The Yowie Factor

a simple estimate of load rate during climber fall arrest

Dave Custer
Center for Sports Innovation
Massachusetts Institute of Technology
an overview of this talk

the starting point

the simple model

some complications

final thoughts
motivation

both ice and ice screws exhibit reduced strength with increased load rate. A 100 X increase in rate halves the strength.

objectives

- determine degree of “impact”
- make impact accessible
- guide safety system design
methods

determine the load rate based on a (very) simple rope model

compare the simple model to complicated models (Wexler and Pavier)

compare the simple model to data (Mägdefrau)
expected kinematics behavior

Rope Behavior
Modeled Kinematics
UIAA Drop, m=80 kg, l=2.8 m, ff=1.7
Pavier Moduli & $\lambda$, $M_1=35$ kN, $M_2=20$ kN, $\lambda=3$ kNs

Tension (kN)
Tension (kN) (free fall)
Velocity (m/s)
Velocity (m/s) (free fall)
Position (m)
Position (m) (free fall)

time (s)
the simple model

the potential energy of fall height is exchanged for energy stored in the stretched rope spring:

\[ mgh = \frac{1}{2} m\dot{y}_{\text{max}}^2 = \frac{1}{2} ky_{\text{max}}^2 \]

simple mass/spring oscillation:

\[ f = -ky = m\ddot{y} \]

- \( f \): force on climber
- \( \dot{f} \): load rate
- \( F \): fall factor \((h/l)\)
- \( g \): gravity acceleration
- \( \gamma \): tension ratio
- \( h \): fall height
- \( k \): rope's spring constant
- \( l \): rope length
- \( \lambda \): damping coefficient

- \( m \): climber's mass
- \( M \): rope modulus
- \( \dot{y} \): rope stretch
- \( \ddot{y} \): climber velocity
- \( \dddot{y} \): climber acceleration
- \( y \): climber jerk
- \( \omega \): oscillation frequency
- \( Y \): yowie factor

(based on Wexler work)
the simple estimate of load rate

kinematics solutions:

\[ \omega = \sqrt{\frac{k}{m}} \]

\[ y = -\sqrt{\frac{2mgl^2F}{M}} \sin(\omega t) \]

\[ \dot{y} = -\sqrt{2gFl} \cos(\omega t) \]

\[ \ddot{y} = \sqrt{\frac{2gFM}{m}} \sin(\omega t) \]

\[ \dddot{y} = \frac{M}{m} \sqrt{\frac{2gF}{l}} \cos(\omega t) \]

\[ \dot{\theta} = \frac{d\dot{f}_{(i)}}{dt} = M \sqrt{\frac{2gF}{l}} \cos(\omega t) \]
inclusion of potential energy of stretch

kinematics solutions:

\[ \omega = \sqrt{\frac{k}{m}} \]

\[ y = -\sqrt{\frac{2mgI^2F}{M}} \sin(\omega t) \]

\[ \dot{y} = -\sqrt{2gFl} \cos(\omega t) \]

\[ \ddot{y} = \sqrt{\frac{2gFM}{m}} \sin(\omega t) \]

\[ \dddot{y} = \frac{M}{m} \sqrt{\frac{2gF}{l}} \cos(\omega t) \]

\[ f = \frac{df_{(i)}}{dt} = M \sqrt{\frac{2gF}{l}} \cos(\omega t) \]

\[ F \]

\[ Y \propto M \sqrt{\frac{F}{l}} \]

\[ a \text{ slight adjustment of boundary conditions leads to a 15%-20\% increase in force, but no change in frequency:} \]

\[ mgh + mg y_{\max} = \frac{1}{2} ky_{\max}^2 \]

\[ \dot{y} = g \left(1 + \sqrt{1 + \frac{2MF}{mg}}\right) \]

\[ \ddot{f} = mg \sqrt{\frac{M}{lm}} \left(1 + \sqrt{1 + \frac{2MF}{mg}}\right) \cos(\omega t) \]
complications

- rope damping
- carabiner friction
- belayer behavior
- energy absorbing systems
how does one compare a spring to a damped spring system?

I have no simple answer; rather, 4 complicated ones.
The addition of damping increases load rate.
“critical” damping produces a load rate midway between the top spring and the two springs in series.

Yowie, ISEA 2006 12
carabiner friction

1) increases the force on the top anchor but does not change the proportionality of the yowie factor

2) decreases the effective rope length and thus also increases the fall factor; expect a 20% increase in load rate & reduced proportionality
belayer behavior

the belayer can reduce the energy absorbed by the rope by allowing rope to slip through the belay device and by being lifted up. the reduced energy results in reduced force, increased time, and thus reduced load rate.

\[ mgh + mgy_{\text{max}} = \frac{1}{2} k y_{\text{max}}^2 \]

\[ \ddot{y} = g \left( 1 + \sqrt{1 + \frac{2MF}{mg}} \right) \]

\[ \dot{f} = mg \sqrt{\frac{M}{lm}} \left( 1 + \sqrt{1 + \frac{2MF}{mg}} \right) \cos(\omega t) \]
energy absorbing systems

EAS vs. no EAS
force on anchor/time plot

EAS reduce load rate
model compared to data

Mägdefrau data
square root of fall factor/length vs load rate

- single rope, \( r^2 \approx 0.04 \)
- half rope pair, \( r^2 \approx 0.5 \)

model and data correlate only loosely
conclusions as a rule of thumb: \[ Y \propto M \sqrt{\frac{F}{I}} \]

- Climbers can apply the rule by protecting the belay and using low “modulus” ropes.
- Use EAS and a dynamic belay.
- In the future, the rule might be used to guide the design of better ice screws and perhaps the use of plastic anchor components.
acknowledgements

• title slide photo by Luca Marinelli
• many thanks to the folks at MIT’s aero/astro department and center for sports innovation
• kudos to Susan Ruff for patient editorial comment

abridged bibliography


questions?

$$Y \propto M \sqrt{\frac{F}{I}}$$
stray EAS graphs
Rope Behavior
Modeled Kinematics
UIAA Drop, m=80 kg, l=2.8 m, ff=1.7
Pavier Moduli & λ, M1=35 kN, M2=20 kN, λ=3 kNs
alternate expression

\( Y \propto \frac{M}{I} \sqrt{h} \)