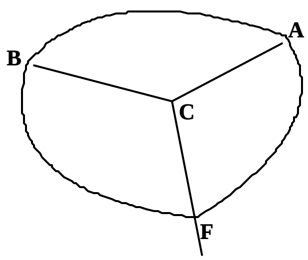
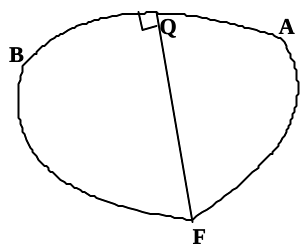
Huygens' Calculation of the Crystallographic Angles

To find the facet angles Huygens used one measurement and spherical trigonometry. The first figure below represents the three crystal facets that intersect at point *C*. The plane angles that have *C* as a common vertex are all equal to one another; we want to know the value. To find out, Huygens first measured the interfacial angle, which is the angle between any pair of contingent planes (e.g. the angle between the planes *ACB* and *BCF* in the figure) for which he obtained a value of 105°.
He could now compute the equal facet angles as follows.





First, he drew a sphere with arbitrary radius centered about point *C*. The crystal facets *ACB, ACF, BCF* each contains the sphere center, and so they will intercept it in great circle arcs *arcAB, arcAF, arcBF* respectively. These equal arcs measure the facet angles.

The resulting spherical triangle, *sphABF*, has spherical angles *spA, spB, spF* that are respectively equal to the angles between the corresponding pair of facets (e.g. *spB* is the angle between the planes *ACB* and *BCF*). These interfacial angles are all equal to one another (being 105°), so we have an equilateral spherical triangle, *sphABF*.

We can now apply trigonometry. From angle *F* of equilateral triangle *sphABF* drop a perpendicular *arcFQ* along the surface of the sphere to the side *arcAB*, which will accordingly be bisected at *Q*. According to the cosine formula of spherical trig., the following relation holds generally between the spherical angles *spA, spF, spQ* and the *arcAF* that is opposite angle *spQ:*

$$\cos(spQ) = -\cos(spA)\cos(spF) + \sin(spA)\sin(spF)\cos(arcAF)$$

In Huygens' case cos(*spQ*) vanishes since *spQ* is a right angle, *spA* is equal to the interfacial angle 105°, and *spF* is 52°30' because *arcAB* is bisected at *Q*. Putting in the values, we find that *arcAF* will be 101.865°, or 101°51'54". Huygens just rounds the result to 101°52'.