Sam Kendig
6.946j/8.351j/12.620j - Structure and Interpretation of Classical Mechanics
Problem Set 1

- Exercise 1.2: Degrees of freedom

  a) Three juggling pins: 18
  (Each juggling pin has a full 6 degrees, so 3 have 18)

  b) Spherical pendulum: 2
  (Position can be determined as θ, ϕ from vertical, distance is fixed)

  c) Spherical double pendulum: 4
  (Same as above, with second mass determined by two angles off of first pendulum)

  d) Point mass on rigid curved wire: 1
  (Position given by distance from an end of the wire)

  e) Axisymmetric top with fixed tip: 3
  (Tip position is fixed, configuration is the two angles of the axis from vertical, and a rotation angle about the axis)

  f) Non-axisymmetric top with fixed tip: 3
  (Same as above, symmetry does not reduce degrees of freedom)

- Exercise 1.3: Generalized coordinates

  a) Three juggling pins: $(x_i, y_i, z_i, \theta_i, \phi_1, \omega_i)$ for i goes from 0 to 2
  (x, y, z are cartesian coordinates of the base of each pin, while θ, ϕ, ω are angles of rotation with respect to an axis)

  b) Spherical pendulum: (θ, ϕ)
  (Two angles from vertical axis)

  c) Spherical double pendulum: $(\theta_0, \phi_0, \theta_1, \phi_1)$
  (θ_0, ϕ_0 determine angle from first pendulum to vertical, θ_1, ϕ_1 determine axis from second pendulum to extension of the first pendulum)

  d) Point mass on rigid curved wire: s
  (s is the distance along the curved wire measured from a fixed point at the end)

  e) Axisymmetric top with fixed tip: (θ, ϕ, ω)
  (θ, ϕ determine angle from the vertical axis, ω is angle of rotation)

  f) Non-axisemmetric top with fixed tip: (θ, ϕ, ω)
  (θ, ϕ determine angle from the vertical axis, ω is angle of rotation)

- Exercise 1.4: Lagrangian actions

  Given the Lagrangian $L(t, x, v) = \frac{1}{2}mv^2$, with x being the constant-velocity path of a particle, show that the action on the solution path is $\frac{m}{2}\frac{(x_b - x_a)^2}{t_b - t_a}$:

  $S_x[q](t_a, t_b) = \int_{t_a}^{t_b} L_x \circ \Gamma[q]$
  $S_x[q](t_a, t_b) = \int_{t_a}^{t_b} \frac{1}{2}mv^2 dx$
  $v = \frac{x_b - x_a}{t_b - t_a}$
  $S_x[q](t_a, t_b) = \frac{m}{2}\frac{(x_b - x_a)^2}{(t_b - t_a)^2}(x_b - x_a)$
  $S_x[q](t_a, t_b) = \frac{m}{2}\frac{(x_b - x_a)^2}{(t_b - t_a)}$

- Excercise 1.5: Solution process

  ```
  (define win2 (frame 0.  :pi/2 0.  1.2))
  ```

  ```
  (define ((parametric-path-action Lagrangian t0 q0 t1 q1)
  intermediate-qs)
  (let ((path (make-path t0 q0 t1 q1 intermediate-qs)))
  ;;display path
  (graphics-clear win2)
  (plot-function win2 path t0 t1 (/ (- t1 t0) 100))
  ;;compute action
  (Lagrangian-action Lagrangian path t0 t1)))
  ```

  ```
  (find-path (L-harmonic 1.  1.)  0.  1.  :pi/2 0.2)
  ```

  The above code draws the test path in each iteration until it finds the local minimum of the Lagrangian.