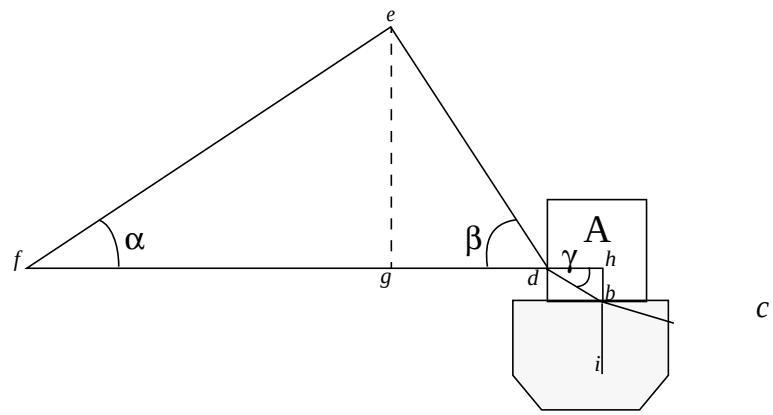
Wollaston's apparatus

Here we go through the geometry, and geometrical optics of Wollaston's apparatus. We show that when there is total internal reflection within the prism at the interface of prism *A* and the material below, and with the lengths *fe* and *de* chosen in the appropriate ratio, the distance *fg* is proportional to the material's index of refraction.

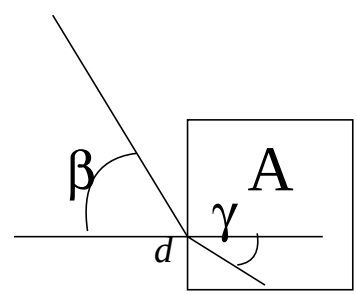


From the figure: $\sin\alpha = eg/fe$ *and* $\sin\beta = eg/de$

so $de \cdot \sin\beta = fe \cdot \sin\alpha$

Now, refraction at the air/prism interface gives:

$$\frac{\sin\beta}{\sin\gamma} = \frac{n_{prism}}{n_{air}}$$



so

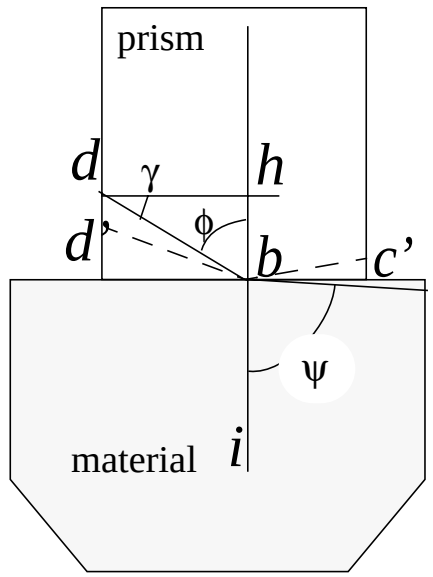
$$\left(\frac{n_{prism}}{n_{air}}\right) \cdot \sin\gamma = \left(\frac{fe}{de}\right) \cdot \sin\alpha$$

Now choose: $\left(\frac{fe}{de}\right) = \left(\frac{n_{prism}}{n_{air}}\right) = \frac{1.583}{1.0}$ so $\sin\gamma = \sin\alpha$

Then, with $\gamma = \alpha$ we have (ef/fg) = (db/dh)

Now, refraction at the material/prism interface gives:

$$\frac{\sin\psi}{\sin\phi} = \frac{n_{prism}}{n_{material}}$$



We will have total internal reflection within the prism when $\psi = \pi/2$

But $\sin\phi = \cos\gamma = \frac{dh}{db}$

So when this is the case, we have

$$\frac{1}{\sin\phi} = \frac{db}{dh} = \frac{n_{prism}}{n_{material}}$$

Thus, putting this together with the last equality on the previous page, we have, finally

$$\frac{ef}{fg} = \frac{n_{prism}}{n_{material}} \qquad or \qquad \boxed{n_{material} = fg}$$

Since *ef* was chosen proportional to n_{prism} .

The way to think of this in terms of procedure is to imagine one *does* have total internal reflection within the prism, e.g., the dashed line *d'bc'* . *I*n this state you will see light entering from the other side of the prism at c'. Then, moving your line of sight up toward *d* , you will reach the point "...where perfect reflection terminates..." This is the state defined by $\psi = \pi/2$.