Analytical model of a rope and karabiner system

By Philippe Bertreau and Chris Pipe

1. Introduction

Various climbing accidents involving the failure of karabiners with their gate open have been reported. Karabiner gates are kept closed by the restoring force of a mechanical spring, and, in the case of locking karabiners, with the additional mechanism of a threaded sleeve that 'locks' the gate shut (see below). Karabiners that fail with their gate open clearly either belong to the former category or the latter in an unlocked position. There is also another class of karabiners with wire gates instead of solid gates, although we have no information on whether they have been found to fail with their gate open in a climbing environment.

That karabiners fail with their gate open is not surprising given that the tensile strength of a karabiner with an open gate is typically 7 kN as opposed to 25 kN for a karabiner with the gate closed. What is less obvious is how the gate was open when the karabiner failed. The two possible scenarios involve the gate being open before the fall which caused failure of the karabiner, or whether the act of falling itself in some way led to the opening of the gate and the subsequent failure of the karabiner. The aim of this analytical study is to investigate whether the karabiner gate could by excited and induced to open by oscillation of the rope.

| Straight gate karabiner | Locking karabiner | Wire gate karabiner |

3. Estimate of physical properties

3.1 Natural frequency of a karabiner gate

The restoring force in the karabiner gate is assumed to be perfectly elastic and the natural frequency of the system, *i.e.* the frequency at which the response of the system is greatest in relation to the forcing function, is found from the equation of motion:

$$J_0\ddot{\theta} = -C\theta$$,

where J_0 is the moment of inertia and C is the spring constant. This second order differential equation with constant coefficients has a solution of the form:

$$\theta(t) = \theta_{max} \sin(\omega t + \phi)$$,

where θ_{max} is the amplitude of oscillation, ω is the natural frequency, and ϕ is the phase shift. Thus we can calculate ω :

$$\omega = \sqrt{\frac{C}{J_0}}.$$

Estimate of spring constant C

The strength of the spring was found by measuring the force $F=Mg$ required to hold the spring open at an angle θ_0. This was carried out by attaching a known mass M to the end of the karabiner gate using dental floss and θ_0 was estimated from a digital photo using Adobe Photoshop. Thus C is estimated as:

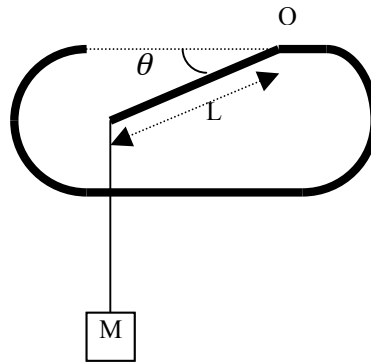$$C = \frac{MgL\cos(\theta_0)}{\theta_0} = 0.63 \pm 0.8\,\text{Nm.rad}^{-1}$$

Figure 1

Estimate of moment of inertia J_0

The moment of inertia J_0 of the gate about the hinge (point 0 shown in figure 1) is given by:

$$J_0 = \frac{ml^2}{3}$$

as we take the gate to be a cylinder in first approximation.

l is the length of the solid cylinder part of the karabiner gate. l=4.0cm±0.1mm

The mass m of the solid cylinder is given by $m = l\pi r^2 \rho$ with r the radius of the gate, and ρ its density. r=6mm±0.5mm and for the gate we took $\rho = \rho_A = 2,700$ kg/m^3. We thus get m=12.2±2.5 g
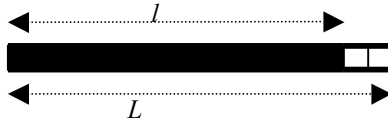
Figure 2: top view of karabiner gate

Eventually, J$_O$=6.51±1.72 10^{-6} kg.m^2 and using the formula $\omega = \sqrt{\dfrac{C}{J_O}}$ we calculate $\omega =$ 49.5±10 Hz.

3.2 Natural frequency of a rope

Plucked string analogy

The natural frequency ω of a thin plucked string can be found from :

$$\omega = \frac{\pi}{l}\sqrt{\frac{T}{\lambda}}$$

where l is the length of the rope, T is the tension and λ is the mass per unit length. For a typical climber of 80 kg falling 3 m (*i.e.* from 1.5 m above the last point of protection) on a 60 m rope weighing 10 kg, this suggests a natural frequency of 72 rad·s^{-1}. By reducing the distance fallen, the natural frequency can be increased : therefore for a fall of distance 0.7 m the natural frequency is around 311 rad·s^{-1}.

Climbing rope as a Hookean spring

A climber - climbing rope system could also be modeled as a mass on the end of a Hookean spring. Here the natural frequency ω is given by :

$$\omega = \sqrt{\frac{k}{m_{\text{climber}} + m_{\text{rope}}}}$$

where k is the elastic stiffness of the rope and ($m_{\text{climber}} + m_{\text{rope}}$) is the total mass of the system. A standard 60 m sports climbing rope which specified a 7% extension for the first fall of force 0.8 kN was used to estimate the spring constant, giving $k = 187$ N·m^{-1}. For an 80 kg climber, this gives us a natural frequency $\omega \sim 1.5$ rad·s^{-1}. It can be immediately seen that this gives a natural frequency which is an order of magnitude less than when the rope is plucked.

4. Discussion of results

The key results are summarized in Table 1. As can be seen the natural frequency of a rope can coincide with that of the karabiner although it is unlikely that such a small fall would dissipate enough energy to cause a karabiner to fail, even if the gate were open. However, the calculation presented in the previous section are only approximate and maybe for a larger fall the gate and rope frequencies are sufficiently close for the two to interact, opening the karabiner and making it fail.

It is clear from our results that the frequencies likely to open a karabiner gate are caused by a plucking motion rather than the Hookean spring mode of oscillation.

Table 1. Key results

Natural frequency of karabiner gate	311 rad·s^{-1}
Natural frequency of rope (plucked string, 0.7m)	311 rad·s^{-1}
Natural frequency of rope (Hookean spring)	1.5 rad·s^{-1}

From the calculations of the gate frequency it can be seen that the natural frequency of a wire gate karabiner is significantly higher due to the lower mass. From this point of view a wire gate is less likely to open due to rope oscillations in falls with significant loading.

5. Conclusions

That a karabiner gate can be opened and then the karabiner fail due to the oscillatory motion of a climbing rope during a fall is not conclusively shown in this study. However, what is seen is that the natural frequency of the rope and karabiner gate can become close in certain fall geometries. Perhaps this is sufficient to leave reasonable doubt and warrant further research in this area in order to reach a better understanding of karabiner - rope interactions. Meanwhile, it is suggested that climbers use wire gate karabiners!