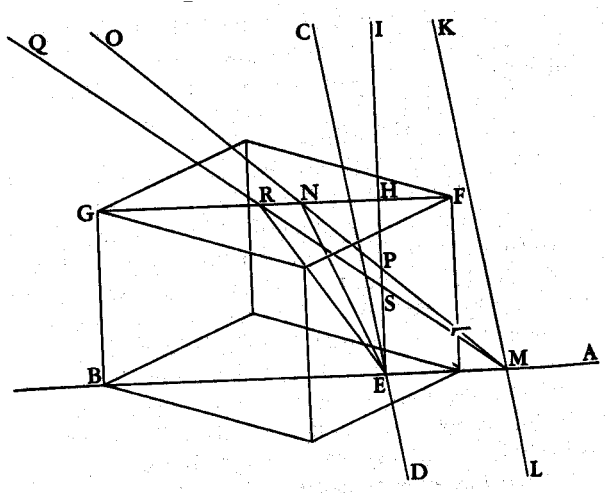
Ordinary Ray Refraction Data - Iceland Crystal

Students carried out the measurements described in paragraph 12 of Chapter V. of Huygen's *Treatise on Light* (English Trans., S.P. Thompson). They then, in accord with the next paragraph, calculated what we call the index of refraction (for the ordinary ray) of the crystal. Paragraphs 12 and 13 read:

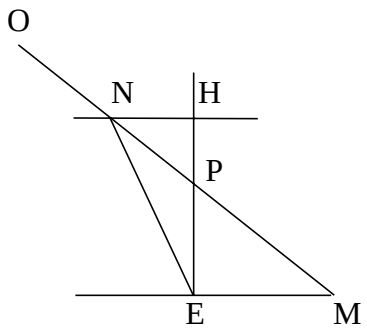
12. The mode of making these observations exactly is as follows. Upon a leaf of paper fixed on a thoroughly flat table there is traced a black line AB, and two others, CED and KML, which cut it at right angles and are more or less distant from one another according as it is desired to examine a ray that is more or less oblique. Then place the Crystal upon the intersection E so that the line AB concurs with that which bisects the obtuse angle of the lower surface, or with some line parallel to it. Then by placing the eye directly above the line AB it will appear single only; and one will see that the portion viewed through the Crystal and the portions



which appear outside it, meet together in a straight line: but the line CD will appear double, and one can distinguish the image which is due to regular refraction by the circumstances that, when the Crystal is turned around on the paper, this image remains stationary, whereas the other image shifts and moves entirely around. Afterwards let the eye be placed at I (remaining always in the plane perpendicular through AB) so that it views the image which is formed by regular refraction of the line CD making a straight line with the remainder of that line which is outside the Crystal. And then, marking on the surface of the Crystal the point H where the intersection E appears, this point will be directly above E. Then draw back the eye towards O, keeping always in the plane perpendicular through AB, so that the image of the line CD, which is formed by ordinary refraction, may appear in a straight line with the line KL viewed without refraction; and then mark on the Crystal the point N where the point of intersection E appears.

13. Then one will know the length and position of the lines NH, EM, and of HE, which is the thickness of the Crystal: which lines being traced separately upon a plan, and then joining NE and NM which cuts HE at P, the proportion of the refraction will be that of EN to NP, because these lines are to one another as the sides of the angles, NPH, NEP, which are equal to those which the incident ray ON and its refraction NE make with the perpendicular to the surface. This proportion, as I have said, is sufficiently precisely as 5 to 3, and is always the same for all inclinations of the incident ray.

The results of the construction described in 13 is shown at the right. Huygens does not provide this figure. He gives his result "...the proportion of the refraction will be that of EN to NP... is sufficiently precisely as 5 to 3, (1.66) and is always the same for all inclination of the incident ray."



The *proportion of the refraction* is the ratio of the sine of the angle the incident ray makes with the normal to the surface, angle NPH, to the sine of the angle of the refracted ray, angle NEP (or NEH).

One student, instead of carrying through the geometrical construction to its conclusion, namely the measurement of EN and NP on the figure so drawn, noted the similarity of the triangles NPH and MPE and proceeded to calculate these distances directly from the three independent measurements, EM, NH, and the height of the crystal HE. First, for NE, since triangle NEH is a right triangle

$$EN^2 = NH^2 + HE^2$$

Then, for the similar triangles NPH and MPE, we have

$$\frac{EM}{HE - HP} = \frac{NH}{HP}$$

Whereupon solving for HP, we have

$$HP = \frac{NH \cdot HE}{(NH + EM)}$$

So that, again because NPH is a right triangle,

$$NP^2 = \left[\frac{NH \cdot HE}{(NH + EM)}\right]^2 + NH^2$$

Several questions arise at this point: Why did Huygens not obtain values for NP and EN in this way? Does not measuring line segments off of a graphical construction introduce further error? Or did he scale up his measurements when constructing the figure to avoid introducing this additional error? And what does Huygens mean by "...sufficiently precisely as 5 to 3"?

Students agreed to the following estimates of uncertainty on the basic measurements:

NH to +/- 20% This included the difficulty of making the sightings called for in paragraph 12.
EM to +/- 5% due to the resolution of the scale.
EH to +/- 7, 8% due to the challenge of measuring the height of the crystal with slanted sides.

The data that follows is chronologically ordered. The students of group alpha calculated EN and NP from the above relationships. Further reduction and analysis remains to be done.

Data of 13 February, 2001.

“Ordinary Ray” Refraction Data

Student	HE	EM	NH	EN	NP
alpha/x	11/16 in	4/16 in	5/16 in.	0.7552 in.	0.382 in.
alpha/dam	5/8 in.	1/4 in.	1/4 in	0.673 in.	0.400 in.
beta/sd	1.3 cm	0.75 mm (?)	0.65 cm.	1.4 cm.	0.85 cm.
beta/sd	1.2 cm	0.5 cm	0.5 cm	1.35 cm	0.8 cm.
gamma/sb	1.5 cm	.8	.8	1.8	1.1
gamma/sb	1.7 cm	.9	.6	1.9	1.1
beta/ld	15.4 mm	15.0 mm	8.8 mm.	17.9 mm	10.5 mm
beta/sd	12.5 (mm?)	0.7	0.7	15 mm	10 mm
alpha/dam	11/16 in.	1/2 in.	3/8 in.	0.713 in.	0.477 in
alpha/x	11/16 in.	4/16 in.	4/16 in	0.7135 in	0.425 in
gamma	11/16 in.	1.40 cm	1.05 cm	2.1 cm	1.3 cm
gamma/sb	.9	.8	.55	1.1	.7
gamma	19/32 in	1.2 cm	0.9 cm.	1.7 cm.	1.1 cm.
beta/sd	1.6 cm	1 cm.	0.9 cm.	19.5 mm	12 mm.
alpha/dam	3/4 in.	1/4 in.	9/32	0.801 in	0.487
beta/ld	17.7 mm.	10 mm.	7.3 mm	19.2 mm	10.5 mm.
alpha/x	15 cm.	6 cm	8	17	11.32
gamma	1.35 cm	1.6 cm	0.95 cm	1.6	0.95

Note: Six different crystals were in circulation around the room. No record was kept of which crystal goes with each row of the above table.