

# FPAKE: Fuzzy Password-Authenticated Key Exchange

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joint work with

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# Motivation



p@\$\$w0rd12

- Want: secure communication
- Over insecure, unauthenticated channel
- Shared secret: password
- The password is...
  - Low-entropy



p@\$\$w0rd12

# Motivation



p@\$\$w0rd12

- Want: secure communication
- Over insecure, unauthenticated channel
- Shared secret: password
- The password is...
  - Low-entropy
  - Possibly noisy



p@\$\$w@rd12

# Motivation



p@\$w0rd12



p@\$w@rd12

- Goal: Agree on high-entropy cryptographic key
- Man-in-the-middle security: Nothing leaks about...
  - Password
  - Key

# Applications

- Mistyped passwords  
e.g. [Chatterjee-Athalye-Akhawe-Juels-Ristenpart-16]



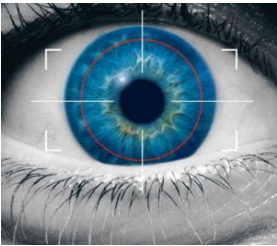
p@\$\$w0rd12



p@\$\$w@rd12

# Applications: Not Just Passwords!

- Mistyped passwords
- Biometric authentication



Bob has a resource Alice is trying to access



# Applications: Not Just Passwords!

- Mistyped passwords
- Biometric authentication
- Location-based authentication  
e.g. [Han-Harishankar-Wang-Chung-Tague-17]



“radiation stinks  
has 3 potholes”



“radiator springs  
has 4 potholes”



# Related Work



are the passwords **low-entropy**?

**low-entropy**: can hit correct password by brute-force enumeration

do the passwords  
have **noise**?



# Related Work



are the passwords **low-entropy**?

do the passwords  
have **noise**?

	Low-entropy password	High-entropy password
Exact match		
Fuzzy match		



# Related Work



	Low-entropy password	High-entropy password
Exact match		privacy amplification [Maurer-97, ...]
Fuzzy match		



# Related Work



	Low-entropy password (no leakage allowed)	High-entropy password (some leakage ok)
Exact match		privacy amplification [Maurer-97, ...]
Fuzzy match		



# Related Work



	Low-entropy password (no leakage allowed)	High-entropy password (some leakage ok)
Exact match	PAKE [Bellare-Pointcheval-Rogaway-00, Boyko-MacKenzie-Patel-00, ...]	privacy amplification [Maurer-97, ...]
Fuzzy match		

Secure against off-line dictionary  
attacks against the password



# Related Work



	Low-entropy password (no leakage allowed)	High-entropy password (some leakage ok)
Exact match	PAKE [Bellare-Pointcheval-Rogaway-00, Boyko-MacKenzie-Patel-00, ...]	privacy amplification [Maurer-97, ...]
Fuzzy match		information reconciliation [Renner-Wolf-04, ...] robust fuzzy extractors [Boyen-Dodis-Katz-Ostrovsky-Smith-05, ...]



# Related Work



	Low-entropy password (no leakage allowed)	High-entropy password (some leakage ok)
Exact match	PAKE [Bellare-Pointcheval-Rogaway-00, Boyko-MacKenzie-Patel-00, ...]	privacy amplification [Maurer-97, ...]
Fuzzy match	generic multi-party computation without authenticated channels [Barak-Canetti-Lindell-Pass-Rabin-05]	information reconciliation [Renner-Wolf-04, ...] robust fuzzy extractors [Boyen-Dodis-Katz-Ostrovsky-Smith-05, ...]



# Fuzzy PAKE



	Low-entropy password (no leakage allowed)	High-entropy password (some leakage ok)
Exact match	PAKE [Bellare-Pointcheval-Rogaway-00, Boyko-MacKenzie-Patel-00, ...]	privacy amplification [Maurer-97, ...]
Fuzzy match	This paper - Fuzzy PAKE	information reconciliation [Renner-Wolf-04, ...] robust fuzzy extractors [Boyen-Dodis-Katz-Ostrovsky-Smith-05, ...]

## Our Contributions

- Security definition
  - Two efficient constructions
- of Fuzzy Password Authenticated Key Exchange



# Fuzzy PAKE



	Low-entropy password (no leakage allowed)	High-entropy password (some leakage ok)
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## Our Contributions

- Security definition
  - Two efficient constructions
- of Fuzzy Password Authenticated Key Exchange

I learned nothing  
about the key or  
the passwords

# FPAKE Definition



p@\$\$w0rd12  
high-entropy key k



p@\$\$w@rd12  
high-entropy key k

If the passwords are “similar enough”  
the parties agree on a key

# FPAKE Definition

I learned nothing  
about Bob's  
password



"curiouser"

I learned nothing  
about Alice's  
password



"can we fix it?"

If the passwords are not "similar enough"  
the parties do not agree on a key

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I learned nothing  
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I learned nothing  
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“can we fix it?”

If the passwords are not “similar enough”  
the parties do not agree on a key

- **Definitional Goals:**
  - Composability (essential for key agreement!)
  - Security against offline dictionary attacks by:
    - Malicious participant
    - Man in the Middle
- **Approach: Generalize UC functionality for PAKE**  
[Canetti-Halevi-Katz-Lindell-MacKenzie-05]



# Fuzzy PAKE



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## Our Contributions

- UC security definition
  - Two efficient constructions
- of Fuzzy Password Authenticated Key Exchange

# Constructions

- Using PAKE + Robust Secret Sharing (RSS)
- Using Yao's Garbled Circuits (YGC)
  - Not generic 2PC - specialized, secure variant of dual execution!

FPAKE construction	PAKE/RSS	Yao's Garbled Circuits
Notion of similarity	Hamming	Any
# rounds	2	5
# exponentiations	$2n + \text{constant}$	$3n + \text{constant}$

$n$  = number of password characters

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FPAKE construction	PAKE/RSS	Yao's Garbled Circuits
Notion of similarity	Hamming (same-length passwords)	Any
# rounds	2	5
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# Fuzzy PAKE from PAKE/RSS



$\text{pw}_A = \text{p}@\$\$w0rd12$

$\text{pw}_B = \text{p}@\$\$w@rd12$

Pick a random session key  $K$

“magical encryption”  
of  $K$  that tolerates  
errors in the  
encryption key  $\text{pw}_A$

$C = \text{Enc}(\text{key}=\text{pw}_A, \text{msg}=K)$

- **Problem:** ciphertext enables offline dictionary attack!
- **Solution:**

1. **expand** each character using PAKE!
2. **magical encryption** using resulting **character keys**

$K = \text{Dec}(\text{key}=\text{pw}_B, C)$



# Fuzzy PAKE from PAKE/RSS: the Expansion Step



$\text{pw}_A = \text{p}@\$\$w0rd12$

$A = [A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 A_9 A_{10}]$



$\text{pw}_B = \text{p}@\$\$w@rd12$

$B = [B_1 B_2 B_3 B_4 B_5 B_6 B_7 B_8 B_9 B_{10}]$



...



...





# Fuzzy PAKE from PAKE/RSS: the Expansion Step



$\text{pw}_A = \text{p}@\$ \$\text{w}0\text{rd}12$

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$B = [B_1 | B_2 | B_3 | B_4 | B_5 | B_6 | B_7 | B_8 | B_9 | B_{10}]$

- It is important to hide whether agreement happened!
- Otherwise, locations of matching characters will leak
- **New UC definition:** implicit-only PAKE
  - no key confirmation: participants do not know whether agreement was successful
  - “EKE2” [Bellare-Pointcheval-Rogaway-00] is implicit-only



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$B = [B_1 | B_2 | B_3 | B_4 | B_5 | B_6 | B_7 | B_8 | B_9 | B_{10}]$

Robust Secret Sharing  
of  $K$   
+  
One Time Pad

$C = \text{Enc}(\text{key}=A, \text{msg}=K)$

Similar to Code-Offset [Juels-Watenberg-02]

$K = \text{Dec}(\text{key}=B, C)$

# Constructions

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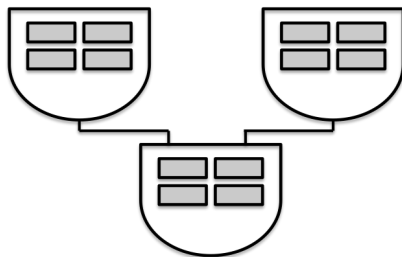
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# Yao's Garbled Circuits for FPAKE



garbler



evaluator

—————→ output

# Yao's Garbled Circuits for FPAKE



$pw_G = p@\$\$w0rd12$

garbler

$pw_E = p@\$\$w@rd12$



evaluator

Circuit determining  
whether  $pw_G$  and  $pw_E$   
are “close enough”;  
outputs session key if yes

output

# Yao's Garbled Circuits for FPAKE



$pw_G = p@\$\$w0rd12$

semi-honest  
garbler

$pw_E = p@\$\$w@rd12$



malicious  
evaluator

Circuit determining  
whether  $pw_G$  and  $pw_E$   
are “close enough”;  
outputs session key if yes

→ output

Yao's Garbled Circuits are an asymmetric 2PC protocol: they are secure against a malicious evaluator, but only against a semi-honest garbler

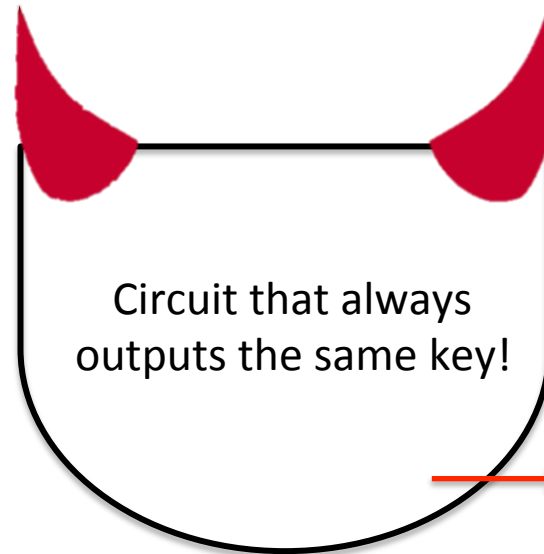
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~~semi-honest~~  
garbler

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Yao's Garbled Circuits are an asymmetric 2PC protocol: they are secure against a malicious evaluator, but only against a semi-honest garbler

# From Semi-Honest to Malicious

Correctness	Privacy	Computation Overhead

# From Semi-Honest to Malicious

Transformation	Correctness	Privacy	Computation Overhead
None			
Commit-and-Prove			
Cut-and-Choose			
Gate-wise Cut-and-Choose			(including pre-processing)
...			

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Transformation	Correctness	Privacy	Computation Overhead
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Commit-and-Prove			
Cut-and-Choose			
Gate-wise Cut-and-Choose			(including pre-processing)
...			
<b>Dual Execution [Mohassel-Franklin-06, Huang-Katz-Evans-12]</b>		1 bit leakage	Only 2x!

1 bit of leakage about a low-entropy password can be crucial!

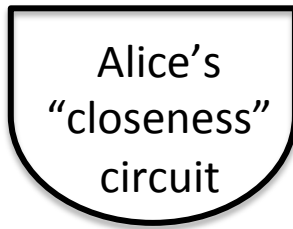
We modify dual execution specifically for Fuzzy PAKE to avoid leakage when it matters

$pw_G = p@\$\$w0rd12$

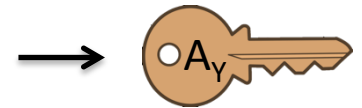
$pw_E = p@\$\$w@rd12$



# Dual Execution for FPAKE



circuit that  
outputs yes/no  
yes/no key

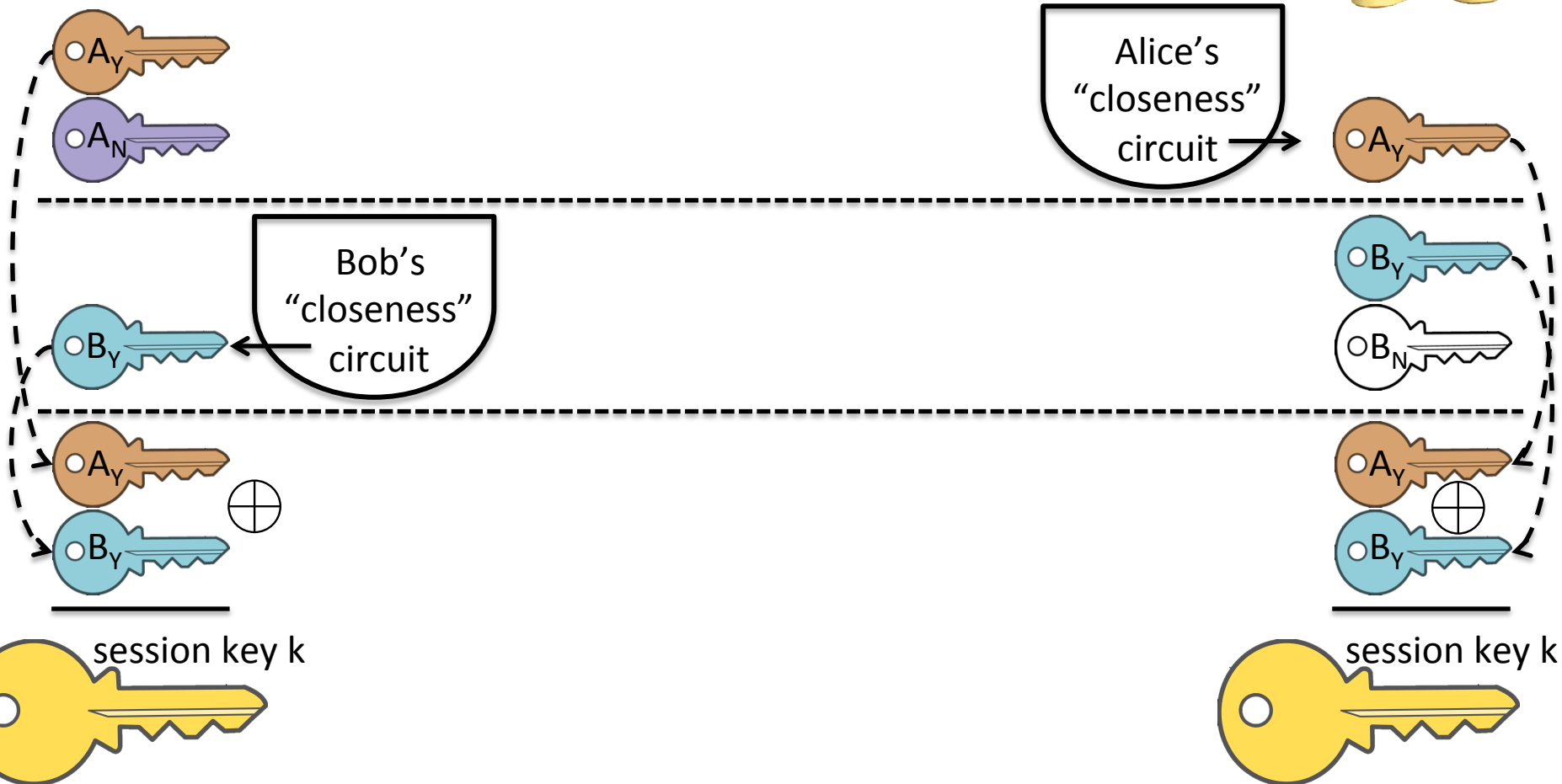


$pw_G = p@\$\$w0rd12$

$pw_E = p@\$\$w@rd12$



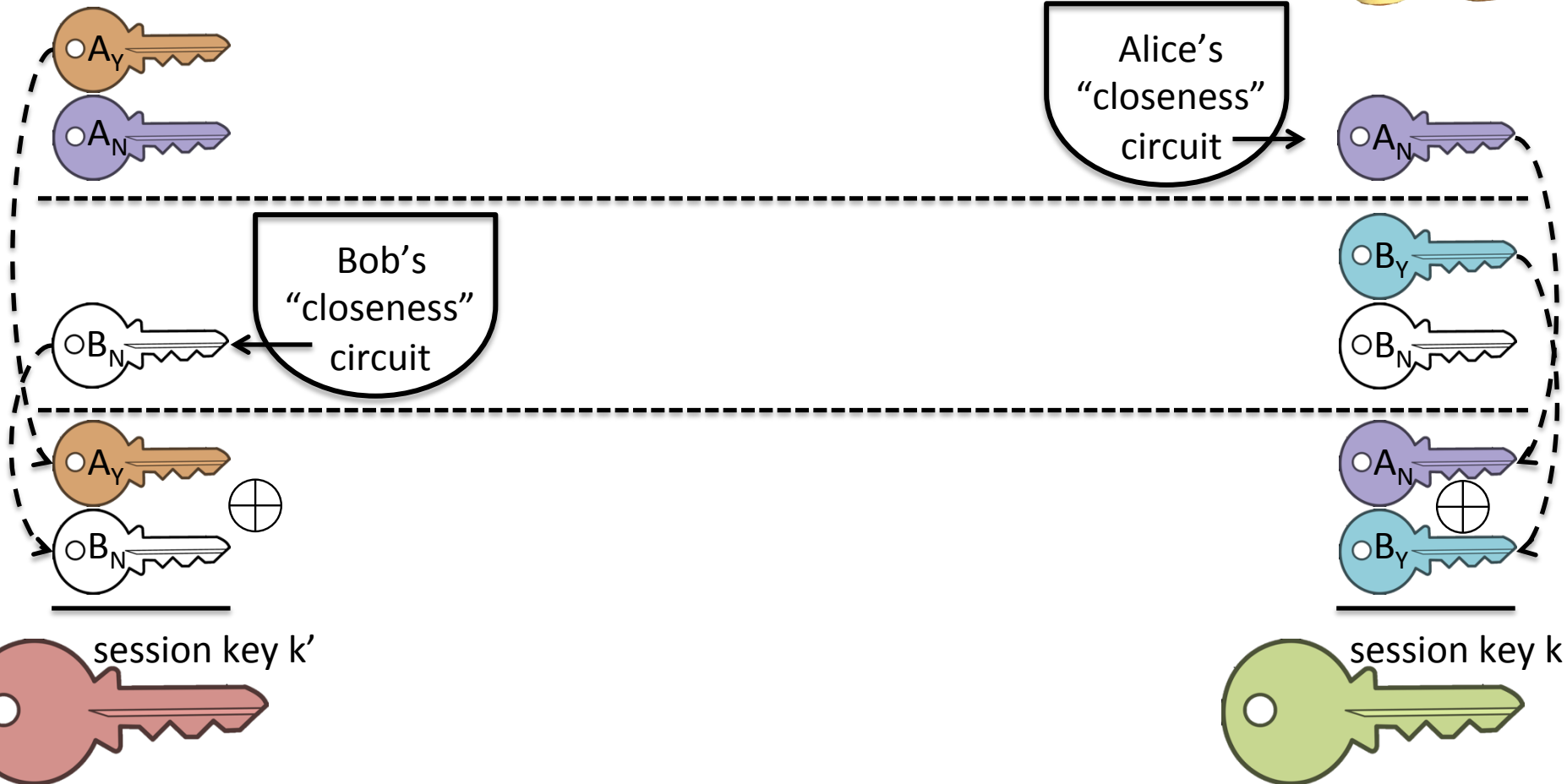
# Dual Execution for FPAKE



$pw_G = \text{"curiouser"}$

$pw_E = \text{"can we fix it"}$

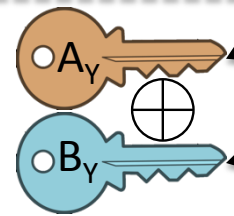
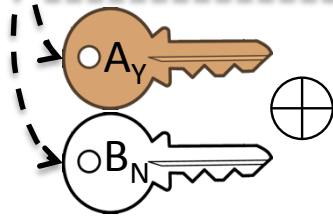
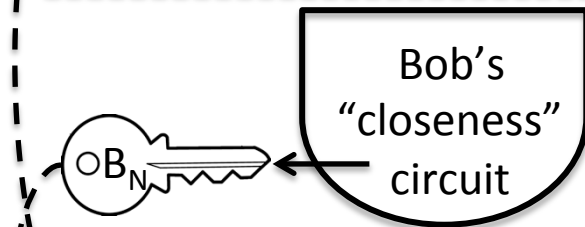
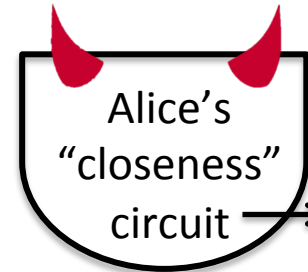
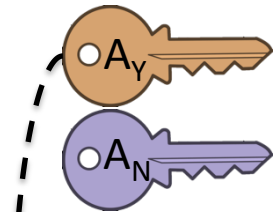
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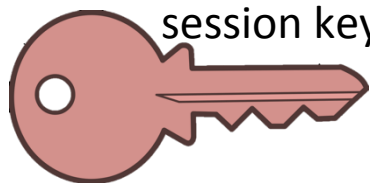
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# Dual Execution for FPAKE



session key  $k'$

session key  $k$



$pw_G = \text{"curiouser"}$

$pw_E = \text{"can we fix it"}$

# Dual Execution for FPAKE



No clue about  
!

Alice's  
"closeness"  
circuit

Bob's  
"closeness"  
circuit

session key  $k'$

session key  $k$



# Dual Execution: Privacy-Correctness Tradeoff for Boolean Functions



[MF'06, HKE'12] Dual Execution	Correct output	Comp. output	Privacy
	"yes"	"yes" or "cheating"	1-bit leakage
	"no"	"no" or "cheating"	1-bit leakage
Our FPAKE Dual Execution	Correct output	Comp. output	Privacy
	"yes"	"yes" or "no"	1-bit leakage
	"no"	"no"	yes

This is the perfect tradeoff for fuzzy PAKE!

- Only care about security against adversary who doesn't know a close-enough password – the "no" case



# Conclusion



	Low-entropy password	High-entropy password
Exact match	PAKE	privacy amplification
Fuzzy match	This paper - Fuzzy PAKE	information reconciliation, robust fuzzy extractors

## Our Contributions

- UC security definition of Fuzzy PAKE
- Two efficient constructions:
  - PAKE + Robust Secret Sharing (more efficient)
  - Yao's Garbled Circuits (more general)