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EXAMINING THE INTERLOCKING STRUCTURE OF THE CORPORATE NETWORK

An Application of a High-Density Clustering Model on a Graph

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James M. Lattin

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Principal Investigator: Stuart Madnick

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I am indebted to Professor Joel Levine of Dartmouth College for his help in obtaining the interlock data from the BARON archive, and for his energetic and insightful guidance throughout my association with the corporate network.

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This report describes the use of a high-density clustering technique to examine the interlocking structure of a social network formed by the overlapping memberships of the 250 largest industrial and financial corporations in the United States. The technique, developed to aid systems designers partition complex design problems into well-defined, manageable subproblems, is conceptually appealing and useful for examining very large problems. The report also compares the clustering trace of regions of "high-density interlocking" in the network provided by the technique to the clustering trace of a model network of "random" interlocking, and finds them very similar.

EXECUTIVE SUMMARY

The directors shared by the boards of the *Fortune* 800 define a set of "corporate interlocks" which enables the investigator to view the corporate data as a large network. In past analyses, investigators have treated the interlock as a similarity measure, and have sought to picture the corporate network in some Euclidean space of minimal dimensionality, in order to better understand its properties and explain the structure of this large social network. Two weaknesses beset this approach: one, a multidimensional scaling technique may not provide an appropriate representation of the corporate network and two, there has been no attempt to determine the extent to which the interlocking patterns in the network represent a purposeful, socially significant phenomenon.

In this analysis, we investigate the interlocking phenomenon among corporations using a high-density clustering model defined on a graph. The model enables us to locate the regions of "high-density interlocking" in the corporate network: regions where any group of firms is quite heavily interlocked, and where any one firm in the group is not highly linked outside the group. Using this model, we form the tree of high-density clusters, using a very rapid and computationally efficient maximal spanning tree algorithm, and examine the patterns and structure of corporate interlocking. Our preliminary results indicate that the high-density clustering model is conceptually appealing, requiring much less time and computational expense than the methods traditionally employed.

Also, we use the high-density clustering technique to examine the structure of a "random" interlock network, where corporations choose directors from different groups without regard to their individual identity or corporate memberships. The results indicate that clustering structures for the two networks (based on the interlock density measure) are not appreciably different.

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1. Introduction

In this report, we critically assess some past approaches to the analysis of a specific social network, the network of interlocking corporate directorates in the U.S. Interlocking corporate directorates, where one or more individuals are common to the boards of any two companies, have long been a concern of sociologists interested in understanding the structure of large social networks. The problem has been a long-standing one due to the intractability of the data: the largest industrial and financial corporations in the United States form an interlock network not only vast in size but almost entirely connected. Mariolis and Schwarz (Mariolis[1975]) compiled an archive of the 797 largest U.S. corporations as reported in the May 1970 issue of Fortune magazine. Of these firms, there are 62 isolates (firms not sharing a director with any of the other 797), four isolated pairs, and one isolated triple. The remaining 724 corporations form a single, interconnected network with over 4000 links. If it were true that the interlocked corporations formed several smaller, unconnected network components, then we might be able to focus our analysis on one considerably less complex component at a time. Without such good fortune, the problem of understanding the phenomenon of interlocking begins with choosing the correct approach to investigate this immense information structure.

In an early paper, Levine[1972] examined a subset of 84 corporations taken from a committee report on the trust activities of commercial banks (Patman[1968]). His goal was to represent the data using a method of "utmost objectivity" in order to define the major outlines and important characteristics of the network using only the information implicit in the interlocks. In that way, he hoped to avoid the potentially biasing "hunches" that accompany prior knowledge about the corporations under study. His first attempt was to draw the network with pencil and paper, representing each corporation as a node and each interlock as an arc weighted by the number of directors in common. When this approach proved unwieldy for even a small number of corporations, Levine turned to a non-metric multidimensional unfolding approach (Guttman and Lingoes[1970]) using the number of shared directors as a measure of the similarity between two corporations. With this approach, Levine hoped to gain more information about the nature of interlocking by configuring the network in some manageable-dimensional space, with interconnected groups of nodes set closely together, and unconnected nodes set far apart. These groups of nodes set closely together might then form the basis for further study on the determinants of corporate interlocking.

In this paper, we suggest that the multidimensional unfolding technique is not an appropriate method for representing the corporate interlock data:

1. The small number of distinct similarity measures

(over 60% of the over 4000 links have a similarity measure of either 1, 2, or 3) provide a limited amount of information about the full context of corporate interlocking. In some cases, this information is inconsistent with the formation of a structure in small-dimensional Euclidean space. For example, two companies that have no direct interlock may be linked directly to a third company (an <u>indirect</u> interlock). The non-metric multidimensional unfolding technique attempts to determine a solution where the companies with positive similarity are close together, while the companies with zero similarity are at least as far apart as any in the space. (In a more recent paper, Levine[1979] employs a centroid scaling approach which allows the possiblity of a "non-choice" between two companies proximal in the solution space, but the technique involves the determination of eigenvectors for a large matrix, and is prohibitively expensive for large problems).

2. When the dimensionality of the solution space exceeds three, the results are difficult to envision and harder to interpret.
3. When applied to problems involving several hundred items, the non-metric multidimensional scaling technique is also computationally expensive and time consuming.

We also assert that in previous studies there has been no attempt to assess the extent to which the structure of the multidimensional representation of the corporate data really reflects the idea of purposeful social interlocking. There is no convenient way to compare the structural differences between different networks, to see whether or not the characteristics of one suggest socially significant interaction. As reflected in point 2 above, there is really no convenient way to examine the structural characteristics of even one large network.

In this paper, we approach the network model and the interlock data with the same goal as Levine: "to 'understand' a large network in a crude, almost a-theoretical sense of being able to represent it, to discern its major outlines, and to distinguish important links from those which are not."[p.14]. To do so, we employ a high-density clustering model defined on a graph, developed as a part of a systematic design methodology for the design of large software systems (see e.g. Andreu and Madnick[1977], Huff and Madnick[1978], Wong[1980], and Lattin[1981]).

The clustering technique determines regions of "high-density" in the graph, i.e. groups of highly or heavily linked nodes separated by other such groups by relatively few, weak links. Systems designers have used the high-density clustering model to focus on the global features of their design specifications, by modeling the design problem as a graph, with the problem's functional requirements as nodes and interdependencies between requirements as arcs. The high-density regions of such a design graph suggest well-defined sub-tasks that exhibit good design characteristics. Just as systems designers use the high-density clustering model to focus on a complicated set of design specifications, so do we use the model to explore the structure of the corporate network, looking for well-defined regions of highly interrelated nodes that appear to stand apart from the rest.

This new approach offers several advantages for our analysis:

1. The high-density clustering model considers the immediate context within which any two companies interact, thereby taking into account any indirect interlocking activities. The technique considers not only the direct relation between two firms within the network, but also the interaction of each with surrounding companies.
2. The results are presented in a hierarchical clustering trace, which requires no unconventional display and facilitates quick scanning and interpretation.
3. The technique is quite computationally efficient, and especially so for relatively sparse graphs.

The new approach also permits us to compare the structural characteristics of different networks. In order to examine the extent to which the clustering structure of the corporate network revealed by the high-density technique is somehow socially significant, we compare its structure to that of a network model of "random" directorate choice (e.g. without regard to directorate membership or director identity). The results are so similar to the actual network as to cast some doubt on the theories involving spheres of corporate influence that have been suggested in the past. The nature of the model, however, which so closely represents the actual network, does provide some theoretical insight into the formal processes of corporate interlocking.

In later sections, we present the high-density clustering approach in more detail, and the results of applying this technique to some large subsets of the Fortune 800. In the following section, we present an extended weighting measure, based on directorate interaction, to accommodate both direct and indirect interlocks between firms.

2. Extending the Network Conceptualization

Levine chose a very simple, discrete weighting function to describe the strength of the link between two companies, using only the number of shared directors. The rationale implicit in that choice is that two companies with one director in common are proximate with respect to some underlying "social distance." Companies sharing more than one director are even closer together, in that they are more likely to arrive at the same choice of directors.

We can extend this model if we are willing to assume that the extent to which directors interact in the boardroom context reflects the social proximity of the interacting firms. Rather than focus upon the frequency that companies draw the same directors, we focus instead upon the extent of the overlap between boards, i.e. the proportion of the directors of any one company who are chosen to be directors of another company. The rationale in this case is that corporations choose from an evoked set of directors, subject to certain constraints, determined by a unique corporate perspective. The greater the proportional overlap in corporate membership, the "closer" two firms are in perspective.

We face an implicit asymmetry when we attempt to assign a weight to the link between two firms. Consider, for example, two firms with boards of quite different size. Almost all of the directors of board C_i, a relatively small board, are also members of the relatively large board C_j. Nearly the entire directorate of board C_i is within the realm of the corporate perspective of C_j, through direct choice by that company. On the other hand, only a small proportion of the membership of board C_i is within the realm of the perspective of C_i. Thus, from the point of view of C_i, the perspective of C_j is quite salient; from the point of view of C_j, the perspective of C_i is less consequential. Because of the potential proximity of the two firms in this situation, regardless of the return effect, we choose to weight heavily the link between two such firms.

We can formalize this weighting scheme for the network of direct interlocks as follows. Let $\mathcal{C} = \{C_1, C_2, C_3, \ldots, C_N\}$ be the set of corporate boards under consideration, which form the N nodes of the network. Each element C_i of $\mathcal{C}$ is in turn a set of corporate directors $\{d_m, \ldots, d_n\}$. Let $\mathcal{A}$ be the set of arcs joining the nodes representing directly interlocked boards, and let w_{ij} denote the weight on the arc between boards C_i and C_j, where

$$w_{ij} = \frac{|C_i \cap C_j|}{\min\{|C_i|, |C_j|\}}, \quad \forall (i,j) \in \mathcal{A} \tag{1}$$

and where $|C_i|$ denotes the cardinality of the set C_i. The value w_{ij} is limited to the interval $(0,1.0]$. When $w_{ij} = 0$ there is no arc between C_i and C_j.

Bearden _et_. _al_. [1974] proposed a weighting scheme similar to (1), using $(|C_i|*|C_j|)$^1/2 in the denominator instead of $\min\{|C_i|, |C_j|\}$. Although Bearden's scheme eliminates the asymmetry of the weighting function, it is not as readily interpretable in terms of the idea of membership overlap. In practice, there is a difference between the two schemes, but it does not appear to be substantial.

Once we have assumed that there is some this social significance to boardroom interaction, we must consider explicit representation of indirect interlocks in the corporate network. If C_i and C_j are indirectly interlocked, then the two firms are both choosing directors from the same realm, which is the choice set of C_k. Clearly, an indirect link does not involve as great an overlap of corporate perspectives as does a direct link. The proximity of the corporate perspectives of C_i and C_j is no longer reflected by a direct overlap in board membership, but only by their mutual closeness to the intervening board C_k. We represent this diminished proximity with a multiplicative measure: the weight of an indirect link between C_i and C_j through C_k is proportional to the product of the direct link between C_i and C_k and the direct link between C_k and C_j. This proximity is also inversely proportional to the size of the intervening board C_k: the greater the membership of C_k, the less likely it is that the perspectives of C_i and C_j are highly similar.

We operationalize the weighting scheme for the extended network as follows. We let $\mathcal{A}$ denote the extended set of arcs in the network representation, including links between all pairs of boards that are either directly linked, indirectly linked, or both. Then w_{ij} becomes

$$w_{ij} = \frac{|C_i \cap C_j|}{\min\{|C_i|, |C_j|\}} +$$

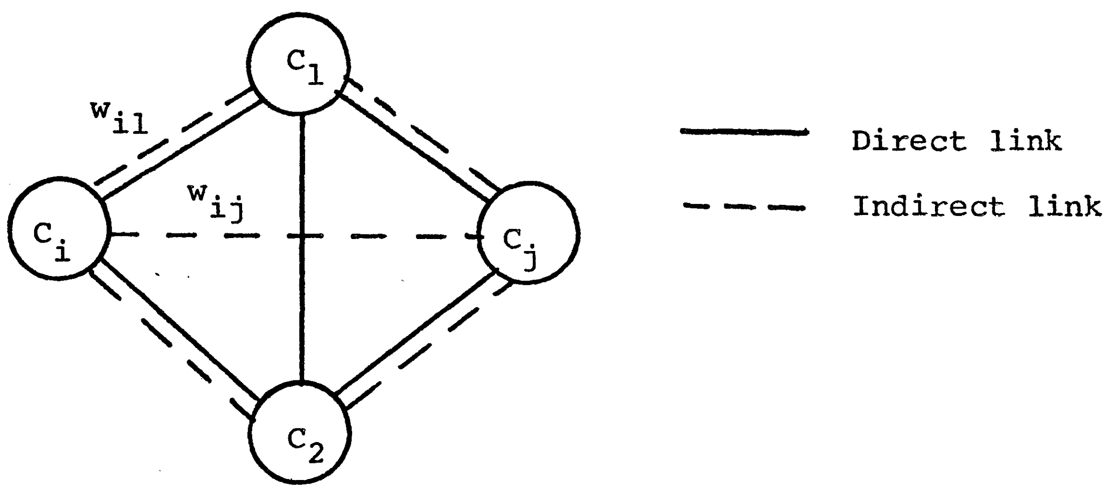
$$\sum_{C_k \in \mathcal{C}} \left(\frac{|\bar{C}_i \cap C_k|}{|C_i|}\right)\left(\frac{|\bar{C}_j \cap C_k|}{|C_j|}\right)\left(\frac{|\bar{C}_i \cap C_k| + |\bar{C}_j \cap C_k|}{|C_k|}\right), \forall (i,j) \in \mathcal{A}_{(2)}$$

where $\bar{C}_i$ and $\bar{C}_j$ denote $C_i - C_i \cap C_j$ and $C_j - C_i \cap C_j$, respectively. Unfortunately, in theory, w_{ij} need no longer remain in the interval (Ø,1.Ø]. An example appears in Figure 1. In practice, however, the overlap in directorate membership is never so extensive.

---Figure 1 about here---

With this extended scheme, we are able to examine a network with indirect interlocks as well as direct interlocks.

$$C_i = \{d_1, d_2, d_3, d_4, d_5, d_6\}$$
$$C_j = \{d_7, d_8, d_9, d_{10}, d_{11}\}$$
$$C_1 = \{d_1, d_2, d_3, d_4, d_5, d_7, d_8, d_9, d_{10}, d_{12}\}$$
$$C_2 = \{d_2, d_3, d_4, d_5, d_6, d_8, d_9, d_{10}, d_{11}, d_{13}, d_{14}\}$$



e.g.

$$w_{i1} = 5/6 + \frac{1}{6} \cdot \frac{3}{10} \cdot \frac{4}{11} = 0.85$$

$$w_{ij} = \underbrace{0}_{\text{direct}} + \underbrace{\frac{5}{6} \cdot \frac{4}{5} \cdot \frac{9}{10} + \frac{5}{6} \cdot \frac{4}{5} \cdot \frac{9}{11}}_{\text{indirect}} = 1.15$$

Figure 1

Hypothetical example showing that the weighting scheme involving indirect links might lead to an arc weight greater than 1.0.

3. The High-Density Clustering Technique

We now begin to examine the network for patterns and structure, using a technique for identifying regions of "high-density interlocking." Each region, which is a cluster of firms that are quite heavily interlocked among themselves and yet not highly linked to firms from outside the cluster, represents a subset of corporations whose similar perspectives are reflected in the widespread overlapping of their directorates. Understanding and explaining these high-density clusters and the relationships among the clusters is our first step toward understanding interlocking as a social phenomenon.

We choose the high-density clustering model on a graph (Wong[1980]; see Lattin[1981] for implementation and performance evaluation) because of its apparent advantages over other network/graph decomposition techniques:

1. The technique does not require *a priori* specification of the number of subgraphs; rather, it identifies regions of high-density and thereby suggests to the investigator the appropriate number of subcomponents to the graph (see e.g. Kernighan and Lin[1970], Christofides and Brooker[1976], both of which require *a priori* specification of subgraph size or the number of subgraphs).

2. The technique utilizes a maximum spanning tree algorithm, which operates very rapidly on large, relatively sparse graphs (see e.g. references in point 1 above; also McCormick[1972], Huff[1979] for heuristics that take no account of the sparsity of the graph).
3. The high-density clustering model does not rely on a goodness-of-partition measure, which is difficult to specify without somehow favoring extreme partitions (see Wong[1980] for a critique of these methods).
4. The high-density technique provides a convenient clustering trace which facilitates comparison of clustering structures of different networks.

Wong[1980] proposes the following density measure for weighted graphs:

$$d_{ij} = \frac{2w_{ij} + \sum_{k} (w_{ik} + w_{kj})/2}{|N_i \cup N_j|} \qquad \forall (i,j) \in \mathcal{A}, \qquad (3)$$

where

d_{ij} = the density on the arc between node i and j,

w_{ij} = the weight on the arc between node i and node j,

N_i = the neighborhood of node i: i.e. node i and all nodes k such that $(i,k) \in \mathcal{A}$,

C = all nodes k (distinct from i and j) such that

(i,k) and $(k,j) \in \mathcal{A}$.

$|N_i \cup N_j|$ = the cardinality of the union of the neighborhood of node i with the neighborhood of node j; i.e. the number of nodes (including i and j) linked to i or j.

(In the context of the interlock data, node i represents the board C_i).

The measure is best envisioned as a ratio of the number of nodes linked to both i and j (weighted) to the number of nodes linked to either i or j (unweighted). Using this measure, a graph with arc weights in the interval (0,1.0] will have arc densities in the same interval. A relatively high value of d_{ij} indicates that C_i (node i) and C_j (node j) are linked principally to the same set of firms (nodes), and that the overlap in board membership is proportionately large (the weights on each arc are high). A low value of d_{ij} indicates either that C_i and C_j are linked to very few common firms, or that the proportional overlap in corporate membership is in no case very large, or both.

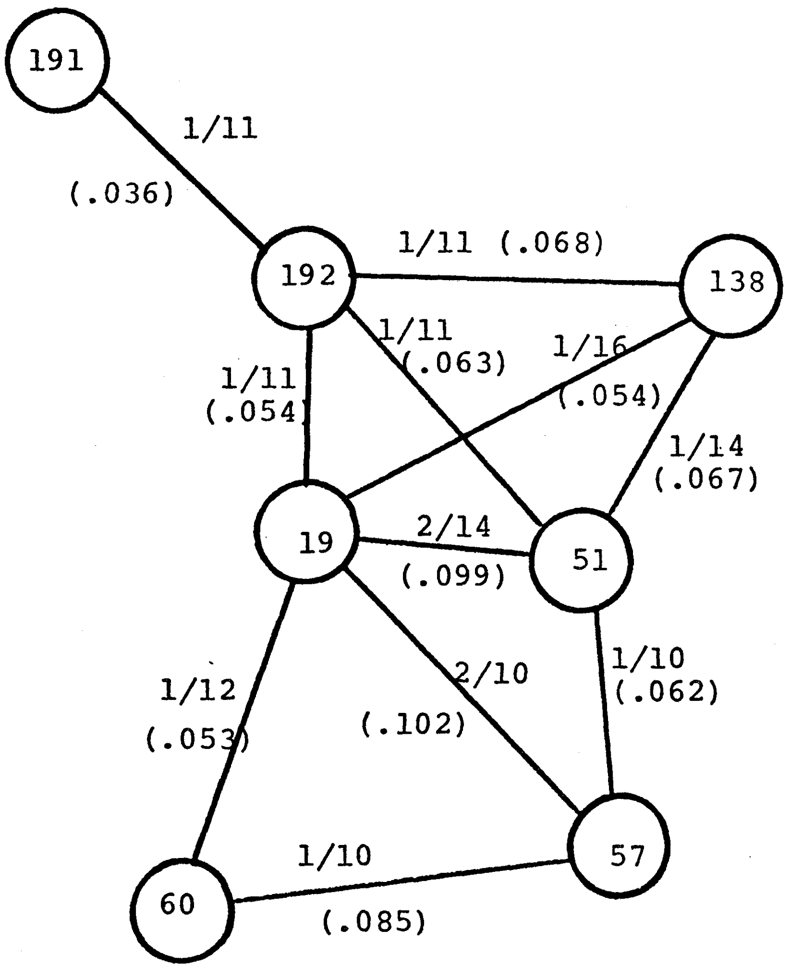
In order to illustrate the concept of an arc density, we select a small subset of seven interlocked corporations from the data compiled by Mariolis and Schwarz, which are currently available on-line in the BARON archive at Dartmouth College. Figure 2 shows the subset configured as a weighted network according to the weighting scheme in (1). The densities for each arc, calculated according to (3), are in parentheses.

---Figure 2 about here---

Any value $d^* \in [0, 1.0]$ defines a <u>density contour</u>, delimiting the high-density clusters at level d*. These are the regions of "high-density interlocking" in the corporate network, where firms are linked to other firms within the group with a density at least d*. These density countours have a hierarchical nature; each lower contour encircles all the nodes in the contours above it. Figure 3 shows an example of the hierarchy of high-density clusters using the seven node network from Figure 2.

---Figure 3 about here---

Had we selected a much larger subset of corporations for the illustration in Figure 2, the depiction of density contours might have become quite difficult. For large graphs, we appeal to a more concise representation, which is the modified form of the standard clustering tree output shown in Figure 4 (see e.g. Hartigan[1975] for standard output). Although this representation contains less information than that of Figure 3, it is somewhat easier to see that General Telephone(19), Continental Can(51), and Textron(57) are clustered at a much higher density level than any of the other firms in the sample. A vertical line at density level $d^* = 0.0675$ delimits the high-density clusters at that level ({19, 51, 57}, {138, 192}) from the low-density nodes ({191}). The fact that the point between Warner Lambert(138) and Otis Elevator(192) does not appear as a



19:	General Telephone
51:	Continental Can
57:	Textron
60:	Sperry Rand
138:	Warner-Lambert
191:	Budd
192:	Otis Elevator

Figure 2

A network of seven industrial corporations chosen from the 1970 Fortune 800. Arc weights are calculated according to the weighting scheme in (1). Arc densities are shown in parentheses.

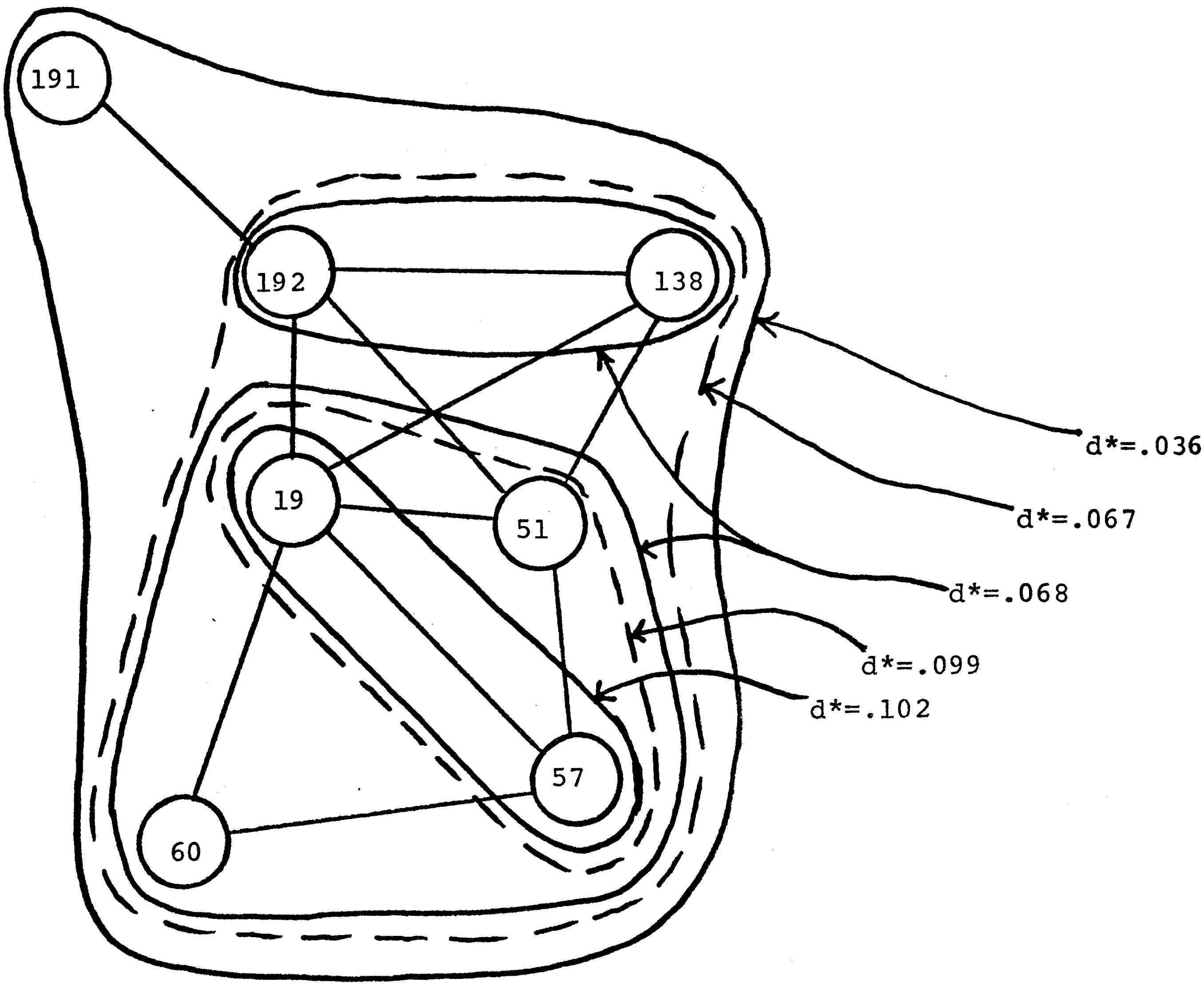


Figure 3

An example showing the hierarchical nature of the density contours for the example network of Figure 2.

sharp peak to the right of the line d* = .0675 in the figure indicates that the pair may not stand by itself as a strong, well-defined cluster.

---Figure 4 about here---

We cannot, however, say anything further about the interlocking behavior of these seven firms in a more global setting. In the context of the entire <u>Fortune</u> 500 industrials, any one of the seven might be more heavily interlocked outside the sample than inside it. Adding another 493 firms to this network might change its characteristics -- and the conclusions we draw from them -- completely.

Figure 4

Clustering trace for example network of seven industrials. Vertical line delimits high-density clusters at level d* = .0675.
Weighting scheme used involves only direct links.

4. Characteristics of the *Fortune* Network

Because we use the full extent of the interlocking activity of a firm in our density measure, we must attempt to present the network in the most complete form possible. Otherwise, we run the risk of distorting certain areas in the network by failing to include firms that might contribute either to the weight assessed to the link between two firms (through a direct or an indirect interlock) or to the density measure on some arc (i,j) (e.g. by increasing the number of firms in the neighborhood of either i or j). In theory, this requires the inclusion of all existing firms into the network; in practice, we compromise by establishing some boundary around a reasonably large number of firms, and hope that in this collection we capture the main effects of the interlocking phenomenon.

One choice of a suitable bounded set has been the *Fortune* 800: the largest 500 industrials (by sales), 50 banks (by assest), 50 insurance companies (by assets), 50 retailers (by sales), 50 transportation companies (by operating revenues), 50 utilities (by revenues), and 47 miscellaneous companies listed by *Fortune* magazine. Recognizing that this is a first cut at the data with the new approach, we limit ourselves to the largest 200 industrials and largest 50 commercial banks, and hope soon to extend our investigation to the entire *Fortune* 800 network.

The name and identifying number of each of these 250 firms appear in Figure 5. Of the network of 200 industrials alone, 29 firms are single isolates (marked by an asterisk (*) in Figure 5) and the remaining 171 form a single connected network. When the 50 commercial banks are added to the network, only 20 firms are isolates (4 banks, 16 industrials, marked by a sharp (#) in Figure 5) and the remaining 230 form a single net. In each case, these firms may not be true outliers, in the sense that their only ties to the connected network are through ties to corporations we have excluded from our sample. We should not, therefore, focus on these firms to investigate the properties of so called outliers. What is significant is that the remaining firms form a single connected network, which in itself indicates that there exists some degree of corporate interaction across all possible collectively exhaustive groups of interlocking firms.

---Figure 5 here---

Certain characteristics of this set of data, discussed also by Levine[1975] for the entire Fortune 800, suggest that the financial institutions may play some qualitatively different role in the structure of the interlock network than do the industrials. The median board membership for the top 200 industrials is 15; among the banks, it is much larger. In fact, almost half of the banks have board memberships of 24 or 25, which is nearly double the median for industrials. The distribution of board size is shown in Figure 6 below.

rials Banks

1 GEN. MOTORS
2 STAND. OIL (N.J
3 FORD MOTOR
4 GEN. ELECTRIC
5 INT. BUSINESS M
6 CHRYSLER
7 MOBIL OIL
8 TEXACO
9 INT. TEL. & TEL
10 GULF OIL
11 WESTERN ELECTRI
12 U.S. STEEL
13 STAND. OIL OF C
14 LING-TEMCO-VOUG
15 DU PONT (E.I.)
16 SHELL OIL
17 WESTINGHOUSE EL
18 STAND. OIL (IND
19 GEN. TELEPHONE
20 GOODYEAR TIRE &
21 RCA
22 SWIFT
23 MCDONNELL DOUGL
24 UNION CARBIDE
25 BETHLEHEM STEEL
26 BOEING
27 EASTMAN KODAK
28 PROCTER & GAMBL
29 ATLANTIC RICHFI
30 NORTH AM. ROCKW
31 INT. HARVESTER
32 KRAFTCO
33 GEN. DYNAMICS
34 TENNECO
35 CONTINENTAL OIL
36 UNITED AIRCRAFT
37 FIRESTONE TIRE
38 PHILLIPS PETROL
39 LITTON INDUSTRI
40 ARMOUR
41 LOCKHEED AIRCRA
42 CATERPILLAR TRA
43 MONSANTO
44 OCCIDENTAL PETR*
45 SINGER
46 GEN. FOODS
47 SUN OIL *#
48 RAPID-AMERICAN*#
49 DOW CHEMICAL*
50 GRACE (W.R.)
51 CONTINENTAL CAN
52 INT. PAPER
53 BURLINGTON INDU
54 BORDEN
55 BOISE CASCADE
56 AMERICAN CAN
57 TEXTRON
58 UNION OIL OF CA
59 MINNESOTA MININ
60 SPERRY RAND
61 TRW
62 REYNOLDS (R.J.)
63 ARMCO STEEL
64 GULF & WESTERN
65 CITIES SERVICE
66 UNIROYAL
67 ALUMINUM CO OF
68 CONSOLIDATED FO
69 REPUBLIC STEEL
70 AMK *#
71 XEROX
72 BENDIX
73 U.S. PLYWOOD-CH
74 SIGNAL COMPANIE*#
75 HONEYWELL
76 ANACONDA
77 FMC
78 RALSTON PURINA
79 COCA-COLA
80 AMERICAN BRANDS
81 ALLIED CHEMICAL
82 AMERICAN STANDA
83 BEATRICE FOODS
84 TELEDYNE
85 OWENS-ILLINOIS
86 RAYTHEON
87 NAT. CASH REGIS
88 CELANESE
89 WEYERHAEUSER
90 GOODRICH (B.F.)
91 NAT. STEEL
92 CPC INTERNATION
93 INLAND STEEL
94 AMERICAN HOME P*
95 GENESCO
96 GRUMMAN*
97 STAND. OIL (OHI
98 GEORGIA-PACIFIC
99 OLIN
100 WHIRLPOOL
101 ASHLAND OIL & R*#
102 PPG INDUSTRIES
103 COLGATE-PAMOLIV
104 GETTY OIL
105 U.S. INDUSTRIES
106 GEN. TIRE & RUB*#
107 AMERICAN CYANAM
108 BORG-WARNER
109 OGDEN
110 EATON YALE & TO
111 KENNECOTT COPPE
112 DEERE
113 MEAD
114 REYNOLDS METALS*
115 STEVENS (J.P.)
116 MARTIN MARIETTA*#
117 CARNATION
118 WHITE MOTOR
119 PEPSICO
120 STANDARD BRANDS
121 NORTON SIMON
122 NAT. LEAD
123 BRISTOL-MYERS
124 LYKES-YOUNGSTOW
125 KAISER ALUMINUM
126 MARATHON OIL
127 CROWN ZELLERBAC
128 AMERADA HESS
129 AVCO
130 GEN. MILLS
131 CAMPBELL SOUP
132 MOTOROLA *
133 ST. REGIS PAPER
134 KIMBERLY-CLARK
135 TEXAS INSTRUMEN*
136 COMBUSTION ENGI
137 SCM
138 WARNER-LAMBERT
139 PFIZER (CHAS.)*#
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141 STUDEBAKER-WORT
142 HEINZ (H.J.)
143 WALTER KIDDE *#
144 AMERICAN SMELTI
145 PHILIP MORRIS
146 WHITE CONSOLIDA*#
147 AMERICAN METAL
148 WHITTAKER*#
149 BURROUGHS
150 HERCULES
151 PULLMAN
152 AMERICAN MOTORS
153 ILLINOIS CENTRA
154 SCOTT PAPER
155 COLT INDUSTRIES
156 INT. UTILITIES*#
157 NAT. BISCUIT
158 NORTHWEST INDUS
159 BABOCK & WILCOX
160 INGERSOLL-RAND
161 DART INDUSTRIES*
162 INTERCO
163 DRESSER INDUSTR
164 NAT. DISTILLERS
165 UNITED MERCHANT
166 ZENITH RADIO
167 IOWA BEEF PACKE*#
168 PHELPS DODGE
169 ANHEUSER-BUSCH
170 JOHNSON & JOHNS*
171 DEL MONTE
172 AVON PRODUCTS
173 DANA *#
174 MERCK
175 CLARK EQUIPMENT
176 SQUIBB BEECH-NU
177 EMERSON ELECTRI
178 HORMEL (GEO.A.)
179 JIM WALTER *#
180 TIME INC
181 PET
182 GILLETTE
183 MCGRAW-EDISON
184 GAF
185 AMERICAN MACHIN
186 LEAR SIEGLER*
187 JOHNS-MANVILLE
188 CONTROL DATA *
189 PILLSBURY
190 OSCAR MAYER*
191 BUDD
192 OTIS ELEVATOR
193 NORTHROP
194 CENTRAL SOYA*
195 QUAKER OATS
196 ARMSTRONG CORK
197 ESSEX INTERNATI*#
198 CRANE
199 NORTH AM. PHILI
200 STERLING DRUG
201 BANKAMERICA
202 FIRST NAT. CITY
203 CHASE MANHATTAN
204 MANU. HANOVER
205 MORGAN (J.P.)
206 WESTERN BANCORP
207 CHEMICAL NEW YO
208 BANKERS TRUST N
209 CONTINENTAL ILL
210 FIRST CHICAGO C
211 SECURITY PACIFI
212 MARINE MIDLAND
213 CHARTER NEW YOR
214 WELLS FARGO & C
215 CROCKER NATIONA
216 MELLON NB & TRU
217 NAT. BANK OF DE
218 FNB OF BOSTON
219 NORTHWEST BANCO
220 FIRST BANK SYST
221 FRANKLIN NEW YO
222 BANK OF NEW YOR
223 FIRST PENNSYLVA
224 CLEVELAND TRUST
225 PNB CORP
226 DETROIT BANK &
227 UNIONAMERICA
228 REPUBLIC NB OF
229 SEATTLE-FIRST N
230 GIRARD CO
231 MANU. NB OF DET
232 HARRIS TRUST &
233 FIRST WISCONSIN
234 PITTSBURGH NATI
235 BANK OF CALIFOR
236 FNB IN DALLAS
237 LINCOLN FIRST B
238 NAT. BANK OF NO#
239 CITIZENS & SOUT#
240 U.S. BANCORP
241 WACHOVIA CORP
242 VALLEY NB
243 NORTHERN TRUST
244 NAT. CITY BANK
245 FIDELITY CORP.
246 SHAWMUT ASSOCIA
247 BANK OF THE COM
248 NCNB CORP
249 BANCOHIO #
250 NAT. BANK OF CO#

Figure 5

Name and identifying number for 200 industrials and 50 banks.

---Figure 6 about here---

Figure 7 shows the distribution of the number of interlocks per firm for the sample of 250. The median number of links is slightly larger for the banks than the industrials, as is the variance, which might be attributable to significant differences in board size. However, as Figures 8 and 9 demonstrate, the nature of bank interlocking activity differs quite substantially from that of the industrials. As shown in Figure 8, the banks do not interlock with each other as a rule, which might be due to some existing legislation or perhaps the perceived extra-sensitivity of that kind of financial interlock. On the other hand, of the 184 industrial corporations in the connected network (excluding the 16 single isolates), all but 20 are linked to at least one bank, indicating that interlocking activity between banks and industrials is quite widespread. Figure 9 shows that while an industrial firm is linked to a median of four other industrials and perhaps as many as 16, a bank is linked to a median of six or seven industrials and in several cases as many as two dozen or more.

---Figures 7, 8, and 9 here---

Because of the noticeable differences in board size and interlocking activity between commercial banks and industrial corporations, we consider both the network of 171 industrials and the network of 230 banks and industrials, and compare the results in order to gain some insight as to the role of the

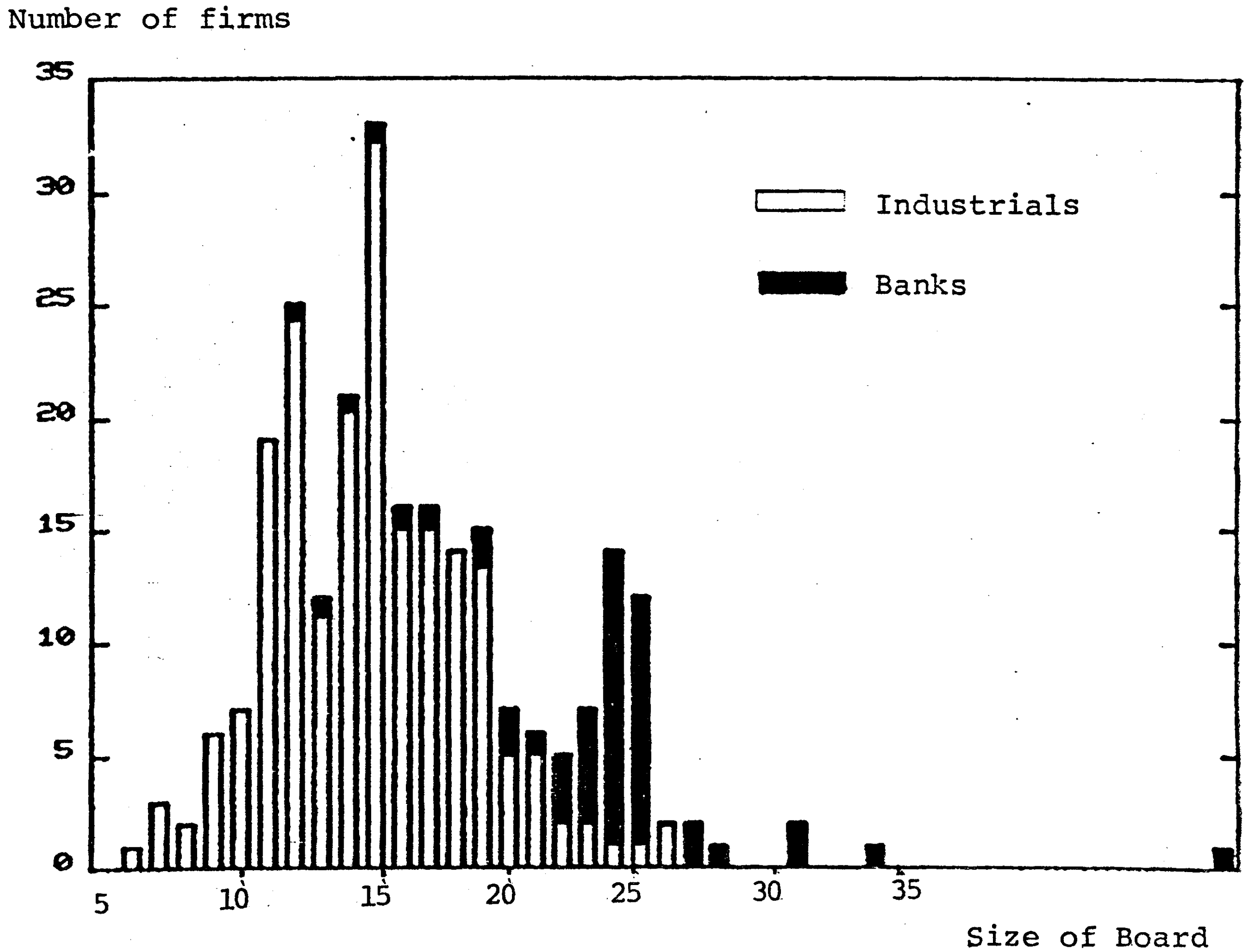


Figure 6

Distribution of board sizes for 250 firms

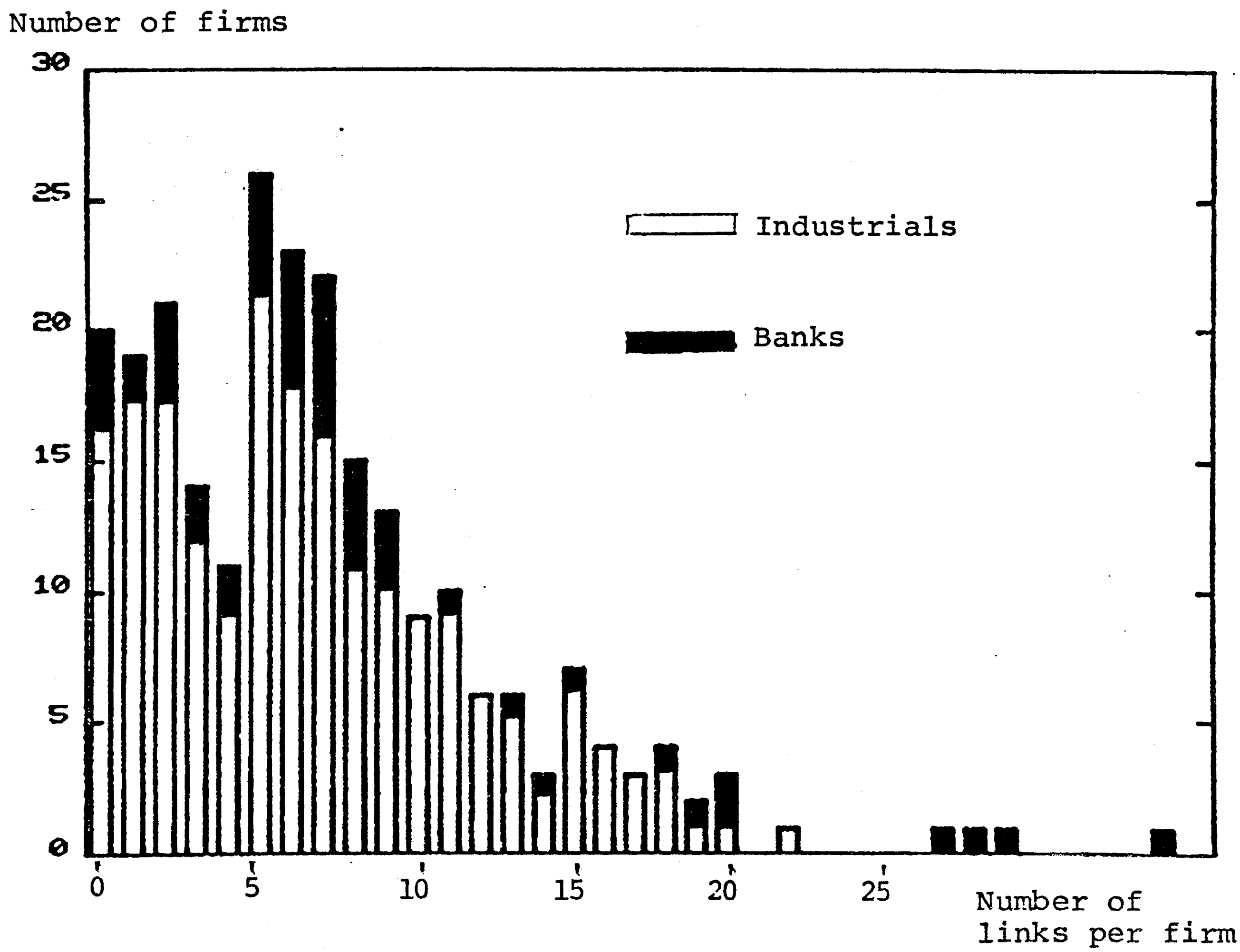


Figure 7

Distribution of number of interlocks per firm

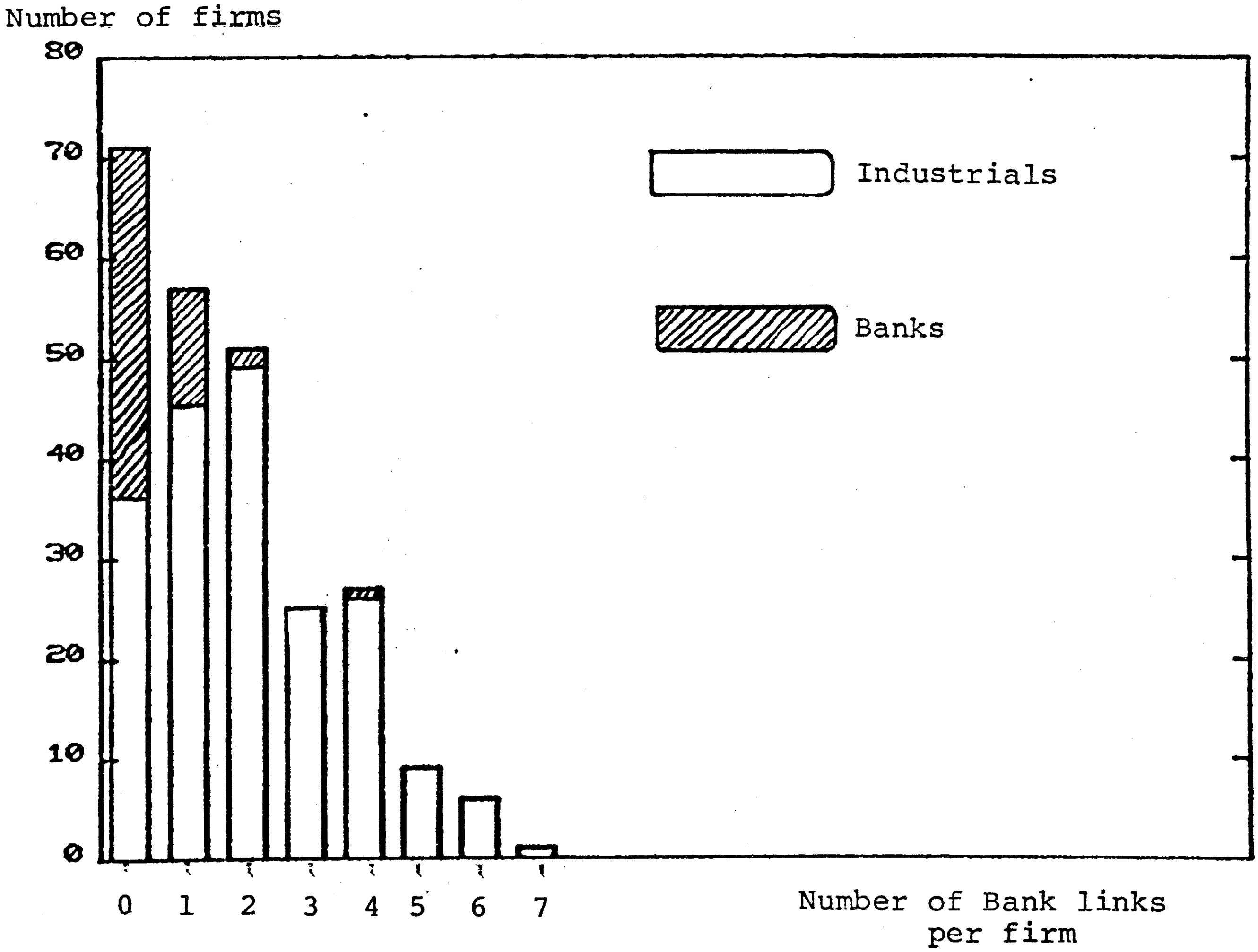


Figure 8

Distribution of number of Bank interlocks per firm

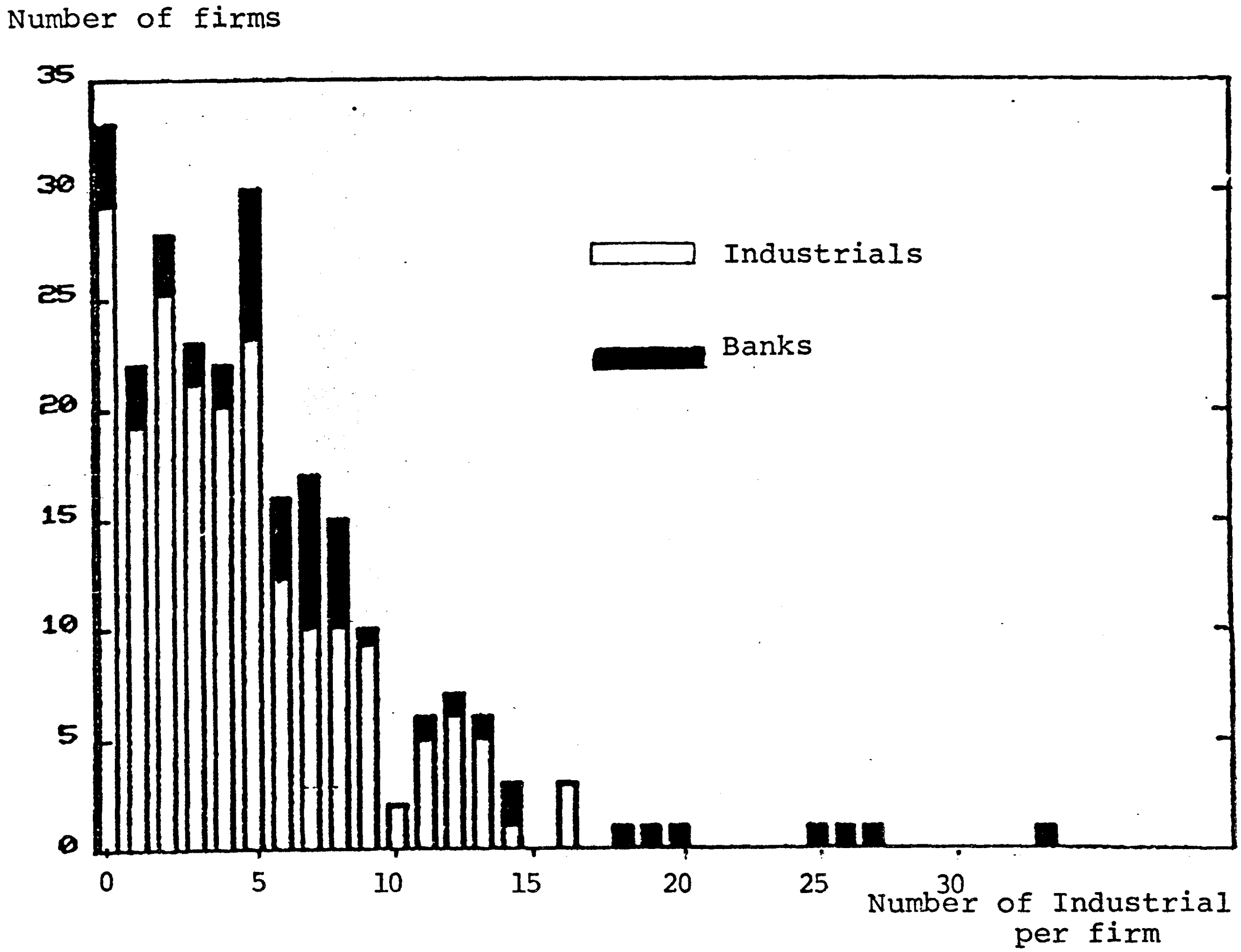


Figure 9

Distribution of number of Industrial interlocks per firm financial-industrial interlock in the overall structure. We also employ the weighting schemes in (1) and (2) in order to assess the extent of the indirect interaction in the network.

5. High-Density Clustering Results

The directory below outlines the presentation of the high-density clustering analyses:

Net-work	Description	Wt. Eqn	# of nodes N	# of arcs A	aver. # incident to node
(A)	indus'ls only	(1)	171	437	5.11
(B)	ind'ls + banks	(1)	230	869	7.56
(C)	indus'ls only	(2)	171	1984	23.10
(D)	ind'ls + banks	(2)	230	5533	44.56

Net-work	Figure holding clustering traces	Figure identifying firms in clusters
(A)	Figure 10A	Figure 11A
(B)	Figure 10B	Figure 11B
(C)	Figure 10C	Figure 11C
(D)	Figure 10D	Figure 11D

The computational efficiency of the high-density clustering technique used here is presented by Lattin [1981]. The formation of each clustering tree is quite rapid, as it is

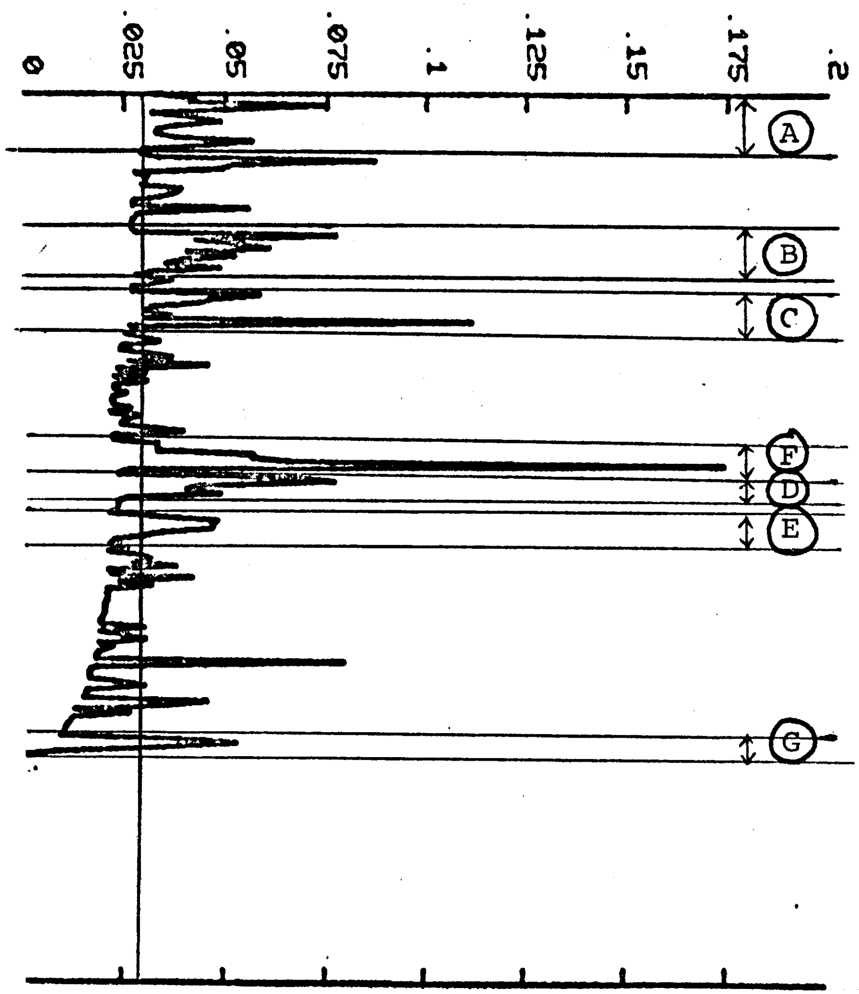


Figure 10A

Clustering trace for 171 industrials.
High-density clusters at level d*=.0300.
Circled letters identify principal clusters.
Weighting scheme includes only direct links.

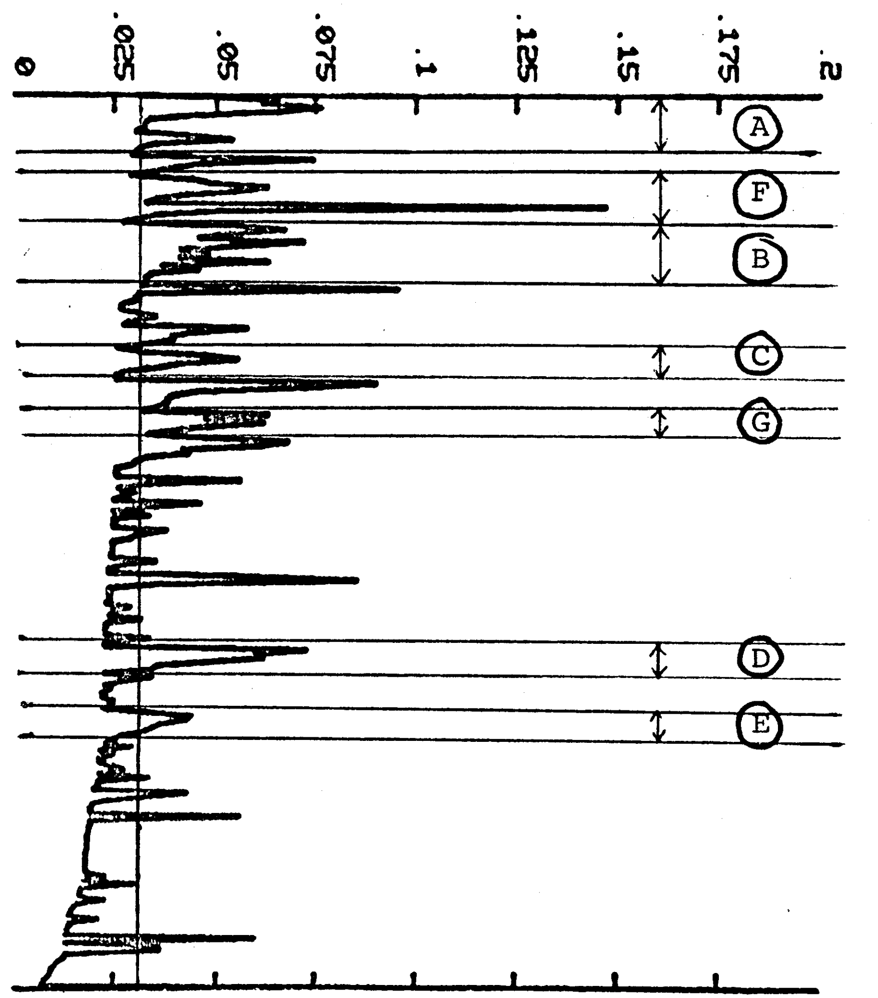


Figure 10B

Clustering trace for 230 industrials and banks. High-density clusters at d*=.0320. Circled letters identify clusters that correspond most closely to Figure 10A. Weighting scheme includes only direct links.

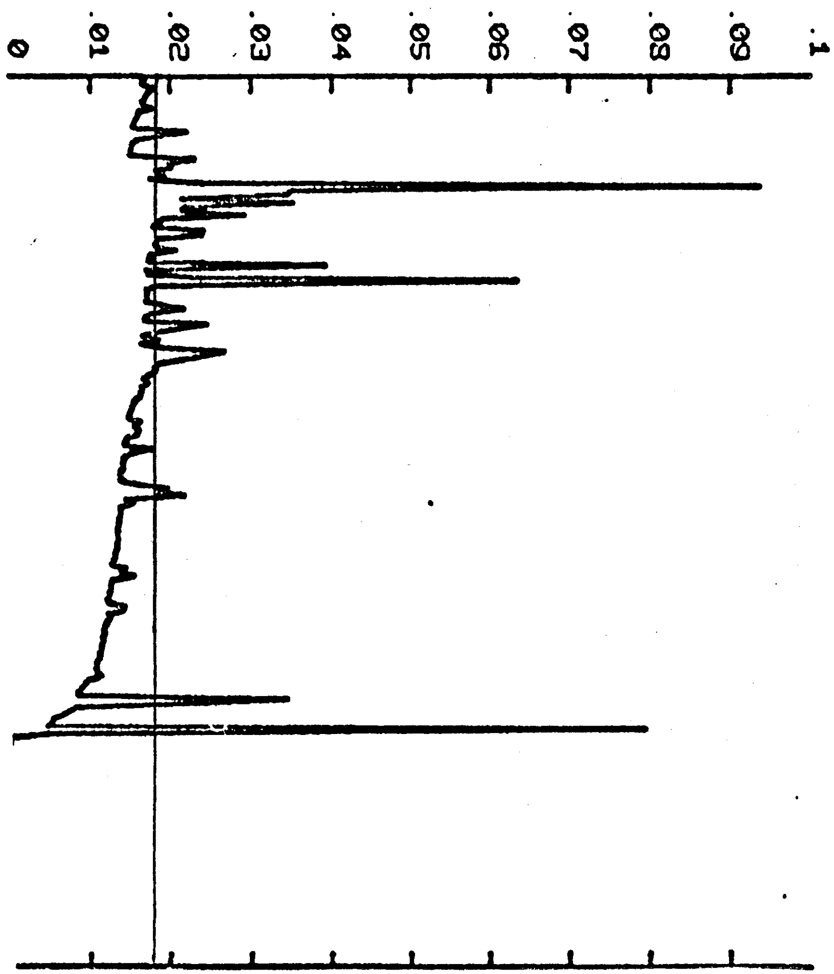


Figure 10C

Clustering trace for 171 industrials.
Vertical delimits high-density clusters
at level d*=.0180.
Weighting scheme includes direct and
indirect links.

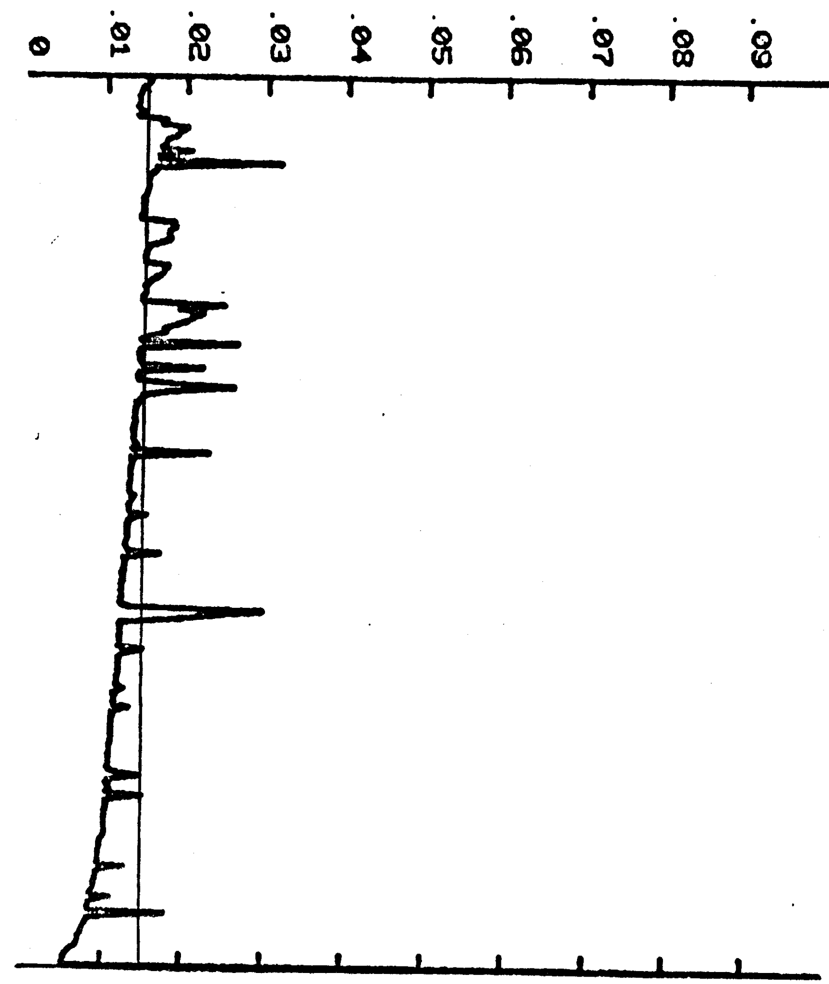


Figure 10D

Clustering trace for network of 230
industrials and banks. Vertical line
delimits high-density clusters at d*=.0145.
Weighting scheme includes direct and
indirect links.

A {1,67,142,10,102,14,46,100,2,7,171,42,158,180,108}
{151,177,169,162,43}
{63,20,28}
{113,90,130,32}
{126,85}
B {99,176,129,69,97,118,37,30,61,27,120}
{58,45}
C {29,123,187,82,87,56,140,71,84}
{147,107}
{199,75,35}
{153,31,195}
{22,178}
{34,73}
{50,160}
{104,150,39}
F {88,76,8,91,149,110,141,159}
D {60,57,19,51,138,192}
E {103,23,25,92,11,66}
{164,198}
{13,89}
{154,65}
{105,163}
{183,68}
{16,125}
{38,200}
G {18,93,40,166}

Total 26 clusters with
106 high-density nodes
leaving 65 low-density nodes
unclustered.

Figure 11A

Each set corresponds to a high-density cluster at level d*=.0300. Each number corresponds to a firm named in Figure 5. Circled letters correspond to the principal clusters identified in Figure 10A.

A {67,142,10,102,216,14,151,17,63}

{46,100,2,7}

{169,177,162,43,134}

F {234,91,149,226,217,110,8,141,159,222,54,72}

B {244,69,97,129,118,224,37,126,85,99,176,61,27,237,120,231}

{186,247}

{90,32}

{219,188,189,75,220,178}

C {87,187,123,29,82,56}

{(243,195,31,22,209,153,24,204) $\overbrace{(232,132,93,18,166,40)}^{G}$

(210,158,180,108,42,171,215)}

{71,84}

{233,140}

{76,88}

{50,160}

{58,206}

{182,218,86,246}

{221,170}

D {51,213,19,57,138,192,60}

{191,223}

E {103,92,23,25,66,11}

{236,19}

{62,241,53}

{228,135}

{68,183}

{200,185,38}

Total 25 clusters with
126 high-density nodes
leaving 104 low-density nodes
unclustered.

Figure 11B

Each set corresponds to a high-density cluster at level d*=.0320. Each number corresponds to a firm named in Figure 5. Circled letters correspond to the principal clusters identified in Figure 10B.

{42,2}

{10,102,14}

{129,69,97,118,20,63}

{110,141,159,54,60,57,19,34,91,149,138,51}

{8,76,88,163,62,53,189}

{99,176}

{56,71,84,140}

{126,85,83}

{30,58,37,39}

{123,29}

{192,92,25,23,157,11}

{43,151}

{177,169}

{200,185,38}

{183,68}

Total 15 clusters with
60 high-density nodes
leaving 111 low-density nodes
unclustered.

Figure 11C

Each set corresponds to a high-density cluster at level d*=.0180. Each number corresponds to a firm named in Figure 5.

{1,202,87,46}

{123,29}

{216,102,244,69,91,126,129,97,118,224,110,186,247,
10,14,142,63,207,234,61,20}

{209,31,243,195,232,132,22,24,204,93,153}

{210,42,158,108,171,18,203,214,166}

{19,57,221,86,170,246,218,213,51,60}

{159,141}

{149,226}

{75,219,188,220,189}

{228,135}

{99,176}

{211,84}

{233,140,190,161}

{225,198}

{230,191}

{68,183}

{200,38}

Total 17 clusters with
84 high-density nodes
leaving 146 low-density nodes
unclustered.

Figure 11D

Each set corresponds to a high-density cluster at level d*=.0145. Each number corresponds to a firm named in Figure 5.

accomplished by an algorithm which runs in O(A) operations. Thus, the technique as implemented on an IBM 370 model 168 requires less than 1 CPU second to form the clustering tree for any one of the above graphs. The calculation of the arc densities, which runs in $O(A^2/N)$ operations, is much more time consuming; network (D) above requires more than a full minute of CPU time. For the sparse graphs (A) and (B), however, the technique requires only about 2 CPU seconds.

The results could be used as a guide for partitioning the network. However, the clustering traces of the networks reveal certain characteristics of the global structure of the network suggesting that the analysis of the phenomenon of corporate interlocking may not be best approached through piecewise analysis of the network data. By simply looking at the clustering traces presented in Figures 10A through 10D, without yet identifying the various firms that make up the clusters at the given density level, we can say several things about the overall properties of the interlock network:

1. There appear to be no large, well-defined high-density clusters in the network of direct interlocks. Instead, the network seems to consist of many small pockets of relatively high-density, each enclosing only a few nodes.
2. The inclusion of the banks in the network of direct interlocks seems not to eliminate these small pockets of high-density, nor does it "flatten" the clustering tree perceptibly.

3. The clustering structure of the network of direct and indirect interlocks seems to indicate only very limited regions where groups of interlocked nodes are not substantially linked outside the group.
4. With the inclusion of indirect interlocks through bank interaction, the density pattern across the network seems to approach uniformity.

While no obvious interlocking structure stands out from these clustering traces, there are nonetheless some patterns among firms that are consistent across the different forms of the network. Seven large, prominent high-density clusters in network (A) are identified in Figures 10A and 11A by the letters A through G. Group D consists of the same set of firms, minus Budd(191), chosen for the example in Figure 2 above. In network (B), where the banks have been included, each one of the seven groups appears more or less intact, and have been identified as such in Figures 10B and 11B.

In five out of the seven of these principal clusters, at least one bank appears as a high-density node within the group. Thus, it seems the groups reappear not because they do not interlock with the banks, but because the bank links support the notion that these particular firms belong together by virtue of their widespread mutual interaction. Group D reappears as a high-density cluster in network (B) with one bank, Charter New York(213), now part of the cluster. Upon inspection, we find that Charter New York is linked to a total of eight industrials, five of which are members of group D: General Telephone(19), Continental Can(51), Textron(57), Warner Lambert(138), and Otis Elevator(192).

At this point it becomes tempting, as it has in past analyses, to begin to draw conclusions about the social interactions among these firms where such "heavy" interlocking seems apparent. Before drawing such conclusions, however, it seems prudent first to examine the structure of a network of "random" social interaction, in order to establish a baseline for comparison.

6. Assessing the Validity of a Decomposition Based on High-Density Interlocking

We should not simply use these high-density clusters to divide up the network for further analyses without investigating the extent to which the clustering structure truly reflects our idea of a region of high-density interlocking. If such a structure might just as well have arisen from some completely random process of director choice, then the shape of the clustering output is misleading us into thinking that the corporate cliques we have isolated really do interact with purpose and organizational significance when in fact they do not. Until we understand the patterns and forms of an undirected, haphazard interlocking process, we cannot begin to isolate those details of the clustering trace that reveal something about the phenomenon of corporate interlocking.

Recognizing once again that this is a first cut at the data, we consider only the network of industrials in our attempt to understand what kind of random choice process might give rise to an interlock network with the same characteristics as network (A). By random, we mean that the decision to link with another firm or choose a specific director is not affected by the identity of that firm or director, or the established associations of that director. The "random" process we specify must give rise to a network that roughly matches the actual network in connectivity (number of disjoint components), distribution of interlocks per firm, distribution of interlocking directors per board, and distribution of board memberships per individual. If we can determine such a process, we can then use the clustering trace of the random network as an indication of its overall structure. We can compare the clustering trace of the actual interlock network against this baseline, and any glaring departures from the random structure should be cause for further investigation.

To start with a simple process, we consider a type of random graph discussed by Erdos[1973] and suggested by Levine[1975] and Pennings[1980] as a possible model for the actual network's underlying structure. In such a graph, an arc connecting any two nodes in the network exists with probability p and does not exist with probability (1-p). We can estimate p by the ratio of the total number of interlocks in the actual network (437) to the total possible number of arcs among 200 industrials (19900), and use some random number generator to construct the model network.

Figure 12 presents the distributions of the number of interlocks per firm for the random network (with p = 0.022) and for the actual interlock network. Clearly, there is a substantial difference between the two in the way they characterize the interlocking activity in the network. In the model network, the distribution is centered around a mode of 4 or 5 links per node with relatively small variance. The number of isolated nodes is almost non-existent compared to the 29 isolates of the actual network. Similarly, very few nodes in the model network are highly linked, while the distribution for the actual network shows a rather prominent tail of highly interlocked firms.

---Figure 12 about here ---

The failure of the random graph model to adequately represent certain aspects of the actual interlock network may be due in part to the artificial boundaries imposed by our selection of firms. Even if the model provided a better representation, however, we would still be at a loss to explain the differential weighting on the arcs or the nature of the interaction among interlocking directors. There is also some evidence to suggest that the first 100 industrials, as ranked by Fortune, are more heavily interlocked than the second 100 industrials, which is a phenomenon not addressed by the Erdos-type graph.

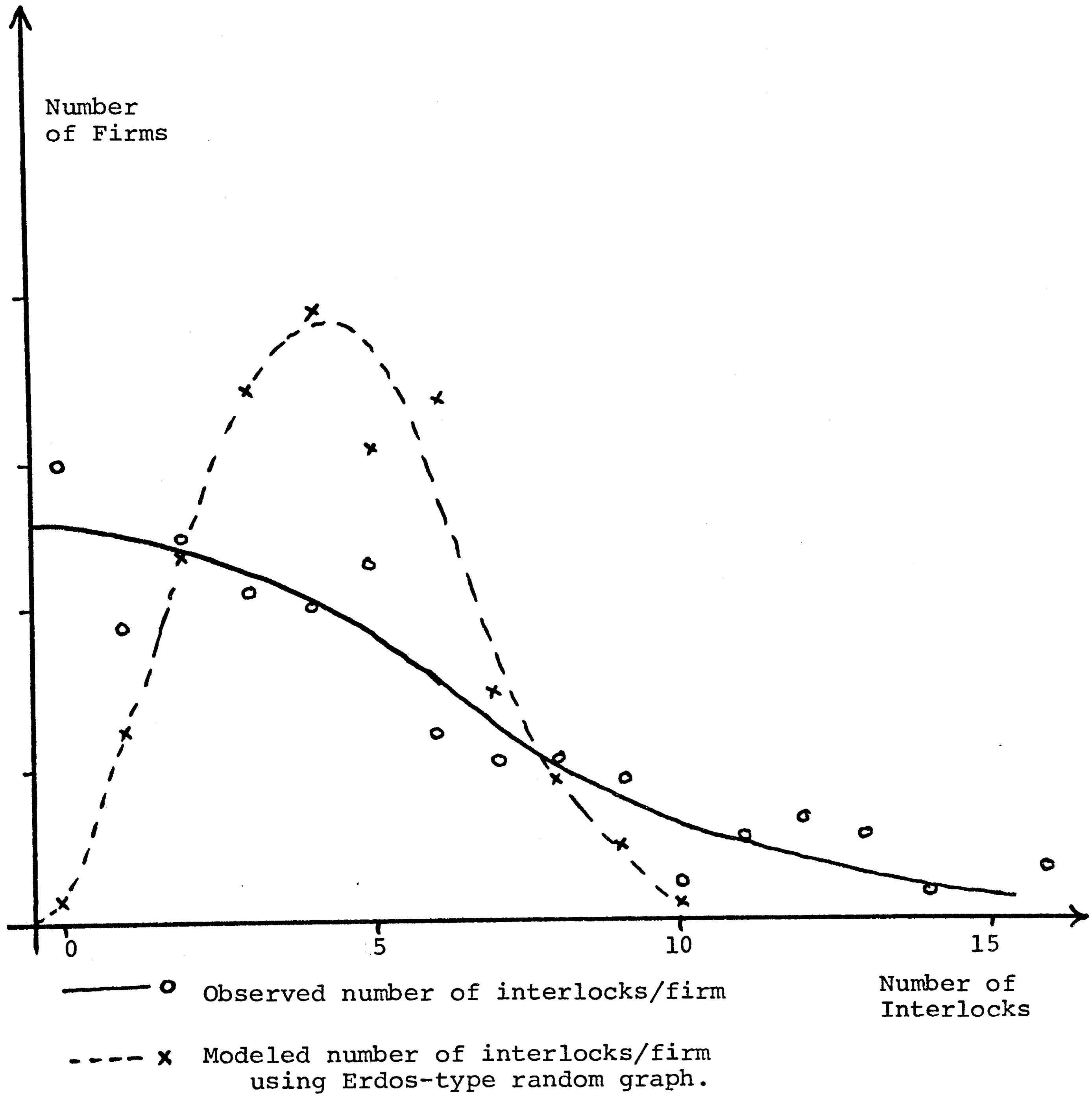


Figure 12

The distributions of interlocks per firm for the actual interlock data and for the Erdos random graph model.

In order to more effectively capture the interaction of firms through specific directors, we shift our focus to a process of corporations choosing randomly (with replacement) from an undifferentiated, finite population of individuals. The nearly normal shape of the distribution of board sized in Figure 6 supports the notion that these boards might have been formed by some sort of a Bernoulli choice process, where a firm makes some fixed number of attempts N to place a individual on its board, succeeding with some probability p and failing with probability (1-p). If the population is small enough, some individuals will be chosen by more than one firm, forming an interlock.

We estimate p and N using the mean and approximate variance of the distribution of industrial board sizes in Figure 6:

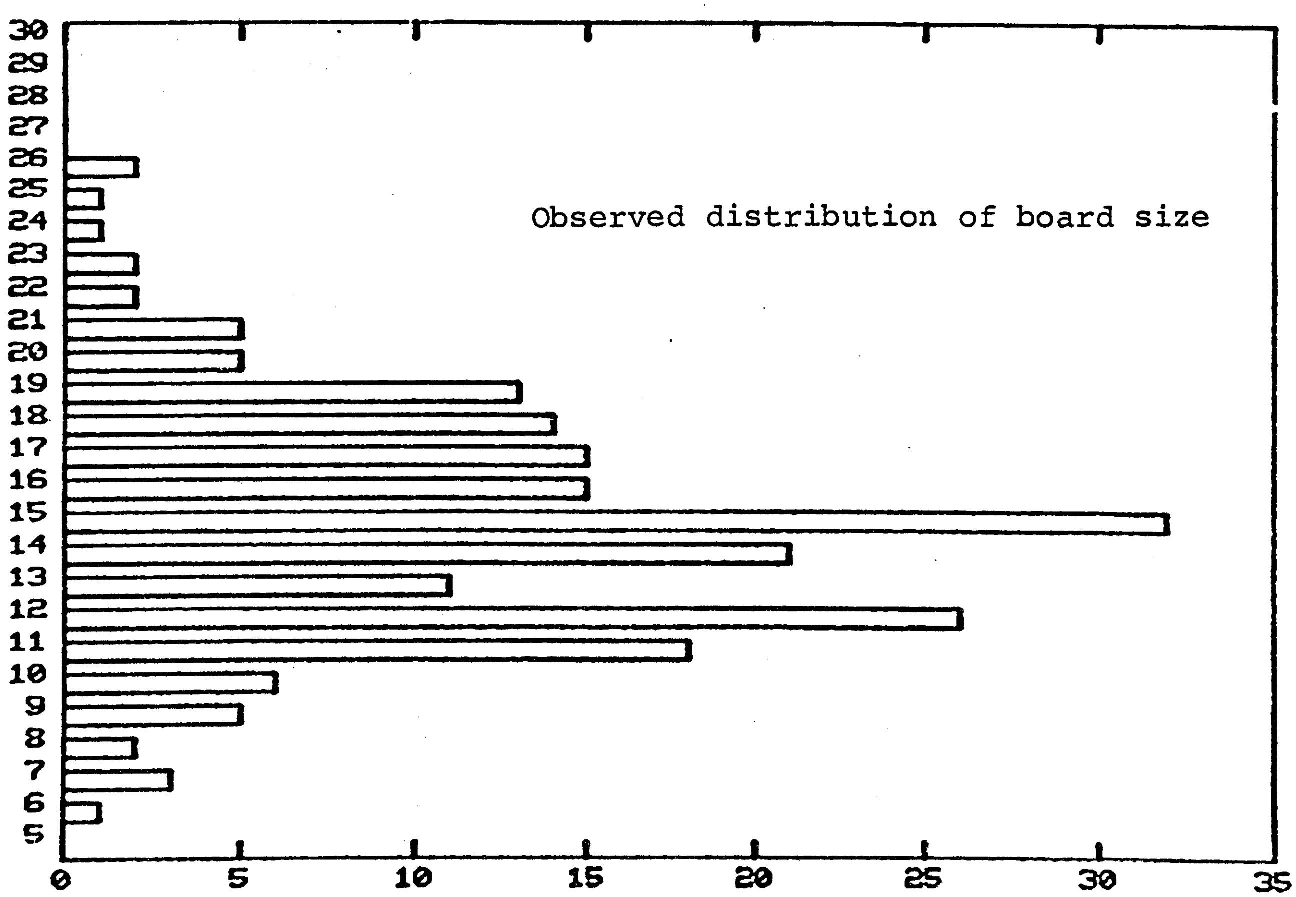
$$\left.\begin{array}{l} Np \cong 15 \\ Np(1-p) \cong 9 \end{array}\right\} \Longrightarrow \left\{\begin{array}{l} p = 0.33 \\ N = 45 \end{array}\right.$$

In order to acheive an expected value of about 2300 directors who are chosen by exactly one corporation, we establish a population of 8000 individuals as potential directors. We then use a random number generator to construct a model network. Figure 13 shows the distributions of board size for the actual interlock network and for the random choice model. Though the size of the modeled boards has somewhat smaller variance, the two distributions appear to be quite similar. The distributions of interlocks per director, however, reveal the inadequacy of this model of undifferentiated choice. As shown in the table in Figure 14, there are some directors in the actual network with 4, 5, and even 6 different board memberships. The undifferentiated choice model cannot give rise to a network where a small percentage of directors exhibit such considerable interlocking activity and the remaining percentage of directors are members of only one board.

---Figures 13 and 14 here---

In order to resolve this inadequacy, we turn to a differentiated choice process, where corporations can now distinguish among the directors they select. The simplest differentiation is a dichotomous one, dividing the population of directors into one small group of high-linking, "high-leverage" directors, and one large group of "common" directors who typically sit on only one board. If the corporations make a disproportionate number of choices from the smaller group, then there may be some high-leverage individuals who are chosen as many as 5 or 6 times.

If each corporation, however, chooses from this small group with equal likelihood, then each would have roughly the same chance of drawing an interlocking director, which is unlikely to give rise to a model network with as many isolates as the actual network. In order to reflect the wide range in the distribution of interlocks per firm, we posit a process of differential access, where some firms are better able to make a choice from the pool of high-leverage directors. Figure 15 shows one possible scheme of



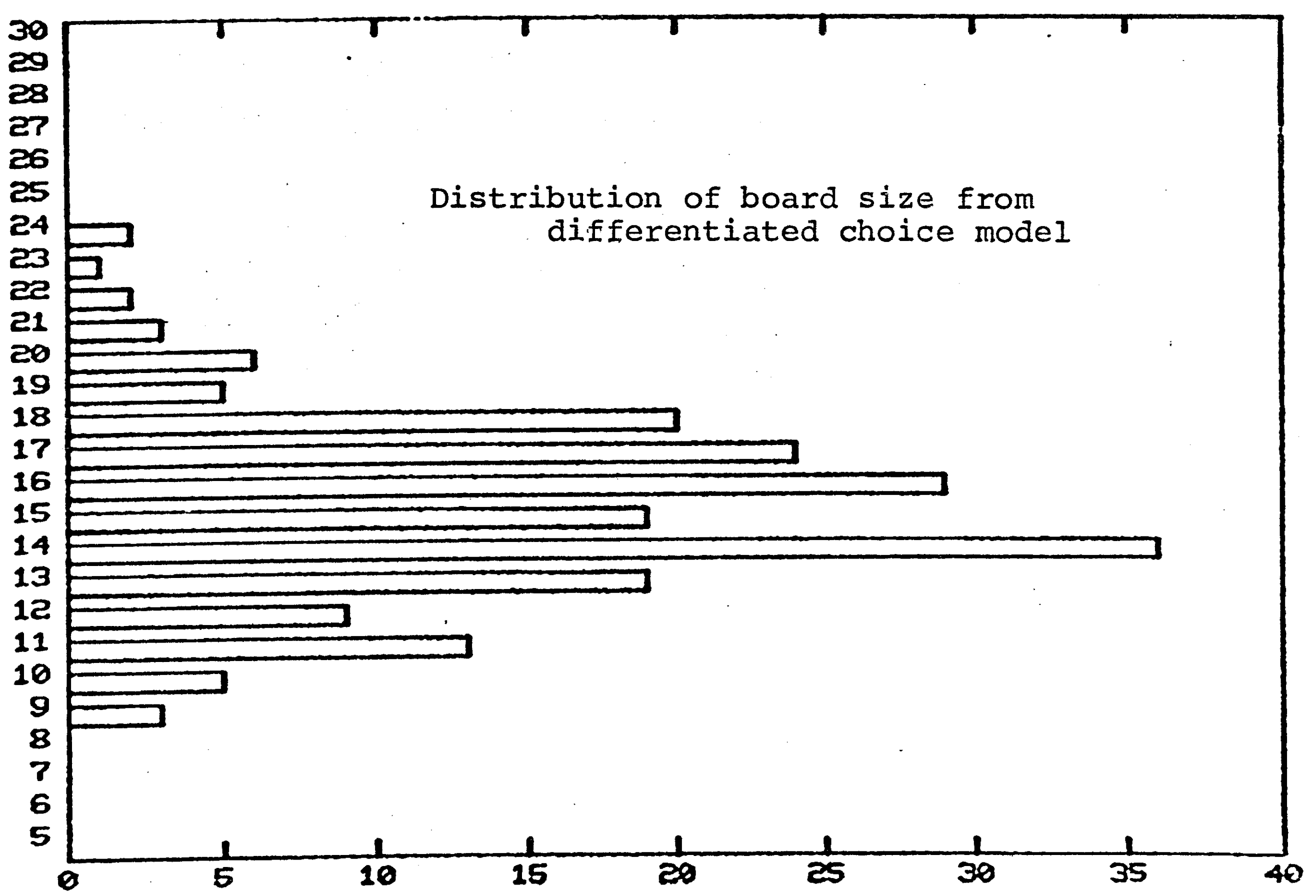


Figure 13

Distributions of board size

Number of interlocks per director	Actual Network	Undifferentiated Choice Model
0	?	5142
1	2315	2308
2	194	325
3	56	0
4	15	0
5	4	0
6	1	0

Figure 14

Table comparing the number of interlocks per director for the actual interlock network and for the undifferentiated choice.

differential access, where a relatively large percentage of firms are almost unable to choose from the set of high-leverage directors, while a very small percentage of firms are almost always able to choose from the smaller group. The fact that not all members of the high-leverage group will be chosen by more than one firm, while some few members of the common group will be chosen twice, illustrates that the distinction between the two groups is somewhat uncertain to the choosing corporations.

---Figure 15 here---

For our model of differential access, we use the same parameters N and p to govern the choice process for the formation of the corporate boards. We use a cumulative density function like the one pictured in Figure 15 to assign each corporation a probability that represents its "power of access," i.e. the proportion of times each firm makes a directorate choice from the smaller set of high-leverage individuals. We also split a population of 12,000 individuals into two groups: one group of 750 high-leverage individuals, and one group of 11,250 common individuals (each group determined in much the same fashion as the single group of 8000 above). Finally, we employ a random number generator to construct a model network.

Figures 16 and 17 present the distribution of interlocking directors per board and board memberships per individual, respectively, for the actual interlock network and for the differential access model. Although the

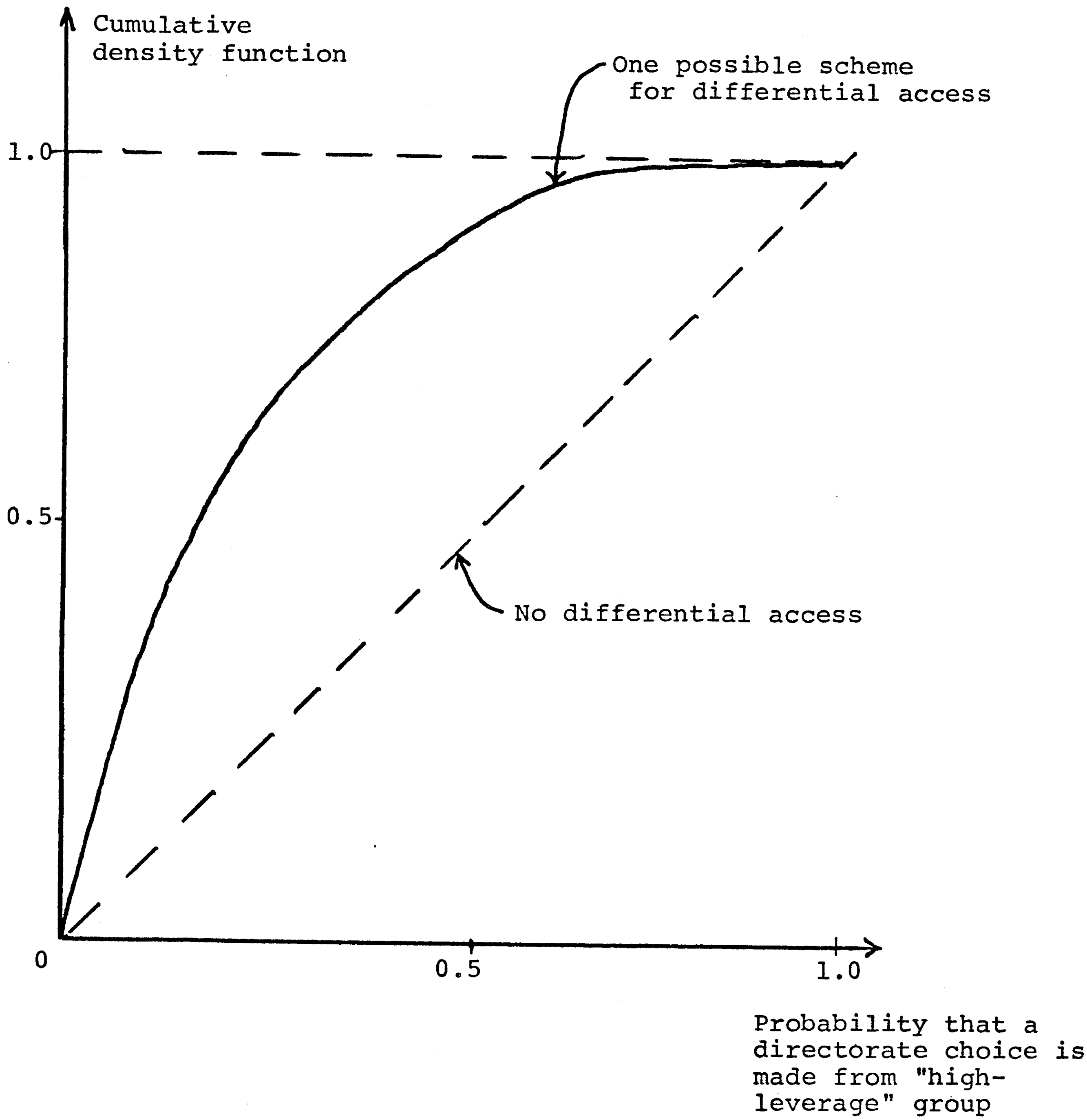


Figure 15

Example showing one possible operationalization
of differential corporate access to individuals.

correspondence is not perfect, which may be due principally to the imperfect calibration of the model parameters, the differential access model seems to capture quite well the important characteristics of the actual network. The director/board and board/director marginals from the model both reflect the extended tails indicating a high level of interlocking activity.

---Figures 16 and 17 about here---

With these conditions satisfied, we turn to examine the structure of the network using the high-density clustering technique. Figure 18 compares the clustering trace of the network created by the differential access model to the clustering output for the industrial network from Figure 10A. With the possible exception of a few isolated points of high-density in the actual interlock network and a single isolated pair of firms in the model network, the clustering structures are remarkably similar. The small differences in density that do exist between the two may be principally attributable to the regional patterns of interlocking noted above, which is not captured in the differential access model.

---Figure 18 about here---

Thus, we should approach with caution the decision to partition the corporate network along the lines suggested by the high-density clustering model. On one hand, the high-density clusters do identify the most heavily interlocked groups of firms in the network, according to the

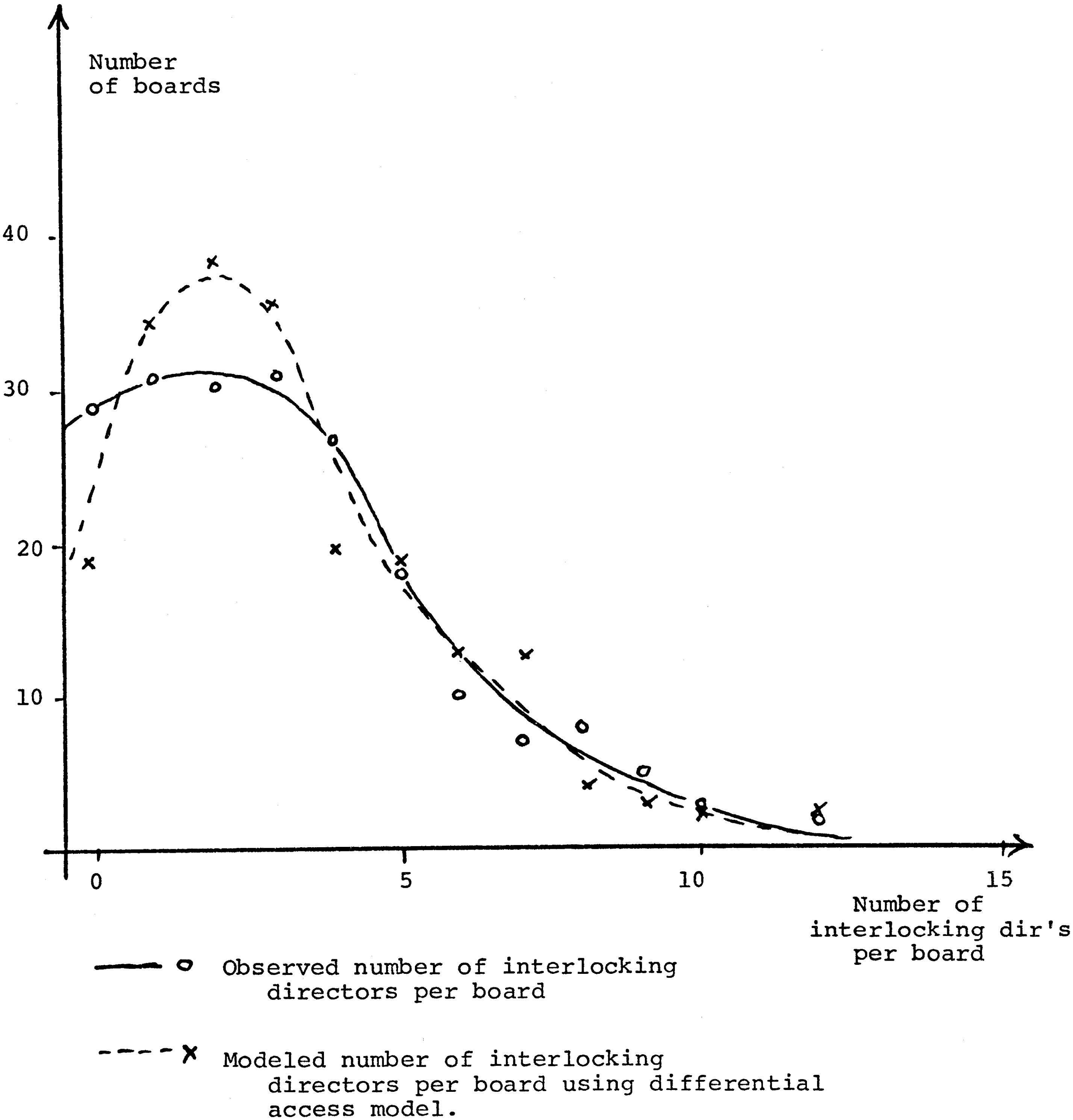


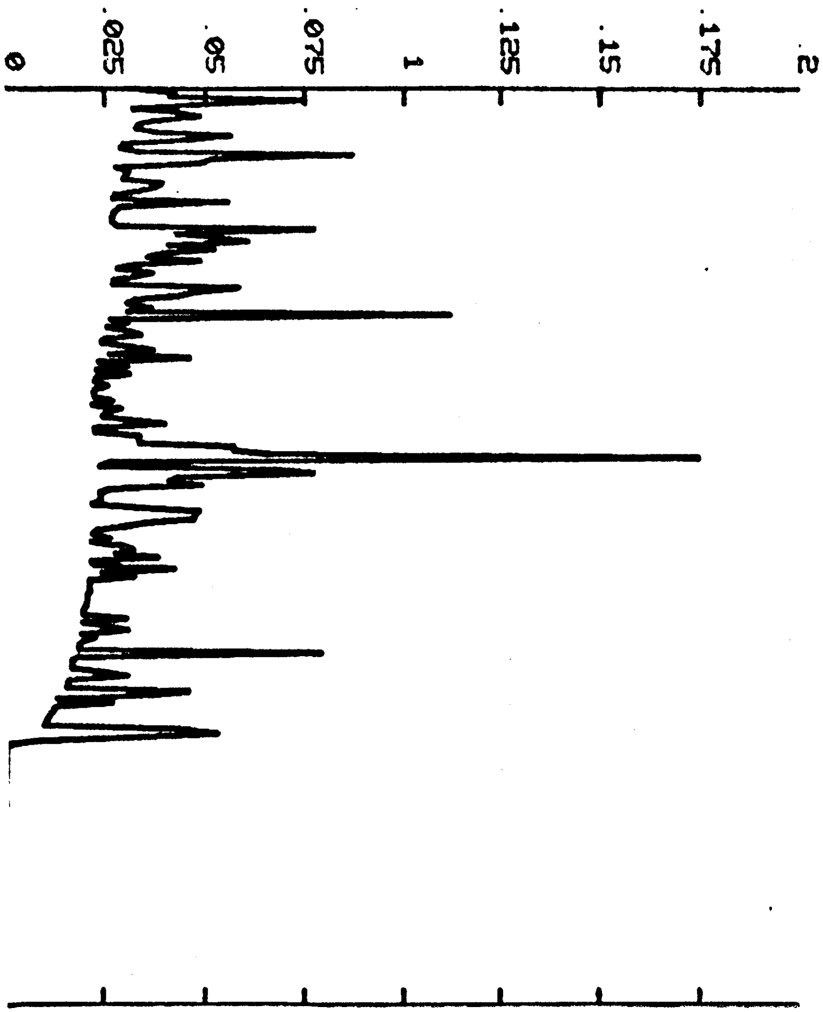
Figure 16

Distributions of interlocking directors per board for the actual network and for the differentiated choice/differential access model.

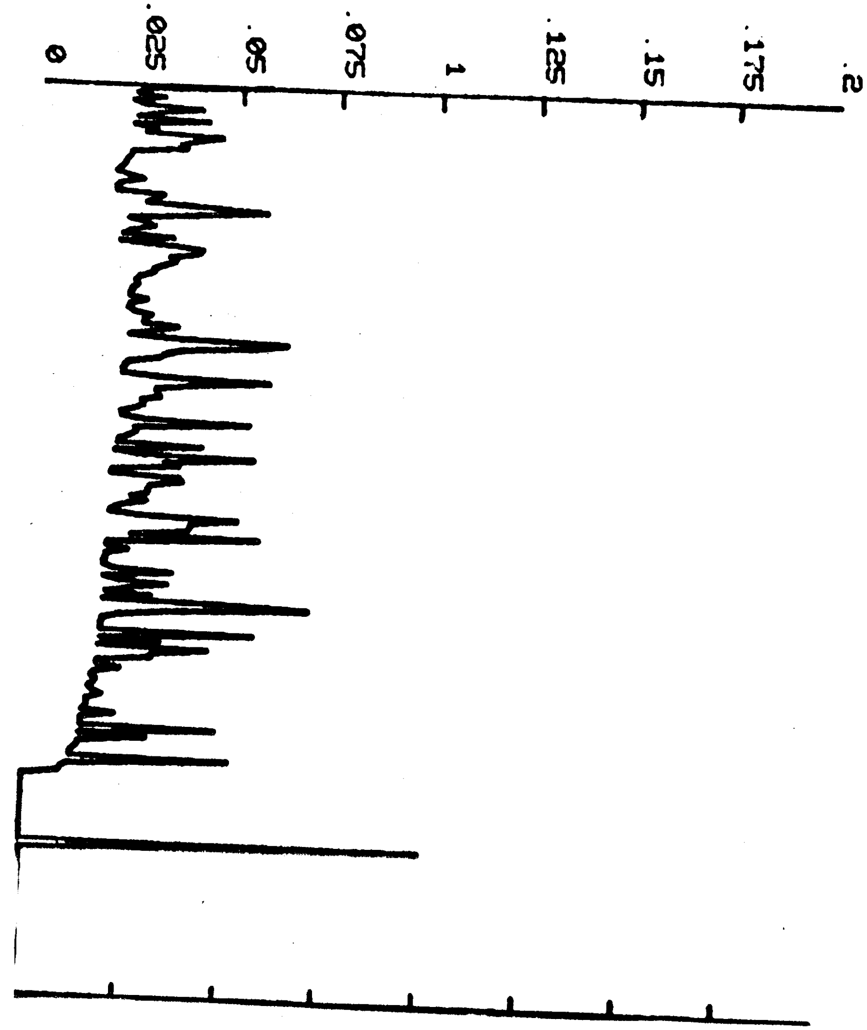
Number of interlocks per director	Actual Network	Differential Access Model
0	?	8339
1	2315	2392
2	194	190
3	56	54
4	15	22
5	4	2
6	1	1

Figure 17

Table comparing the number of interlocks per director for the actual interlock network and for the differential access model.



Clustering trace for the actual interlock network of 171 nodes.



Clustering trace for the differential access model

Figure 18

Comparison of clustering structures.

weighting scheme in (2) and the definition of density in (3). If one wants to break the network into component parts without dismantling the naturally occurring regions of higher-than-ordinary interlocking, then the high-density clusters should be left intact. On the other hand, the fact that the clustering structure of the actual interlock network is not strikingly different from the structure of the random network suggests that there exist no natural lines along which to partition the network.

The results here give rise to two promising directions for future research. One possible approach is to probe further to uncover some structural difference between the random network and the actual interlock network using some enhanced version of the high-density clustering technique. Further improvements to the technique have been targeted in the Systematic Design Methodology, including some form of sensitivity diagnostic to enable systems designers to identify critical design specifications that exert some disproportionate influence on the clustering structure. Applying the diagnostic to the actual interlock network and to the random network might reveal certain differences in the "robustness" of each network's structure. Another approach involves examining the random network with indirect interlocks included, to identify any second order structural differences.

Another promising direction involves comparing the structural changes in the model network and the actual interlock network as the networks grow in size. There may be certain size-dependent characteristics of the interlocking phenomenon not captured by the differential access model. Implicit in this approach is the further development of the random model to include financial-type interlocks in the network.

Each of these approaches requires some means of identifying significant structural differences between clustering traces. Up until now, we have relied upon "eyeballing," but this may overlook certain subtle differences. Further development in this area is indicated.

7. Conclusion

We have presented a new approach to understanding the phenomenon of interlocking among the largest of the corporations in the United States. We adopted the network representation previously employed by investigators and modified the weighting scheme to reflect the approximate proportional overlap of potential directorate choice sets, so as to be able to include indirect interlocks into the network. Rather than attempt to "picture" the network in some multidimensional social space, we sought to investigate the extent to which regions of higher-than-ordinary interlocking appeared in the network. To do so, we used a high-density clustering technique based on a graph.

The nature of the high-density clustering solution raised some questions about the validity of pursuing further understanding of the phenomenon of interlocking by focusing on the characteristics of the high-density components in piecewise fashion. The global clustering structure of the actual network is very close to that of a model network constructed under a policy of random director choice. Until we reach a better understanding of the differences between these structures, it seems best not to impart too much social significance to the composition of the "high-density" groups in the network.

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