CrowdMath: Massive Research Collaboration among High School and College Students

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Enabling Mathematical Cultures
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Room 4-370, MIT
Open to the public
Conference abstracts booklet
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Saturday, May 20
Mathematics

Prof. Pavel Etingof and Dr. Tanya Khovanova with PRIMES students

0:20 am Welcoming Remarks
Prof. Tomasz Mrowka, Head of the MIT Mathematics Department
Prof. Pavel Etingof, PRIMES Chief Research Advisor
Dr. Slava Gerovitch, PRIMES Program Director
Is massively collaborative mathematics possible?

Of course, one might say, there are certain kinds of problems that lend themselves to huge collaborations. One has only to think of the proof of the classification of finite simple groups, or of a rather different kind of example such as a search for a new largest prime carried out during the downtime of thousands of PCs around the world. But my question is a different one. What about the solving of a problem that does not naturally split up into a vast number of subtasks? Are such problems best tackled by $n$ people for some $n$ that belongs to the set $\{1, 2, 3\}$? (Examples of famous papers with four authors do not count as an interesting answer to this question.)

It seems to me that, at least in theory, a different model could work: different, that is, from the usual model of people working in isolation or collaborating with one or two others. Suppose one had a forum (in the non-technical sense, but quite possibly in the technical sense as well) for the online discussion of a particular problem. The idea would be that anybody who had anything whatsoever to say about the problem could chip in. And the ethos of the forum — in whatever form it took — would be that comments would mostly be kept short. In other words, what you would not tend to do, at least if you wanted to keep within the spirit of things, is spend a month thinking hard about the problem and then come back and write ten pages about it. Rather,
A new proof of the density Hales-Jewett theorem

D. H. J. Polymath

(Submitted on 20 Oct 2009 (v1), last revised 16 Feb 2010 (this version, v2))

The Hales-Jewett theorem asserts that for every r and every k there exists n such that every r-colouring of the set \{1,...,n\} contains a combinatorial line of k elements. In particular, we show that a subset of size \((1/2+\epsilon)n\) contains a combinatorial line. In this paper, we give the first elementary proof of the theorem of Furstenberg and Katznelson in 1991 by means of a significant extension of the ergodic proof of Szemeredi's theorem. In particular, we show that a subset of size \((1/2+\epsilon)n\) contains a combinatorial line.
“Was Polymath1 a success? From a mathematical point of view, the answer is unequivocally, yes. ... Whether or not the project was a success as an experiment in “massive-scale” collaboration, requires a more nuanced discussion” (Cranshow & Kittur, “The Polymath Project: Lessons from a Successful Online Collaboration in Mathematics,” 2011)
“The idea would be that anybody who had anything whatsoever to say about the problem could chip in. ... you would contribute ideas even if they were undeveloped and/or likely to be wrong.”

Tim Gowers

“...one significant cultural inhibitor to having contributions (particularly the crazy “blue sky” contributions) to these sorts of projects by professional mathematicians is that, as a whole, we are quite reluctant to say anything on the public record which may end up being wrong, foolish, or naive, lest this damage a hard-earned mathematical reputation.”

Terry Tao
CrowdMath is an open project that gives all high school and college students the opportunity to collaborate on a large research project with top-tier research mentors and an exceptional peer group. MIT PRIMES and Art of Problem Solving are working together to create a place for students to experience research mathematics and discover ideas that did not exist before.

CrowdMath Projects

<table>
<thead>
<tr>
<th>Project Name</th>
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<tbody>
<tr>
<td>MIT PRIMES 2018: Pattern Avoidance</td>
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<tr>
<td>MIT PRIMES 2017: Graph Algorithms and Applications</td>
</tr>
<tr>
<td>MIT PRIMES 2017: The Broken Stick Problem</td>
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With respect, kids are the last people that this should be tested on. After all, the complexity of the problem "we" are talking about is *well* beyond there [sic] capability. And since with an increase in complexity, the dynamic of how one needs to work with others changes (not to mention the problem itself), any simple task that the kids might be given wouldn’t yield any usable information about the productivity of this method. At best it’d produce a false positive/negative.

the best way to test what you guys are talking about is do it with kids and see what happens... set out problems and open source the solutions could produce interesting results 😊
Expected Area of Largest Perimeter

Problem 6a
Radioactive
Oct 16, 2017 by v_enhance

The square \([0,1]^{2}\) is cut into several regions by picking two random points on the border of the square. What is the expected area of the largest piece of perimeter which has not been interrupted by random lines? What is the probability that no piece is longer than \(x\)?

We're playing the game on the unit square, which obviously has area \(4\). A lower bound is to define an uninterrupted piece of perimeter that we can call \(p\) (make it the longest piece). Then we say that for \(n\) random lines the probability that \(p\) is interrupted is approximately \(2p/4\) and that the expected value of \(p\) when intersected is obviously \(p/2\).

This is obviously not a super great lower bound but it should suffice as a start.
Polymath1 (2009)
- Solved the problem
- Did not fundamentally transform the way mathematicians collaborate
- Required high-profile leadership
- Collective credit was an issue
- Reproduced the hierarchies, virtues, and vices characteristic of the math community
- Lacked informality, exploratory aspects, experimentation, play, empathy, comradery (cf. Lorenzo Lane’s talk)

CrowdMath1 (2016)
- Would high school students be able to solve a real research problem collectively?
- What would motivate them to participate?
- Would they accept collective authorship?
- What kind of collaboration patterns would they display?
- Would their collaboration dynamics differ from that of Polymath?
Here are the final Crowdmath 2016 standings. College students are not included in the standings. If you have any questions about your score, please send a message to jgeneson.

Final Crowdmath 2016 Standings

igamgau: 146
abk2015: 73
adityaguharoy: 68
talkon: 52
Th3Prob13mSolv3r: 40
hkumar8200: 22
M5_1792: 15
msinghal: 15
StarFrost7: 15
GovAzul: 15
Polymath1 (2009)

- Professional mathematicians
- Led by Fields medalists
- 39 contributors
- 74% of contributors use true identity
- 3% are female
- High barrier for entry
- Typical blog post: 10 commenters, 100 comments in one week
- Three papers on arXiv.org under the pseudonym D.H.J. Polymath

CrowdMath1 (2016)

- High school students
- Led by grad students and undergrads
- 35 contributors
- 3% of contributors use true identity
- 14% are female
- Classes for newcomers
- Typical forum topic: 25 commenters, 75 comments in several weeks
- Three papers on arXiv.org under the pseudonym P.A. CrowdMath
**Polymath1 (2009)**

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**CrowdMath1 (2016)**

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Motivation

How important is each of the following for your motivation to participate in CrowdMath?

- Open problems
- Learning advanced math
- Students like you
- Access to math resources
- PRIMES entry
- Feedback from mentors
- No criticism

Not at all  Very little  Not sure  To some extent  Extremely important
Motivation Conclusions

- Solving open problems is the greatest motivation
- Discussion with like-minded students is more motivating than feedback from mentors
- Significance of ranking and prize winning is downplayed
- Substantial progress toward the solution and publishing a collective paper are motivating even if it was not due to the student’s personal contribution
Collaboration patterns

To what extent each of the following inspires your thinking on CrowdMath problems?

- My independent study
- Mentors' posts and comments
- Other students' posts
- Talking to outside people

0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%

Not at all  Very little  Not sure  To some extent  To a great extent
Mentor/student dynamics

How active do you think the mentors' involvement in a CrowdMath discussion should be?

- As low as possible
- Somewhat low
- Not sure
- Somewhat active
- As active as possible

Desired role of mentor

0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
Other students’ posts is a greater source of inspiration than mentor’s comments.

Students prefer somewhat reducing the activity of the mentor in favor of inter-student communication.

It’s possible to feel the importance of personal contribution even if visiting the forum only once or twice a week.

Those who feel they contribute a lot to the discussion visit the forum most frequently.
Outcomes

To what extent did CrowdMath help you learn or acquire the following skills?

- How research is done
- Solving problems together
- New math concepts
- Thinking out of the box
- New techniques
- Self-confidence
- Working independently
- Math intuition
- Making conjectures

- Not at all
- Very little
- Not sure
- To some extent
- To a great extent
Do you want to become a research mathematician?

- Definitely: 43%
- Probably: 36%
- No/Not sure: 21%
How did participation in CrowdMath affect your intention of becoming a professional mathematician?

<table>
<thead>
<tr>
<th>Want to be a mathematician?</th>
<th>Intention decreased substantially</th>
<th>Somewhat decreased</th>
<th>Didn't influence at all</th>
<th>Somewhat increased</th>
<th>Increased Substantially</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yes/Probably</td>
<td></td>
<td></td>
<td>18%</td>
<td>46%</td>
<td>36%</td>
</tr>
<tr>
<td>No</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>100%</td>
</tr>
</tbody>
</table>
Outcomes Conclusions

- Students perceive the forum not only as a place where they can learn “how math research is done” but also how to work with other students.
- Forum participation encourages students to pursue the career of a professional mathematician.
Collaboration patterns

- Interviews with mentors
- Interviews with students
Mentor/student dynamics

- I never say “I teach” or “I mentor” - I always say we discuss or work together (mentor A).
- We try not to be the main people talking. We try to make it so that students are talking to each other, rather than we are talking to them (mentor B).
- Open problems are just so difficult. Sometimes mentors don’t know where to go (mentor C).
- The main thing that I try to train is how you make progress on work on the problems with very little guidance (mentor D).
Informal discussion

- Compared with more conventional research groups, it’s less organized. Less controllable. Because as you see, everybody can come and they came. They can pose any ideas they have (mentor C).

- I feel that my contribution was mostly asking questions and making conjectures. I feel that most of my contribution was not rigorous mathematics in nature, but rather providing ideas that others built upon. It was always a humbling experience when someone would provide simple examples to questions I ask, or provide a short proof of something I conjectured (student B).

- From the posts, I think people are not reading their own posts. Before you post them (student F).
“Friendly environment”

- I don’t think there is any criticism (mentor A).
- You can just sit there and no one knows you are there. It’s also an open forum. You can try new ideas. It feels more open to your ideas (student A).
- What I like about CrowdMath most is its anonymity, the fact that people using usernames. I don’t have to present myself as a college student and a woman. Less judgment is attached to it. I really appreciate the friendly environment on CrowdMath (student C).
People can see their ranking, and I guess it kind of motivates participants to post more (mentor A).

I don’t know if the reward points make a difference (mentor B).

Honestly, I don’t think it really does anything (mentor C).

The number of posts is always consistent with ranking. There are a lot of one-sentence or even one-word. ... Encouraging the number of posts indirectly encourages posts of lesser quality (student D).

I believe the ranking system is fun, and once I was very proud of myself when I got third on ranking (student D).

Although it helped drive me to contribute for the sake of wanting to maintain the ranking, I felt that it was intimidating to those beginning to contribute to the project (student B).
Collective authorship

- One thing I was really excited about is originally the paper would have been published under [the pseudonym P.A. CrowdMath] (student A).
- Working on the paper together was certainly ... my best moment (student A).
- Overall, however, I don’t think it would have made much of a difference whether or not I was co-author, because I definitely gained more than having another paper I can list on a CV (student B).
Learning research skills

- Over time I post more ideas and approaches. ... Most of my posts still contain proofs. But now I have more posts with conjectures (student A).

- My participation last year initially was to summarize already known results and solutions. Later, I started to publish what I presumed were proofs, and then somebody challenged my posts. I also challenged other people’s posts (student C).

- CrowdMath taught me that it was okay to post something and then have someone point out that it was completely wrong. I learned that it was okay to post ideas even if they were just ideas and I had no idea what to do with them. It was okay to just ask questions without providing any answers, and that it was okay to post if I didn’t understand something (student B).
Trying on the identity of a mathematician

- CrowdMath is unique to me because it's the only mathematics research I've ever done... it's like wading in the kiddy pool your whole life and then getting pushed into the deep end (student E).

- There is a lot more in math than learning new concepts. ... CrowdMath is nice in a sense it feels good to discover things that are actually new instead of doing other people’s development (student A).

- Research math – when you are proving something, you don’t fully know if it’s true or not. ... It’s a lot more about experimenting and playing with things over time. And getting a really good feel and understanding of what you are doing (student A).

- Contributing to a research project made the idea of doing research less intimidating, and made a career in research something I can seriously consider (student B).
Polymath1 (2009)
- Did not fundamentally transform the way mathematicians collaborate
- Reproduced the hierarchies, virtues, and vices characteristic of the math community
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CrowdMath
- Participants themselves are deeply transformed – from being a mere learner to being a researcher
- Collaboration is valued over competition
- Collective rewards are valued over individual rewards
- Creates the sense of open, informal, non-critical discussion, creative exploration and experimenting in a friendly environment
- Might serve as a model for future collaboration?