## "A Remarkable Rule" - Iceland Crystal

While Huygens found that the extra-ordinary ray did not obey Snell's Law, he did discover the following "remarkable rule" which, I conjecture, gave him hope in his search for an explanation of extra-ordinary refraction (reminds me of Kepler's "discovery" of equal areas in equal times governing planetary orbits, no longer circular):
17. Finally, continuing my observations to discover the nature of this refraction, I learned that it obeyed the following remarkable rule. Let the parallelogram GCFH, made by the principal section of the Crystal, as previously determined, be traced separately. I found then that always, when the inclinations of two rays which come from opposite sides, as VK, SK here, are equal, their refractions KX and KT meet the bottom line HF in such wise that points X and T are equally distant from the point M , where the refraction of the perpendicular ray IK falls; and this occurs also for refractions in other sections of this Crystal.


What follows is a technique for experimentally testing the rule. The method makes use of the fact that the ordinary ray does obey Snell's Law in fixing the inclinations of two rays which come from opposite sides of the normal to the face - making the inclinations equal. (But see the final caveat).


Draw two parallel lines, some distance apart, and a perpendicular which will serve to align the principal section of the crystal. E is the intersection of the lower parallel and the perpendicular. The dashed line shows the location of the extraordinary image of the lower parallel when the sighting down the normal through E.
(Rotate the crystal $180^{\circ}$ if the extraordinary image lies below the lower parallel).

Sighting down the normal through E, mark a the point H on the top face, directly above E. Then, move the eye back until the ordinary image of the line through E is coincident with the line through M sighted outside the crystal. Mark the point N on the top face at the intersection of this line of sight. All this is as was done before.

Holding this line of sight, mark the point $N^{\prime}$ on the top face where the extraordinary ray from the point E intersects. At this
 point we have three marks on the top face of the crystal, $\mathrm{N}, \mathrm{N}$ ' and H .

We now turn the crystal around $180^{\circ}$ and align the mark N (but labeled H again in the figure at the right) on the top face directly over the point E on the paper, sighting along the normal to the face. (Note: Now the extraordinary image of the line through E will lie below E ).

Again, move the eye back until the ordinary image of the line through E is coincident with the line through M sighted outside the crystal. Mark the point on the top face
 at the intersection of this line of sight. This point should already be there, that point labeled H in the previous figure.

Holding this line of sight, mark the point N" on the top face where the extraordinary ray from the point E intersects. Now we have four marks on the top face of the crystal.

Superimposing the two constructions we obtain the figure, left below. Eliminating all but the extraordinary rays yields the figure at the right. Here we have the geometry of extraordinary refraction of two rays whose inclination which come from opposite sides of the normal are equal.


What remains is to redraw the figure such that N' and N " are coincident - as shown at the left.

Finally, bisecting the line segment XT (at M), we need to show that M is the point where the refraction of the perpendicular ray IK falls, i.e., it makes an angle of $6^{\circ} 40^{\prime \prime}$ with the vertical.

To test again, draw two parallells separated by a different distance and go again.

A Caveat: There is some error involved in sighting the intersection of the extraordinary ray from the eye position set by sighting the refraction of the ordinary ray. The technique assumes the eye is at a great distance from the crystal face.

