World War III or World War Z?
The Complex Dynamics of Doom

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“I know not with what weapons World War III will be fought, but World War IV will be fought with sticks and stones.”

— Albert Einstein

“The monsters that rose from the dead, they are nothing compared to the ones we carry in our hearts.”

— Max Brooks, World War Z

1 Abstract

During the history of civilization, humans have been subjected to different threats to human existence: natural disasters, wars, infectious diseases, *etc.* Strongly influenced by recent media and entertainment, the fictional or metaphorical possibility of a zombie apocalypse now occupies the public's imagination. Here we attempted to improve on existing exercise in zombie-human modeling, by implementing the notion of panic into the dynamics of a zombie apocalypse. We modeled and simulated a situation where zombies and humans are cohabitants of a setting similar to Manhattan in terms of population. We utilized ordinary differential equations (ODE) to calculate dynamics between populations of zombies (Z), susceptible individuals (S), panicked individuals (P) and fighters (F). A simplified fixed point analysis of the dynamics uncovers equilibria between the modeled populations, indicating the existence of an optimal distribution of different populations in some situations. We interpret the findings both mathematically and in terms of their "practical" implications for a hypothetical (or pending) zombie apocalypse. Lastly, we comment on the implications of our work for preventative measures, intervention methods and policy, both in the context of a zombie apocalypse and disasters in general.

2 Introduction

Sixty five million years have passed since the Alvarez event, widely understood to have caused the extinction of dinosaurs (Pope et al., 1998). One hundred thousands years ago, modern *homo sapiens* arose and have come to dominate the earth to such a degree that the current geological epoch is informally referred to as the Anthropocene (Stromberg, 2013). Much of the *Homo sapiens*' influence on earth has been concentrated in the last few centuries, as the industrial revolution led to advances in science, medicine and technology, which created limitless possibilities for travel and information exchange. Nonetheless, we remain vulnerable to extinction-level attacks of various kinds. Should an Alvarez-like event occur today, it would likely have an effect analogous to that on dinosaurs 65 million years ago: massive instant death, followed by the overthrow of the social order, the onset of starvation and collapse. But meteorites striking the planet are hardly the only threat to modern *Homo sapiens*. Our species is consistently threatened by warfare, weapons of mass destruction, hurricanes, tsunamis, earthquakes, and viral pandemics. Some have even modeled doomsday scenarios for *Homo sapiens*, some of which predict a pending population collapse within the first half of the 21^{st} century (Meadows, 1974).

The general fear of cataclysmic population collapse from an unforeseen cause has, in the last century, become the object of public fascination. This has engendered fictional depictions of doomsday scenarios of various kinds, books and films that explored post-apocalyptic worlds. In the 1960s, a new fascination arose: the fear of the "Zombie Apocalypse", or rather, doomsday scenarios where zombies threatened the existence of humans (Romero et al., 1968). This fear entailed the rise of zombies, former humans who underwent a transformation into an "undead" state, somewhere between living and dead. To some, this phenomena is purely entertainment; to others, it provides a useful metaphor and social commentary on power, deception, and its effect on our collective psyche. The zombie phenomenon has since exploded around the world, now the subject of several multi-million dollar industries and a thriving artistic subculture.

Even further, this fascination has influenced the way scientists describe their work, as zombie-like behavior is often invoked in other contexts. For example, "zombie-ants" have been reported when infected by the fungus *Ophiocordyceps unilateralis*. Once infected, infected ants abandon the trees where they live and die while the spores of the fungus are still viable (Pontoppidan et al., 2009). So widespread is the interest in the zombie apocalypse that the Center for Disease Control (CDC), created a set of hypothetical zombie preparedness protocols (CDC, 2012). While obviously fictional, the reason why the preparedness protocols are humorous is because many of the social dynamics of a zombie apocalypse resembles other disaster scenarios, where people must be protected, quarantined, and avoid contact with an affected population.

Not surprisingly, mathematical modelers have joined the discussion, using analytical equations to examine hypothetical scenarios where the planet is attacked by zombies, coming to a range of conclusions about the feasibility of a zombie apocalypse (RPM, 2007). In addition, computer scientists have created web-accessible simulations, that allow users to create scenarios on their own (`http://www.kevan.org/proce55ing/zombies/`). Regardless of how one feels about zombies or their widespread popularity in modern culture, they serve as an effective teaching tool for exploring questions, mathematical and anthropological, about how society responds to extinction-level events. Here we create a mathematical model for human-zombie dynamics on the island of Manhattan, in New York City. We utilize an S-I-R style model to examine dynamics under a set of different parameters and interpret findings in the context of epidemiology, policy and the science of extinction-level events.

3 Methods

3.1 Model

Our model is composed of people and zombies. People can be divided as:

- Fighters (F) are the ones that attack and are resistant to zombies.
- People that panic (P) that are scared of zombies and the encounter rate is low.
- People susceptible (S) to an attack of zombies, and compared to P there encounter rate is larger

Zombies are only of one type (Z).

- Zombies (Z) are the creatures able to convert S and P into themselves

The basic interaction in the model consists of a two-body interaction between a "victim" species V and a "contagion" species C. These could be humans and zombies, calm and panicked, or simply healthy and sick. Assuming a uniform spatial distribution of each species (or, more specifically, no correlations in the spatial distribution), then the populations are summarized by aggregate (mean field) population statistics obeying

$$\begin{aligned}\frac{dV}{dt} =& -kVC \\ \frac{dC}{dt} =& kVC. \end{aligned} \tag{1}$$

The coefficient k is a product of a rate of encounters (say, 1000s of encounters per 1000 victims per 1000 infected per day) and a probability of an encounter resulting in a transformation. The equal and opposite form of the right hand sides of the equations indicate that every loss of a victim is a gain on infected, so that total number is a constant. Indeed, $d(V+C)/dt = 0$, so $V + C = T_0$, where T_0 is the initial total population. The system of equations then simplifies to

$$\frac{dC}{dt} = kT_0C - kC^2. \tag{2}$$

Despite the nonlinearity of this equation, it can be solved exactly. The result is

$$C(t) = \frac{T_0}{2}\left[\tanh\left(\sqrt{kT_0}(t - t_{1/2})\right) + 1\right], \tag{3}$$

where $t_{1/2}$ is the time at which the population is 50/50 between the two species. This solution starts at zero in the distant past, then transitions smoothly but rather suddenly to the value T_0. In the typical application of this two species model to the case of an initially very small infection, then the solution is characterized by a long incubation time (long in the same sense that the initial infection is small) and a short onset time controlled by k.

In the full model with many such interactions between different species, there will be ranges of parameters and coefficients where the dynamics of the whole are dominated just a single two-species interaction, at least for a period of time. In this case, the full set of nonlinear coupled differential equations can be understood as a simple system whose collective behavior is well described as the sum its parts. However, generically, all transformations are in play at the same time, nonlinearly modulating each other's rates, and the behavior of any part can only be solved in full consideration of the whole.

3.2 Equations

The differential equations of the model are:

$$\frac{dS}{dt} = b_R\left(1 - \mu_S\right)S + b_R\left(P + F\right) - \left[\left(\beta_P + \beta_F + \beta_Z\right)Z + \Omega P + \Lambda F\right]S, \tag{4}$$

$$\frac{dP}{dt} = \left(\Omega P + \beta_P Z\right)S - \left(\alpha Z + \mu_P\right)P, \tag{5}$$

$$\frac{dF}{dt} = \left(\Lambda F + \beta_F Z\right)S - \mu_F F, \tag{6}$$

$$\frac{dZ}{dt} = \left(\alpha P + \beta_Z S\right)Z - \left(VF + \mu_Z\right)Z, \tag{7}$$

where the parameters are defined in Table 1. These equations are integrted by the forward Euler method.

4 Results

Figures 1–4 describe the dynamics of a zombie apocalypse on the Manhattan island across a range of parameters, scenarios and timescales. Figure 1 displays the results of a model with low encounter rate, high panic contagion rate (0.90), run for just over a century. One can observe how the susceptible population drops abruptly within a decade, as the zombie and panic populations rise. A local maxima for the panicked population (P) population occurs after approximately one decade, before the zombies eventually begin to find, and convert, those in panic. This leads to an increase in zombies that peaks around 17 years post-spotting the first zombie. Also during this first decade is a slow growth in the

Parameter	Description
S	number of susceptibles
P	number of susceptibles that are panicked
F	number of susceptibles that have converted into fighters
Z	number of zombies
b_R	birth rate of S
Ω	rate of encounter S and P $\times$ probability of panicking
Λ	rate of encounter S and F $\times$ probability of becoming a fighter
β_P	rate of encounter S and Z $\times$ probability of panicking
β_F	rate of encounter S and Z $\times$ probability of becoming fighter
β_Z	rate of encounter S and Z $\times$ probability of becoming a Zombie
α	rate of encounter P and Z $\times$ probability of becoming a Zombie
V	rate of encounter F and Z $\times$ probability of killing a Zombie
μ_i	mortality rate of i

Table 1: Parameters used in the Equations (4–7).

fighter population, which eventually causes the decline in the zombie population. At approximately 25 years, we observe a new rise in the panicked population, even though the actual number of zombies is quite low. This is because of the high panic-conversion rate in this iteration of the model, where panic can spread even with very few zombie conversions. This trend—few zombies causing a lot of panic—is one that is consistent across iterations of the model.

Figures 2 and 3 occur at an even shorter timescale, the course of days. Figure 2 displays the dynamics for a low panic encounter scenario, where population density is high, but the propensity for panic is low. In this scenario, there is zombie spike at nearly 28 days, with an eventual spike in zombies followed by a rise in fighters, leading to a zombie collapse. Figure 3 shows a high panic scenario, with a low population density (encounter rate). The higher panic causes a longer-term and far more costly zombie scenario. Fighters eventually win, but only after a much longer "battle" with zombies.

Figures 4 and 5 describe the dynamics over the course of 2.5 millennia. Figure 4 shows the dynamics of the model depicted in Figure 1, and Figure 5 the dynamics with a much lower panic contagion rate of 0.50 (panic spreads less rapidly). Figure 4 demonstrates zombie uprisings just about every century, with a stable equilibrium arising only after about 2500 years of co-existence. Figure 5 communicates a similar oscillations but with a much larger variance, secondary to the relatively low (0.50) panic contagion rate, that pushes the zombie population to the brink of extinction at various points. The inability to full exterminate the zombies, however, leads to their resurgence, until a standing equilibrium arises at approximately 2000 years.

Figures 6–8 demonstrates interactions between populations in our dynamic model. Figure 6 corresponds to the complexity patterns observed between populations in the iteration described in Figure 1; here there are fixed points where

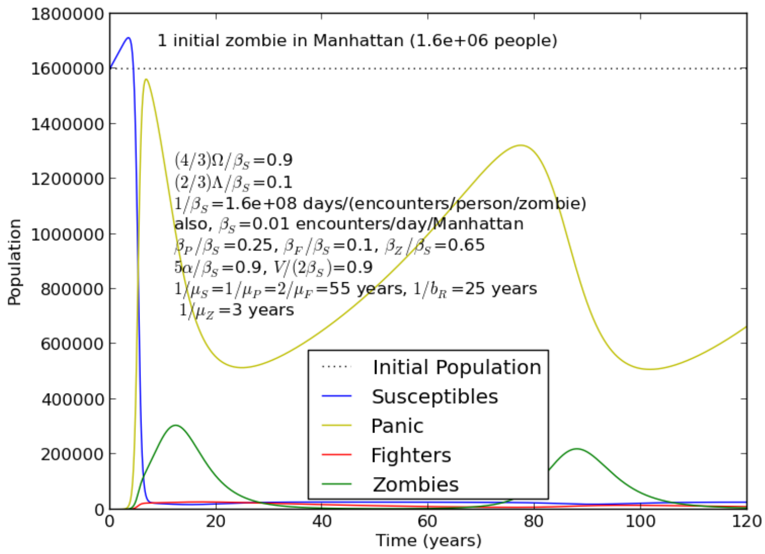

Figure 1: A model with low encounter rate and high panic contagion rate.

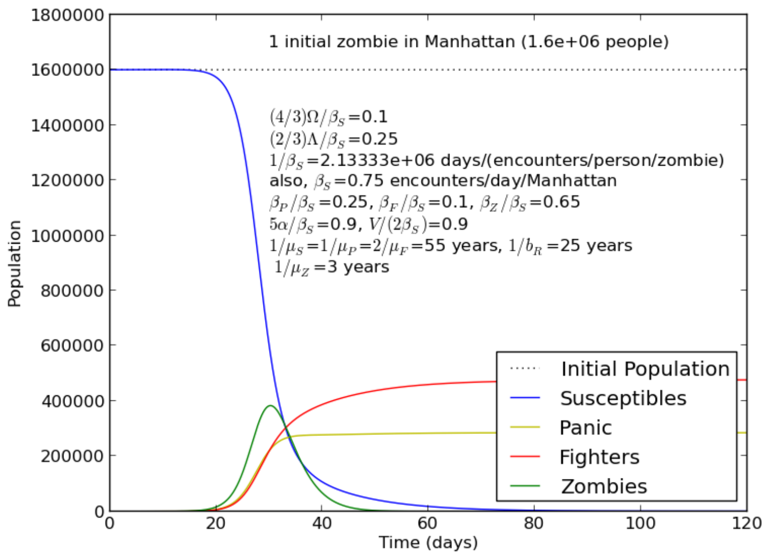

Figure 2: A model with high encounter rate, low panic contagion, and high fighter training rate.

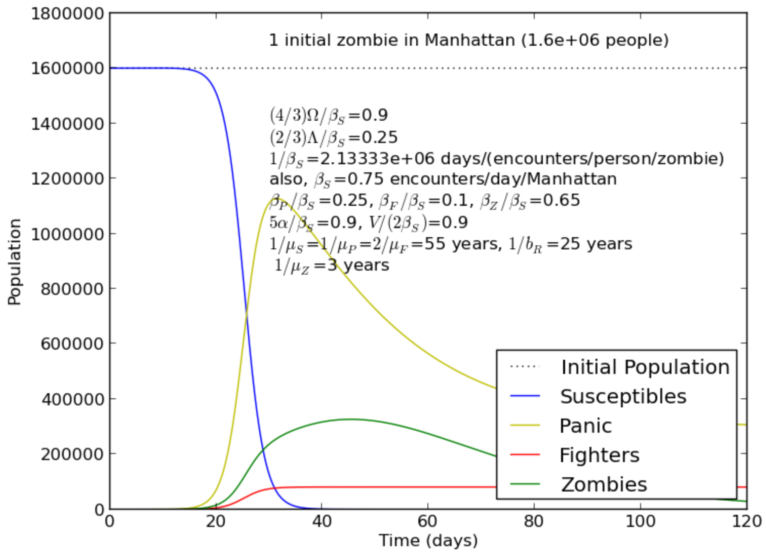

Figure 3: The same as Figure 2, except with a higher panic contagion rate.

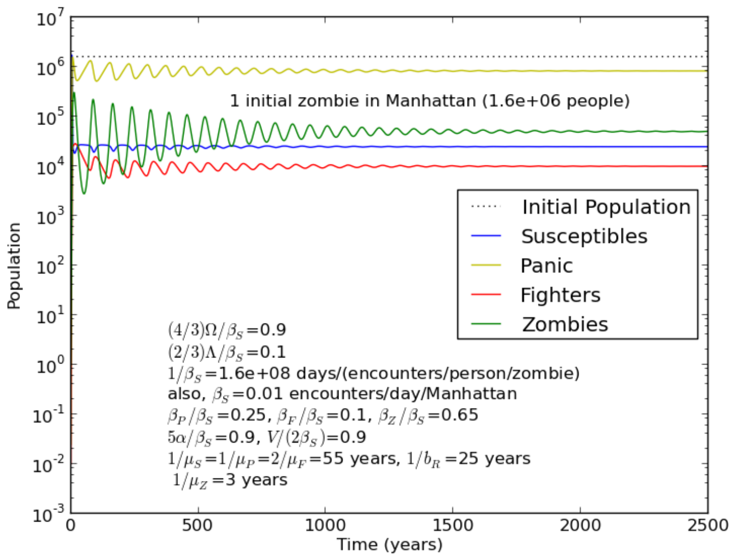

Figure 4: Same model as in Figure 1, plotted over a longer time scale.

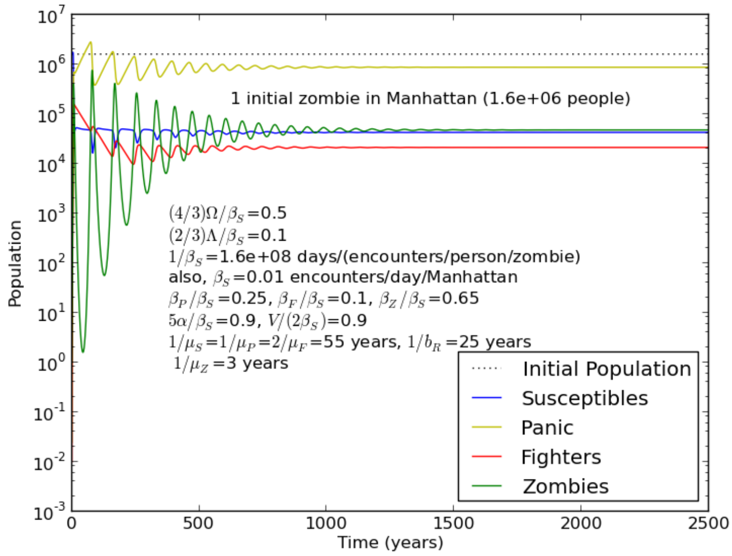

Figure 5: Same model as in Figure 4, with a lower panic contagion rate.

the derivatives of the functions go to zero, indicating a long term equilibrium between populations. Figure 7 corresponds to Figure 2, and Figure 8 to Figure 3. In both of the later high encounter cases (0.50 and 0.90), we observe dynamic relationships between zombie populations and the number of fighters and panicked individuals.

5 Discussion

The results make a range a predictions and observations for zombie apocalypse behavior across a range of parameter values. These interpretations differ depending on the scale in which we are observing phenomenon. Over the course of months, high panic scenarios lead to zombie apocalypses with a lower maximum value (number of zombies), but apocalypses that last for much longer, and are ultimately more costly. On the scale of years and millennia, high panic creates oscillatory behavior, where zombies routinely stage comebacks in the population, secondary to the existence of panic. Panic has a very low decay rate, and so persists in populations even after zombies have largely disappeared from existence. This residual panic, however, becomes fodder for these new zombie apocalypses. Even more troubling is the long term analysis of high panic scenarios, which indicate oscillatory behavior over the course centuries, with an equilibrium arising well after 2000 years. A scenario with slightly lower panic (intermediate) also leads to oscillatory behavior, with a lower nadir, and an equilibrium arising nearly 500 years prior to the high panic scenario.

In general, the model demonstrates how the presence of panic in a population responding to the presence of zombies can drive long term dynamics that are

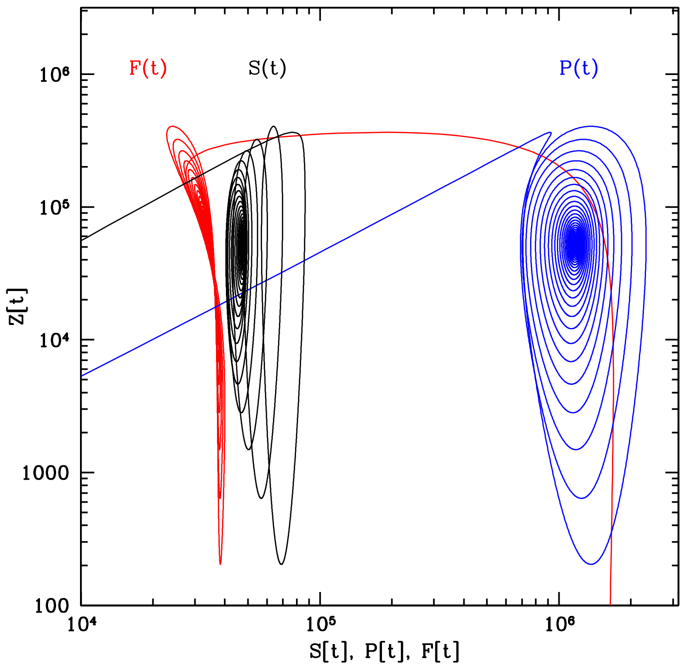

Figure 6: Phase space diagram (Z *vs.* $S + P + F$) for the model in Figure 1.

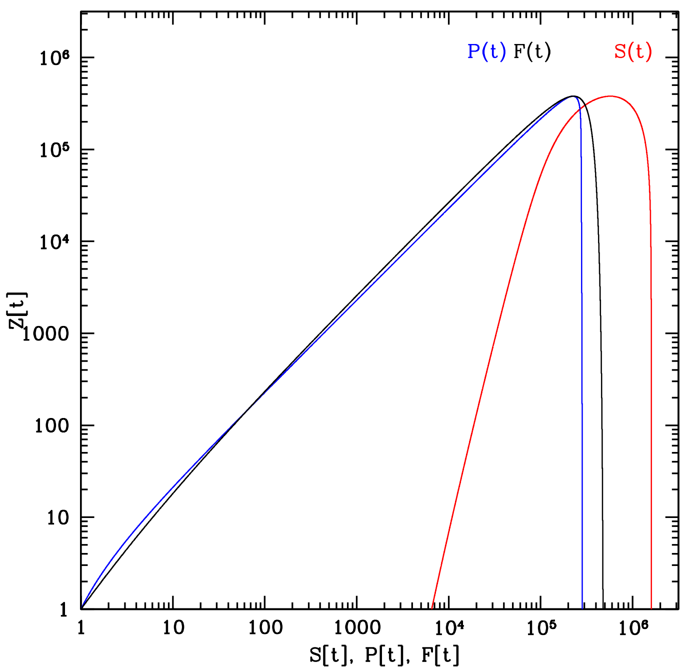

Figure 7: Phase space diagram (Z *vs.* $S + P + F$) for the model in Figure 2.

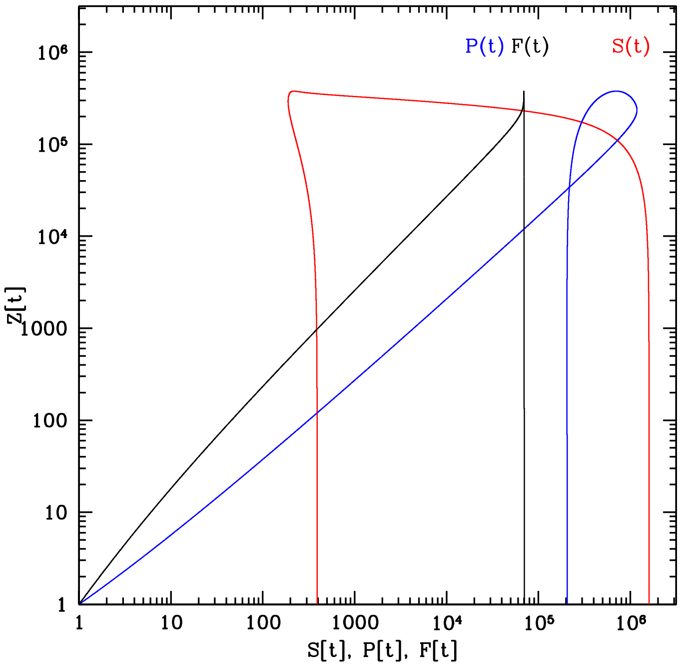

Figure 8: Phase space diagram (Z *vs.* $S + P + F$) for the model in Figure 3.

costly in human lives. That panic is so central to the dynamics can be seen in analogy to historical events where a high degree of panic had negative effects — even widespread death and destruction. The model affirms the use of zombies as an effective teaching tool, and metaphor for the power of panic to have deleterious effects on a population. On the long term scale, the oscillatory behavior of the zombie-panic-fighter dynamics are troubling, indicating that panic, in fact, prevents the full eradication of the agent of destruction (zombies in this case). Policy implications suggest that, in times of crisis, panic should be controlled in addition to to the agent of destruction. Future iterations of the model will explore spatial components of the zombie apocalypse, as well as exploring different relationships between the parameters. For example, there might be scenarios where panic has differing levels as a function of time and space, or where there is an evolved relationship between panicked individuals and fighters (*e.g.* panicked individuals keep active standing armies, which reduce overall panic levels).

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