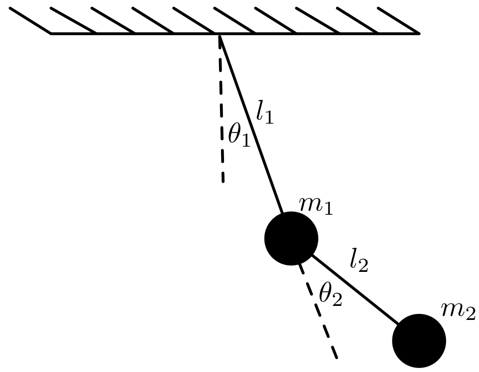
Sam Kendig

6.946j/8.351j/12.620j - Structure and Interpretation of Classical Mechanics

Project 1



$$x_1 = l_1 \sin\theta_1 \qquad y_1 = -l_1 \cos\theta_1$$
$$v_{x1} = l_1 \dot{\theta}_1 \cos\theta_1 \qquad v_{y1} = l_1 \dot{\theta}_1 \sin\theta_1$$
$$v_1 = l_1 \dot{\theta}_1$$

$$x_2 = x_1 + l_2 \sin(\theta_1 + \theta_2) \qquad y_2 = y_1 - l_2 \cos(\theta_1 + \theta_2)$$
$$v_{x2} = v_{x1} + l_2(\dot{\theta}_1 + \dot{\theta}_2)\cos(\theta_1 + \theta_2) \qquad v_{y2} = v_{y1} + l_2(\dot{\theta}_1 + \dot{\theta}_2)\sin(\theta_1 + \theta_2)$$
$$v_2^2 = l_1^2\dot{\theta}_1^2 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2l_1 l_2 \dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)(\cos(\theta_1)\cos(\theta_1 + \theta_2) + \sin(\theta_1)\sin(\theta_1 + \theta_2))$$
$$v_2^2 = l_1^2\dot{\theta}_1^2 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2l_1 l_2 \dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)\cos\theta_2$$

$$T = \tfrac{1}{2} m_1 l_1^2 \dot{\theta}_1^2 + \tfrac{1}{2} m_2 (l_1^2\dot{\theta}_1^2 + l_2^2(\dot{\theta}_1 + \dot{\theta}_2)^2 + 2l_1 l_2 \dot{\theta}_1(\dot{\theta}_1 + \dot{\theta}_2)\cos\theta_2)$$
$$V = -m_1 g l_1 \cos\theta_1 - m_1 g(l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2))$$

$$L = T - V$$

Code:

```
(define ((T-dual-pend m1 m2 g) local)
  (let ((v (velocity local)))
    (let ((v1 (ref v 0))
          (v2 (ref v 1)))
      (+ (* 1/2 m1 (square v1))
         (* 1/2 m2 (square v2))))))

(define ((V-dual-pend m1 m2 g) local)
  (let ((pos (coordinate local)))
    (let ((y1 (ref pos 0 1))
          (y2 (ref pos 1 1)))
      (+ (* m1 g y1) (* m2 g y2)))))

(define L-dual-pend (- T-dual-pend V-dual-pend))

(define ((dual-pend-coords l1 l2) local)
  (let ((theta (coordinate local)))
    (let ((theta1 (ref theta 0))
  (theta2 (ref theta 1)))
      (let ((x1 (* l1 (sin theta1)))
    (y1 (* -1 l1 (cos theta1))))
(let ((x2 (+ x1 (* l2 (sin (+ theta1 theta2)))))
      (y2 (- y1 (* l2 (cos (+ theta1 theta2))))))
  (up (up x1 y1) (up x2 y2)))))))

(define (L m1 m2 l1 l2 g)
  (compose (L-dual-pend m1 m2 g)
   (F->C (dual-pend-coords l1 l2))))

(define (dual-pend-state-deriv m1 m2 l1 l2 g)
  (Lagrangian->state-derivative (L m1 m2 l1 l2 g)))


(define theta1-list '())

(define ((monitor-theta1 win) state)
  (let ((theta1 ((principal-value :pi) (ref (coordinate state) 0))))
    (set! theta1-list (cons theta1 theta1-list))
    (plot-point win (time state) theta1)
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```
    state))

(define theta1-win (frame 0. 50. :-pi :pi))

(define theta2-list '())

(define ((monitor-theta2 win) state)
  (let ((theta2 ((principal-value :pi) (ref (coordinate state) 1))))
    (set! theta2-list (cons theta2 theta2-list))
    (plot-point win (time state) theta2)
    state))

(define theta2-win (frame 0. 50. :-pi :pi))

(define energy-list '())

(define (energy m1 m2 l1 l2 g)
  (compose ((+ T-dual-pend V-dual-pend) m1 m2 g)
   (F->C (dual-pend-coords l1 l2))))

(define energy-win (frame 0. 50. -1e-10 1e-10))

(define ((monitor-energy win) state)
  (let ((e ((energy 1.0 3.0 1.0 0.9 9.8) state)))
    (set! energy-list (cons e energy-list))
    (plot-point win (time state) e)
    state))

(define (monitor theta1-win theta-2win energy-win)
  (compose (monitor-theta1 theta1-win)
   (monitor-theta2 theta2-win)
   (monitor-energy energy-win)))

((evolve dual-pend-state-deriv
 1.0 3.0 1.0 0.9 9.8)
 (up 0.0 (up :pi/2 :pi) (up 0. 0.))
 (monitor theta1-win theta2-win energy-win)
 0.125
 50.0
 1.0e-13)
```

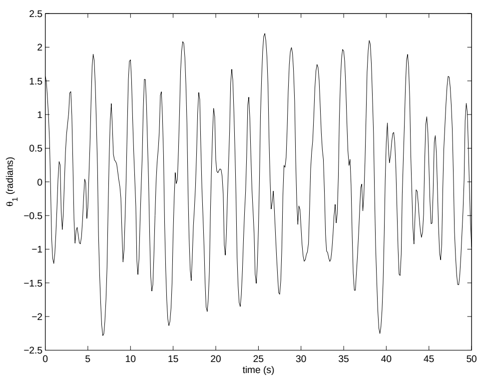
Initial Conditions:

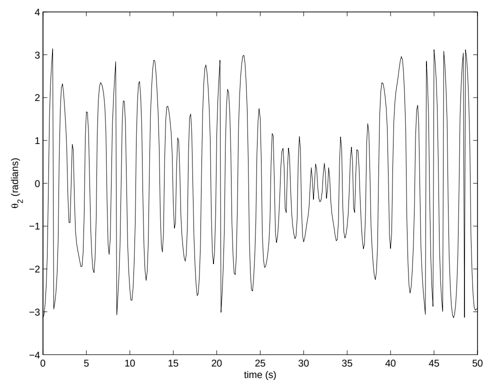
$\theta_1(0) = \frac{\pi}{2}$

$\theta_2(0) = \pi$

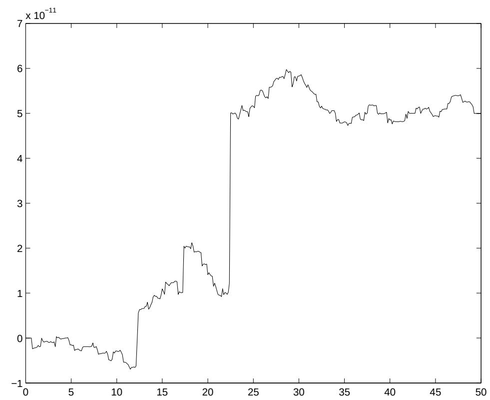
$\dot{\theta}_1(0) = 0$

$\dot{\theta}_2(0) = 0$

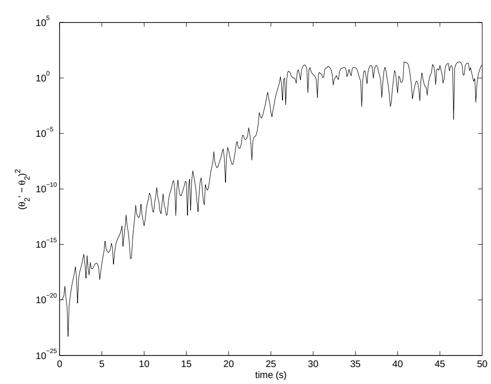




The two graphs above show the two pendulum angles over the 50 second experiment.



The above graph shows the total energy of the system as a function of time. It stays below $10^{-}10$ J, which is a reasonable conservation. The error is induced from errors accrued in numerical integration.

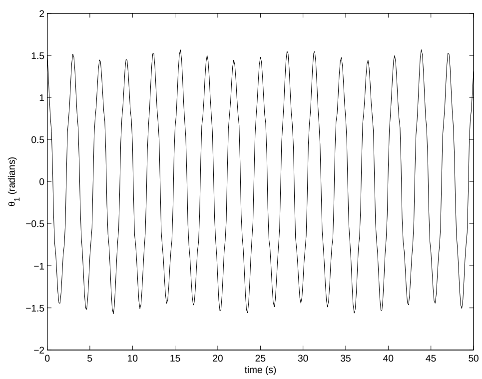


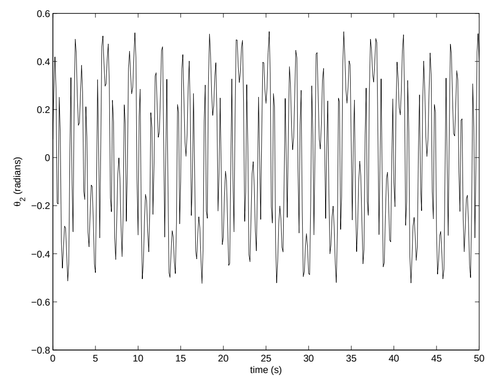
The squared difference between θ_2 in two runs with initial conditions varying by an angstrom is shown above. The log of the error increases roughly linearly with time, until reaching its maximum error of

π^2. This display shows that the system is in its chaotic regime, where small disturbances to the initial conditions produce large discrepancies relatively quickly. In this example, it takes only 30 seconds to be completely chaotic.

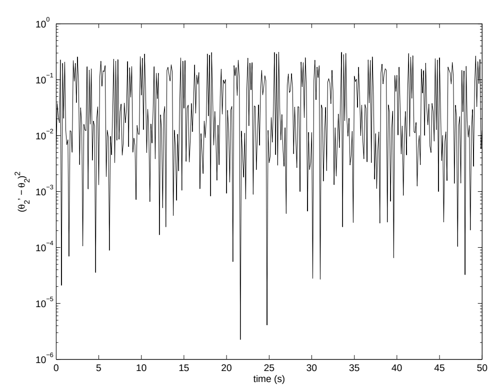
Initial Conditions:

$\theta_1(0) = \frac{\pi}{2}$
$\theta_2(0) = 0$
$\dot{\theta}_1(0) = 0$
$\dot{\theta}_2(0) = 0$





With differing initial conditions, the dual-pendulum system acts almost periodically. The first pendulum has roughly sinusoidal displacement, while the lower pendulum has smaller periodic oscillations.



When varying these sets of initial conditions slightly, we find very little difference in the resulting motion, showing a stable region of the dual pendulum's behavior. The error terms are small, and do not grow with time.