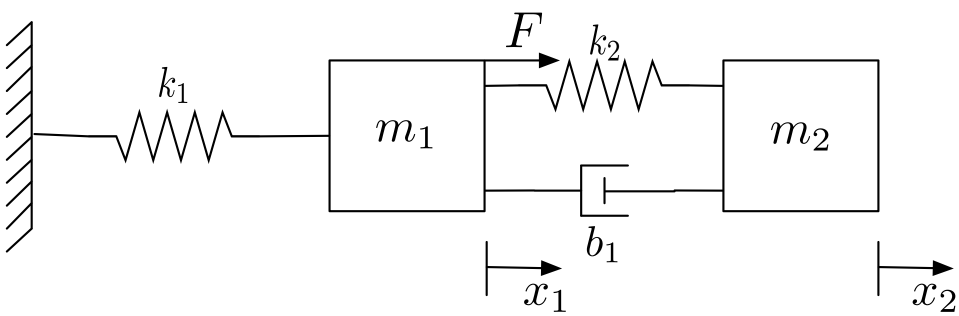
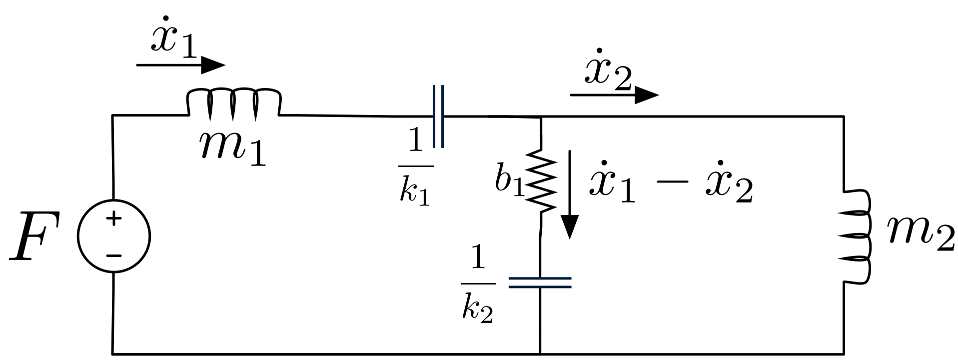
Sam Kendig
6.777/2.751J - Design and Fabrication of Microelectromechanical Devices
Problem Set 3

1. The mechanical system:



This can be transformed into the following equivalent circuit:



2.

$$\dot{x}_2 = m_2 s \frac{(b_1+\frac{k_2}{s})||m_2 s}{m_1 s+\frac{k_1}{s}+((b_1+\frac{k_2}{s})||m_2 s)} F$$

$$\frac{x_2(s)}{F(s)} = \frac{b_1 s+k_2}{m_1 m_2 s^4+(m_1+m_2)b_1 s^3+(m_1 k_2+m_2(k_1+k_2))s^2+k_1 b_1 s+k_1 k_2}$$

3. (a)

$$C = \frac{2\epsilon t(l-g)}{h}$$

$$W(Q,g) = \frac{Q^2}{2C} = \frac{Q^2 h}{4\epsilon t(l-g)}$$

$$dW(Q,g) = VdQ + Fdg$$

$$F = \frac{dW}{dg}|_Q = \frac{Q^2 h}{4\epsilon t(l-g)^2}$$

$$V = \frac{dW}{dQ}|_g = \frac{Qh}{2\epsilon t(l-g)}$$

$$g = g_0 - z = g_0 - \frac{F}{k} = g_0 - \frac{Q^2 h}{4\epsilon k t(l-g)^2}$$

(b)

- Motion is only in the z direction.
- We can ignore fringe capacitance and the capacitance across the gap g.

(c)

$$W^*(V,g) = QV - W(Q,g)$$

$$dW^*(V,g) = QdV + VdQ - (VdQ + Fdg) = QdV - Fdg$$

$$W^*(V,g) = \frac{1}{2}CV^2 = \frac{\epsilon t(l-g)}{h}V^2$$

$$F = -\frac{dW^*}{dg} = \frac{\epsilon t}{h}V^2$$

$$Q = \frac{dW^*}{dV} = \frac{2\epsilon t(l-g)}{h}V$$

$$g = g_0 - z = g_0 - \frac{F}{k} = g_0 - \frac{\epsilon t}{hk}V^2$$

(d)

$$F_{net} = k(g_0 - g) - \frac{\epsilon t}{h}V^2$$

$\frac{\partial F_{net}}{\partial g} = -k < 0 \ \forall \ (g,V)$, so no spring softening.

(e)

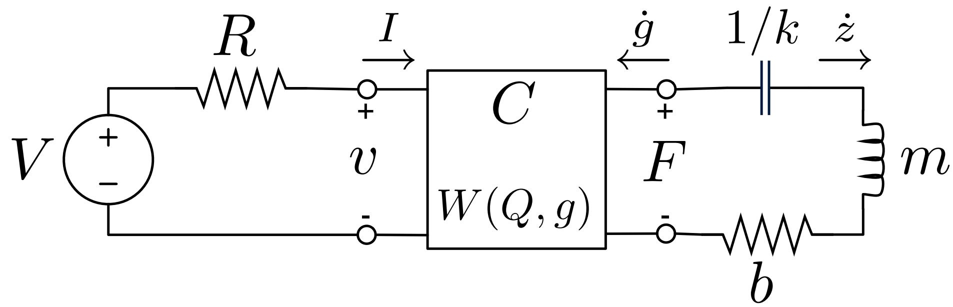
$$F_{net} = k(g_0 - g) - \frac{Q^2 h}{4\epsilon t(l-g)^2}$$

$$\frac{\partial F_{net}}{\partial g} = -k - \frac{Q^2 h}{2\epsilon t(l-g)^3}$$

$\frac{\partial F_{net}}{\partial g} < 0$ for $g < l$.

The spring will soften, but will not experience pull-in because the system is always stable (when $g > l$, our assumptions used to determine the capacitance, and thus the forces, break down due to the lack of any overlap of the plates).

4. (a)



(b)

$$\frac{d}{dt}\begin{bmatrix} Q \\ g \\ \dot{g} \end{bmatrix} = \begin{bmatrix} \frac{1}{R}\left(V - \frac{Qh}{2\epsilon t(l-g)}\right) \\ \dot{g} \\ -\frac{1}{m}\left(\frac{Q^2h}{4\epsilon t(l-g)^2} - k\left(g_0 - g\right) + b\dot{g}\right) \end{bmatrix}$$

(c)

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{R}\left(V - \frac{Qh}{2\epsilon t(l-g)}\right) \\ \dot{g} \\ -\frac{1}{m}\left(\frac{Q^2h}{4\epsilon t(l-g)^2} - k\left(g_0 - g\right) + b\dot{g}\right) \end{bmatrix}$$

$V = \frac{Qh}{2\epsilon t(l-g)}$

$\frac{Q^2h}{4\epsilon t(l-g)^2} = V^2\frac{\epsilon t}{h} = k(g_0 - g)$

$g = g_0 - \frac{\epsilon t V^2}{hk}$

$V = 100 \rightarrow g = 4.99\mu\text{m}$

This value is not changed when considering 200 fingers, as the damping factor b is not involved in calculating the static gap. This makes sense intuitively, as the damping should not influence the static case.