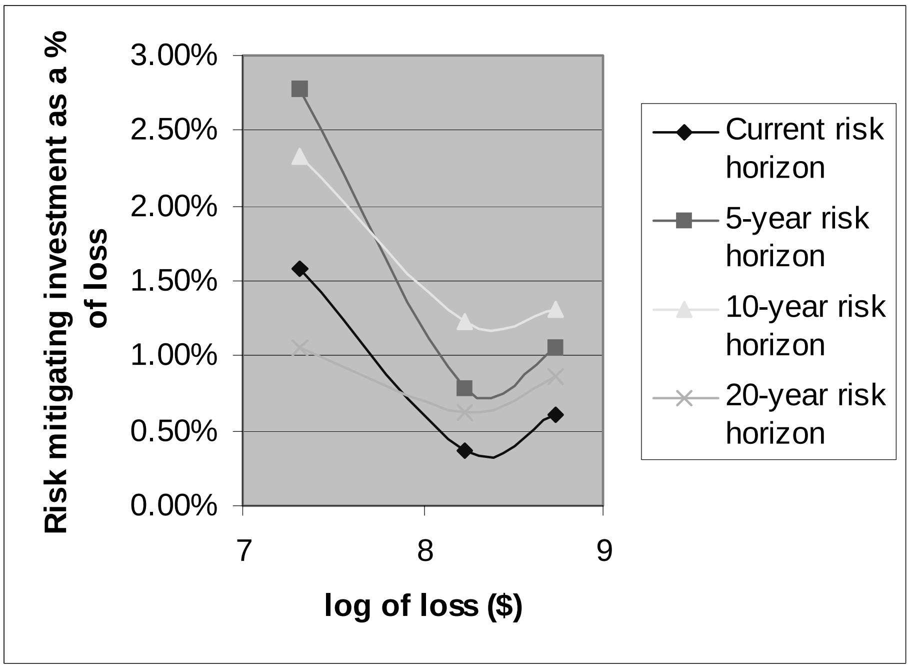
	Range of casualties Lo to L1	Mean # of casualties	monetary value of loss coming from injury (in $ million)	probability of loss between Lo and L1	Optimum investments K* in $ million	Optimum investments K* as % of loss
Current risk	1000-5000	3000	1,500.00	0.019	3.18	0.21%
	5000-10000	7500	3,750.00	0.005	1.87	0.05%
	10000-15000	12500	6,250.00	0.007	5.08	0.08%
5-year forecast	1000-5000	3000	1,500.00	0.034	5.57	0.37%
	5000-10000	7500	3,750.00	0.009	3.92	0.10%
	10000-15000	12500	6,250.00	0.013	8.79	0.14%
10-year forecast	1000-5000	3000	1,500.00	0.028	4.70	0.31%
	5000-10000	7500	3,750.00	0.015	6.13	0.16%
	10000-15000	12500	6,250.00	0.016	11.02	0.18%
20-year forecast	1000-5000	3000	1,500.00	0.013	2.10	0.14%
	5000-10000	7500	3,750.00	0.008	3.16	0.08%
	10000-15000	12500	6,250.00	0.010	7.15	0.11%

Source: Based on Table 1 and analysis in the text

Focusing on the last column of Table 2, a one-time capital investment aimed at mitigating extreme risk ranges from 37 cents per \$100.00 in a 10-year time-horizon, for a potential loss of 1.5 billion dollars, to 5 cents per \$100.00, in the current time-horizon, associated with a loss of 3.75 billion dollars. The larger percentage associated with the smaller loss seems puzzling at first. The key to the puzzle is the associated probabilities. The event with a higher risk mitigation cost has a probability of 3.4% while the event with the lower percentage cost has a probability of only 0.5 percent. Notice also that probabilities rise as time horizon increases! This is a important feature of the cumulative nature of the distribution in that with a longer time horizon, the low-frequency, high impact "tail" events become more likely, a feature of extreme value statistics that is not too dissimilar with the hazard function approach. Finally, a "fat tail" feature is also captured for the CBRN events associated with a higher probability (and thus loss mitigation investments) for an exceptionally extreme event of that would result in 10000 to

15000 injuries. This feature is frequently observed and is the consequence of the fact that extreme events, while rare indeed, are still more likely to occur than would be predicted by the tail end of a normal distribution. Figure 2 illustrated this upturn in the curve by showing the higher percent investments that are needed to protect against exceptionally extreme catastrophes.

Figure 2



4 Comparison with Catastrophic Insurance: Concluding Remarks

How do expenditures aimed at risk mitigation compare with the purchase of catastrophic risk insurance? To gain some insights into the answer to this question consider the fact that in 2002 the Insurance Service Office assigned insurance cost of approximately 10 cents per $100 loss for the highest risk cities, but after discussions with the regulators, these rates were later adjusted downwards to less than 3 cents per $100 of loss (See Auerswald, et. al. 2006, pp. 283). This suggests that loss mitigation costs are as high or higher than the cost of catastrophic insurance. While risk mitigation and insurance are not entirely comparable strategies due to continued risk exposure under risk mitigation strategies, still the high deductibles of catastrophic insurance make the two approaches more comparable than might appear at the first glance. In this respect, several points need to be stressed.

First, risk mitigation investments are one-time actions. For example, a one time capital expenditure of 5 cents per $100 from table 2 to mitigate the risk of an event that could cause $3.75 billion loss in the current time horizon, pays off compared to the purchase of catastrophic insurance after the fifth year. The key of course is that risk mitigation is not full proof so that risk is not eliminated, but only reduced. As mentioned, however, this fact needs to be balanced against the fact that catastrophic risk insurance often entails large deductibles and, therefore, at least up to the deductible level, risk mitigation expenditures remain relevant whether or not catastrophic insurance is purchased.

Second, the availability of catastrophic insurance has become more limited after 9/11 attacks, due to a variety of complicating factors, including (a) the increased risk trend (for evidence see Mohtadi and Murshid 2006d; for discussion see Bogen and Jones, 2006), (b) the large size of the losses, requiring far greater pooling of resources than has been available to insurers and reinsures, (c) the uncertainties associated with calculating the probabilities of catastrophic insurance[6] and (d) the asymmetric information between insurance firms and the insured on the one hand, and the reinsurance firms and insurance firms on the other, resulting in a double moral hazard problem (Auerswald, ibid)

Third, as stated previously, issues such as reputation effects cannot be overlooked. These affects cannot be easily addressed by the purchase of insurance, especially if they are the outcome of inadequate risk mitigation by firms in the first place.

Fourth, the adequate coverage of catastrophic insurance together with the large potential losses and the associated "risk externalities" call for the involvement of the Federal, state and local governments (see Auerswald, et. al. 2006, pp. 284), as this becomes a clear case of market failure. In fact the Terrorism Risk Insurance Act (TRIA) and its eleventh hour extension in December of 2005 is aimed at addressing the gaps in this market. However, many obstacles remain and catastrophic insurance remains a highly imperfect tool with limited or no availability in many cases. Thus, until and unless these issues are resolved risk mitigation becomes a important and essential strategy.

[6] On this point, we hope that this contribution will be of value. We argue that terrorism risk is quantifiable in the same manner as weather risk may be.

Fifth, while risk mitigation is not well correlated with insurance costs, due to moral hazard problems associated with asymmetric information between the insured and the insurer (again see Auerswald, et. al. 2006, pp. 283), it is likely that catastrophic risk mitigation will have a positive effect on the cost of risk financing, if not risk insurance. For example, Moody's risk ratings are highly affected by the ability of firms to prepare for and respond to risk. For all these reasons the importance of catastrophic risk mitigation strategies cannot be overstated and such strategies must be an essential part of firms' overall strategy.

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