

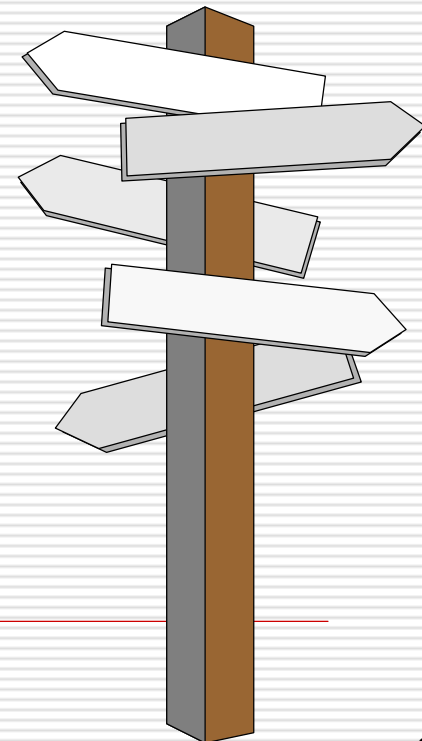
Supply Contracts



Yossi Sheffi
Mass Inst of Tech
Cambridge, MA

Outline

- ✎ Supply contracts
 - ✎ Demand distribution
- ✎ Wholesale contracts
 - ✎ Optimal orders
- ✎ Buyback contracts
- ✎ Revenue sharing contracts



Single period ordering

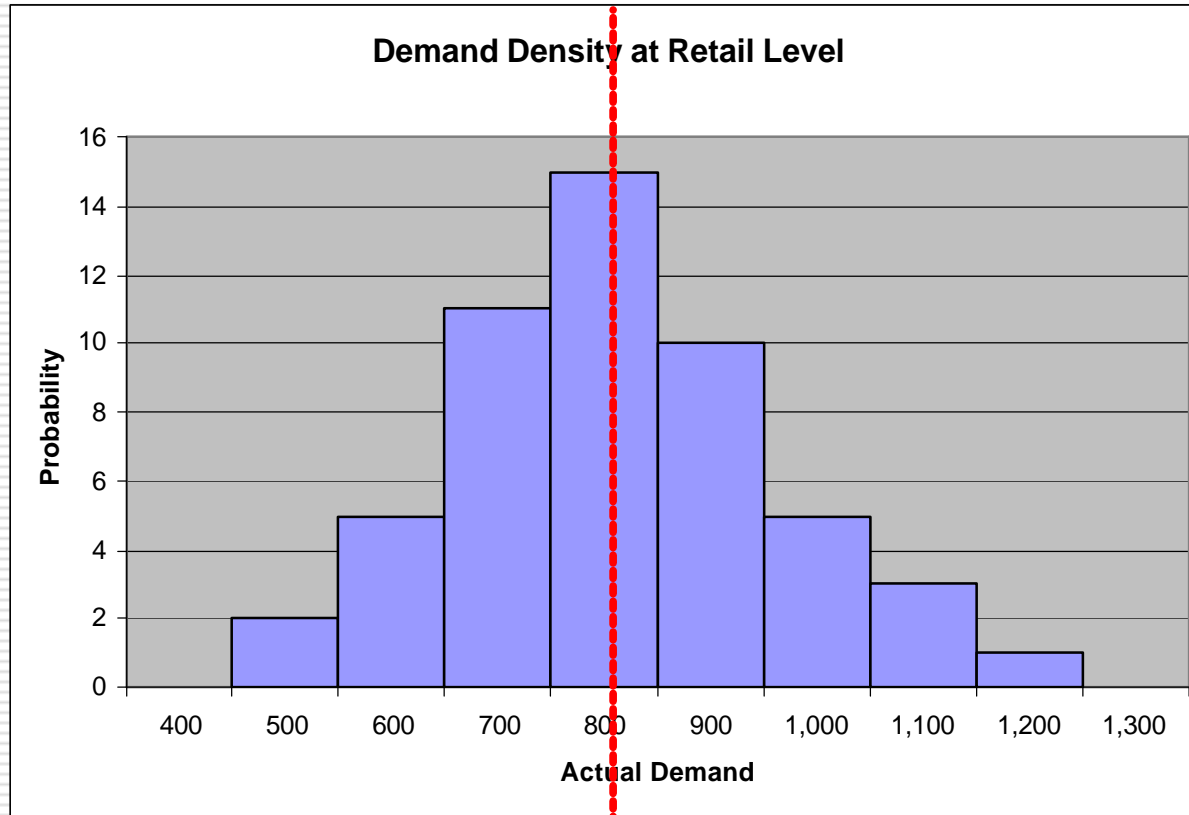
- ✍ Seasonal items
- ✍ Perishable goods
- ✍ News print
- ✍ Fashion items
- ✍ many high tech products
- ✍ Every item where the lead time is long and the selling season is short

The Scenario:



- ✂ Contract is negotiated
- ✂ Retailer places order (a single period)
- ✂ Supplier makes and sends the stuff
- ✂ The selling season takes place
- ✂ Accounting (sales, salvage, etc.)

Consumer Demand

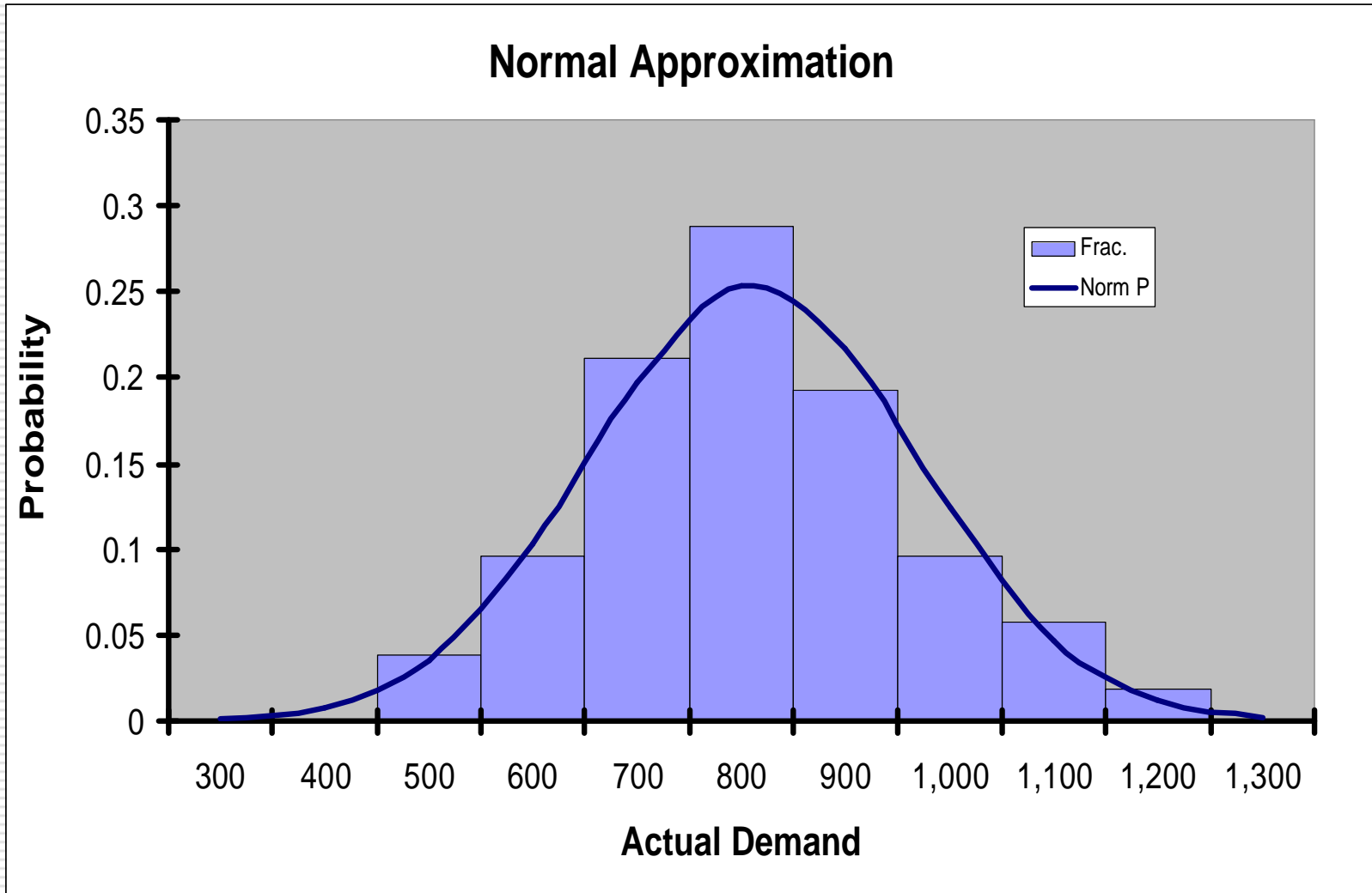


<i>Bin</i>	<i>Freq.</i>
400	0
500	2
600	5
700	11
800	15
900	10
1,000	5
1,100	3
1,200	1
1,300	0

Average: 811.54
Std Dev: 154.23

Average: 811.54
Std Dev: 154.23

Demand Distribution








Notations:

Retail Price	R
Wholesale Price	W
Supplier's Cost	C
Salvage Value	S
Quantity Ordered	Q
Actual Retail Demand	d
Retail Demand Density	$f(D)$
Retail Cum. Demand Distribution	$F(D)$

Supply Contracts

Prices and costs:

-  Supplier has a cost to make/purchase ($C = \$50$)
 -  Supplier is selling and retail is buying at a wholesale price ($W = \$135$)
 -  Retailer is selling for a retail price ($R = \$200$)
 -  Retailer can salvage ($S = \$10$)
-  Retailer is facing a Newsboy problem and supplier's profit is trivial

R=\$200
W=\$135
C=\$50
S=\$10

Wholesale Price Contract

Retailer:

Order:	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	
Demand																
Prob																
400	0.00	\$13,000	\$19,500	\$26,000	\$13,500	\$1,000	-\$11,500	-\$24,000	-\$36,500	-\$49,000	-\$61,500	-\$74,000	-\$86,500	-\$99,000	-\$111,500	-\$124,000
500	0.04	\$13,000	\$19,500	\$26,000	\$32,500	\$20,000	\$7,500	-\$5,000	-\$17,500	-\$30,000	-\$42,500	-\$55,000	-\$67,500	-\$80,000	-\$92,500	-\$105,000
600	0.10	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$26,500	\$14,000	\$1,500	-\$11,000	-\$23,500	-\$36,000	-\$48,500	-\$61,000	-\$73,500	-\$86,000
700	0.21	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$33,000	\$20,500	\$8,000	-\$4,500	-\$17,000	-\$29,500	-\$42,000	-\$54,500	-\$67,000
800	0.29	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$39,500	\$27,000	\$14,500	\$2,000	-\$10,500	-\$23,000	-\$35,500	-\$48,000
900	0.19	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$46,000	\$33,500	\$21,000	\$8,500	-\$4,000	-\$16,500	-\$29,000
1,000	0.10	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$52,500	\$40,000	\$27,500	\$15,000	\$2,500	-\$10,000
1,100	0.06	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$71,500	\$59,000	\$46,500	\$34,000	\$21,500	\$9,000
1,200	0.02	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$71,500	\$78,000	\$65,500	\$53,000	\$40,500	\$28,000
1,300	0.00	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$71,500	\$78,000	\$84,500	\$72,000	\$59,500	\$47,000
Expected Profit:		\$13,000	\$19,500	\$26,000	\$32,500	\$38,269	\$42,212	\$42,135	\$36,577	\$27,365	\$16,327	\$4,192	-\$8,308	-\$20,808	-\$33,308	-\$45,808
Maximum Profit:							1									

Supplier:

Order:	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	
Demand																
Prob																
400	0.00	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
500	0.04	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
600	0.10	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
700	0.21	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
800	0.29	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
900	0.19	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,000	0.10	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,100	0.06	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,200	0.02	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,300	0.00	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
Expected Profit:		\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
Maximum Profit:																!!!

Total Profit: \$30,000 \$45,000 \$60,000 \$75,000 \$89,269 \$101,712 \$110,135 \$113,077 \$112,365 \$109,827 \$106,192 \$102,192 \$98,192 \$94,192 \$90,192
Maximum Profit !!!

R=\$200
W=\$135
C=\$50
S=\$10

Wholesale Price Contract

Retailer:

Order:	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	
Demand																
Prob																
400	0.00	\$13,000	\$19,500	\$26,000	\$13,500	\$1,000	-\$11,500	-\$24,000	-\$36,500	-\$49,000	-\$61,500	-\$74,000	-\$86,500	-\$99,000	-\$111,500	-\$124,000
500	0.04	\$13,000	\$19,500	\$26,000	\$32,500	\$20,000	\$7,500	-\$5,000	-\$17,500	-\$30,000	-\$42,500	-\$55,000	-\$67,500	-\$80,000	-\$92,500	-\$105,000
600	0.10	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$26,500	\$14,000	\$1,500	-\$11,000	-\$23,500	-\$36,000	-\$48,500	-\$61,000	-\$73,500	-\$86,000
700	0.21	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$33,000	\$20,500	\$8,000	-\$4,500	-\$17,000	-\$29,500	-\$42,000	-\$54,500	-\$67,000
800	0.29	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$39,000	\$27,000	\$14,500	\$2,000	-\$10,500	-\$23,000	-\$35,500	-\$48,000
900	0.19	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$46,000	\$33,500	\$21,000	\$8,500	-\$4,000	-\$16,500	-\$29,000
1,000	0.10	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$53,000	\$40,000	\$27,500	\$15,000	\$2,500	-\$10,000
1,100	0.06	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$72,000	\$60,000	\$47,500	\$35,000	\$22,500	\$10,000
1,200	0.02	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$72,000	\$79,000	\$67,000	\$55,000	\$43,000	\$31,000
1,300	0.00	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$72,000	\$79,000	\$86,000	\$93,000	\$100,000	\$107,000
Expected Profit:		\$13,000	\$19,500	\$26,000	\$32,500	\$38,269	\$42,135	\$45,500	\$48,500	\$51,000	\$53,000	\$54,500	\$55,500	\$56,000	\$56,000	\$55,500
Maximum Profit:																

800 ordered and 600 demanded
 $\text{Profit} = 600 \times (\$200 - \$135) + (800 - 600) \times (\$10 - \$135) = \$14,000$

800 ordered and 1000 demanded
 $\text{Profit} = 800 \times (\$200 - \$135) = \$52,000$

800 ordered
 $\text{Profit} = 800 \times (\$135 - \$50) = \$68,000$

Supplier:

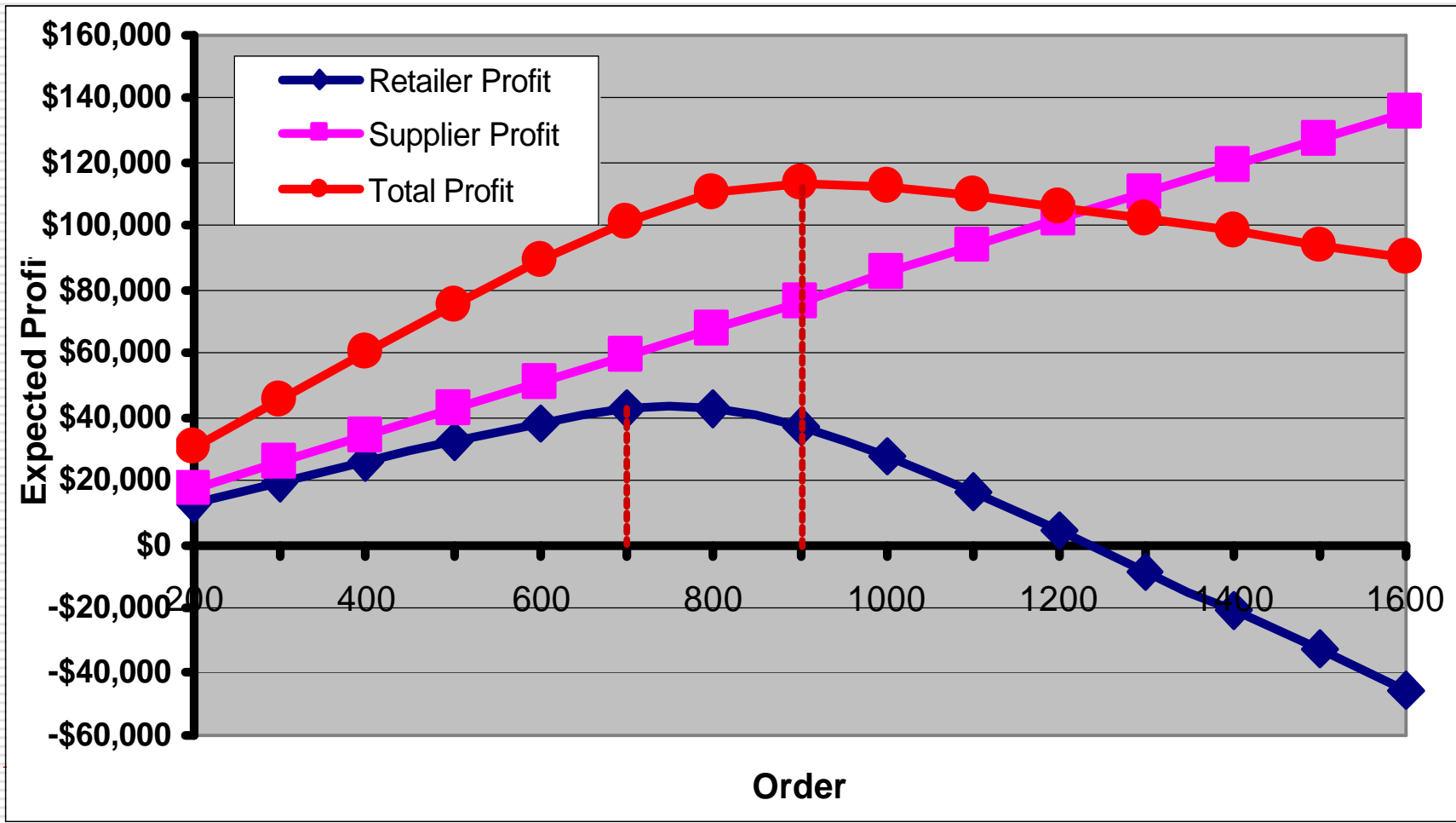
Order:	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	
Demand																
Prob																
400	0.00							\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000	
500	0.04							\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000	
600	0.10	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
700	0.21	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
800	0.29	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
900	0.19	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,000	0.10	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,100	0.06	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,200	0.02	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,300	0.00	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
Expected Profit:		\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
Maximum Profit:																

Total:

Order:	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600
Maxi								\$113,077	\$112,365	\$109,827	\$106,192	\$102,192	\$98,192	\$94,192	\$90,192

Wholesale Price Contract Expected Profits

R=\$200
W=\$135
C=\$50
S=\$10



Optimal Order

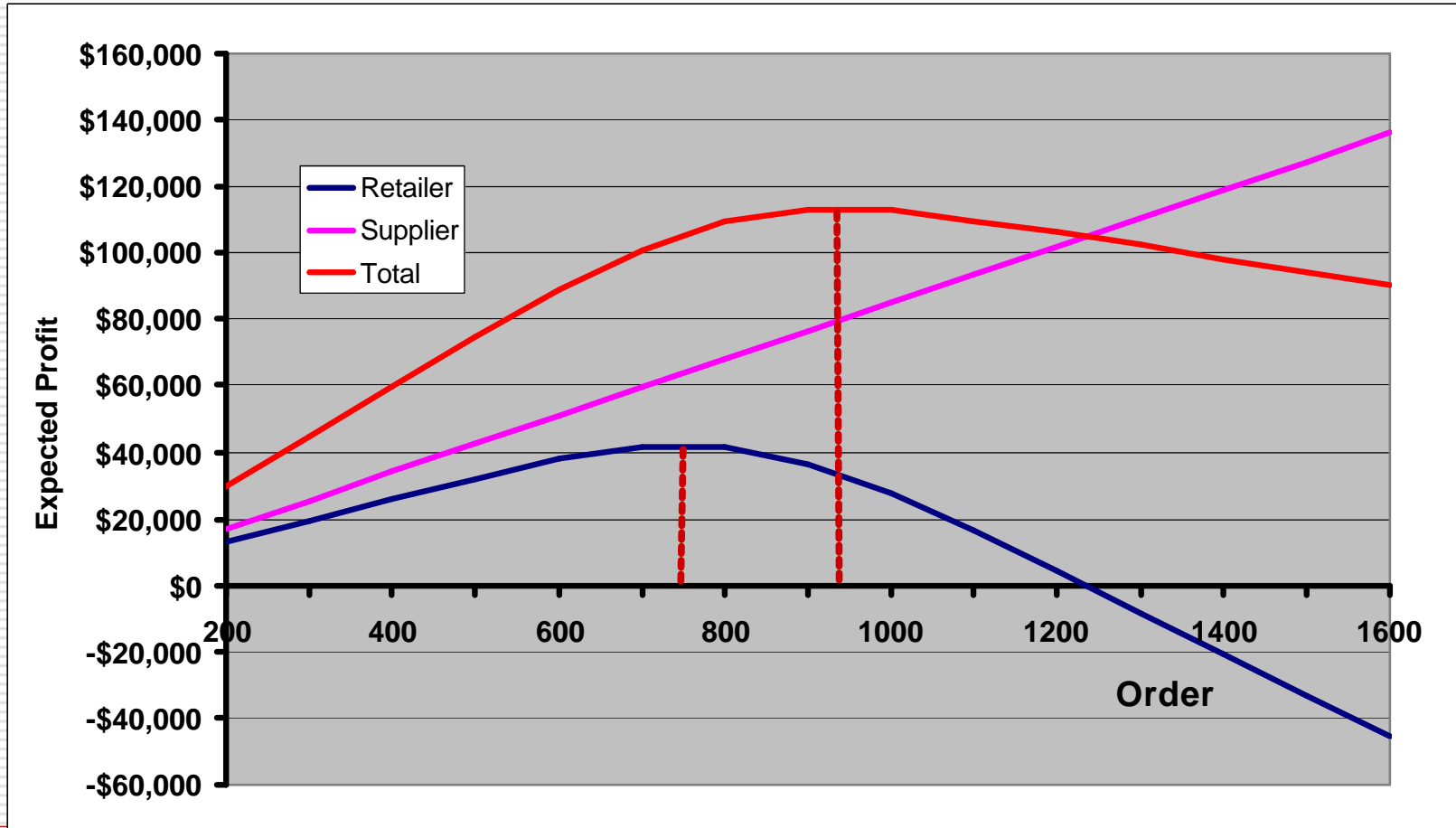
- ✍ The optimal order is Q^*
- ✍ At Q^* the probability of selling one more magazine is the probability that demand is greater than Q^*
- ✍ The expected profit from ordering the $(Q^* + 1)$ st magazine is:
 - ✍ If demand is high and we sell it:
 - ✍ $(R-C) \times \Pr(\text{Demand is higher than } Q^*)$
 - ✍ If demand is low and we are stuck but can salvage:
 - ✍ $(S-C) \times \Pr(\text{Demand is lower or equal to } Q^*)$
- ✍ The optimum is where the total expected profit from ordering one more magazine is zero:
 - ✍ $(R-C) \times \Pr(\text{Demand} > Q^*) + (S - C) \times \Pr(\text{Demand} = Q^*) = 0$

$$\Pr(\text{Demand} > Q^*) = \frac{R - C}{R - S}$$

$$Q^* = F^{-1}\left(\frac{R - C}{R - S}\right)$$

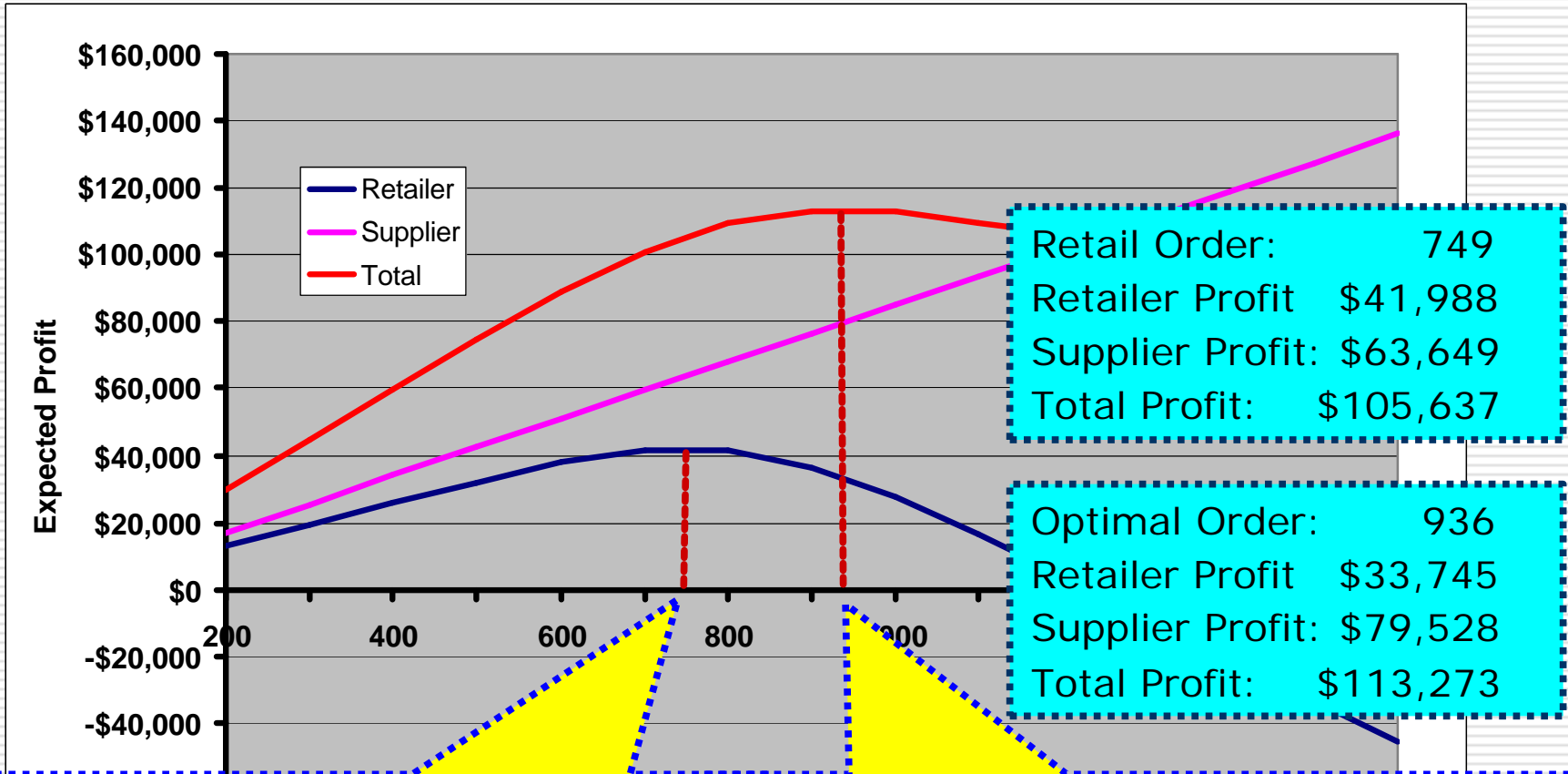
Wholesale Price Contract Exp Profits (Normal Approx)

R=\$200
W=\$135
C=\$50
S=\$10



Wholesale Price Contract Exp Profits (Normal Approx)

R=\$200
W=\$135
C=\$50
S=\$10



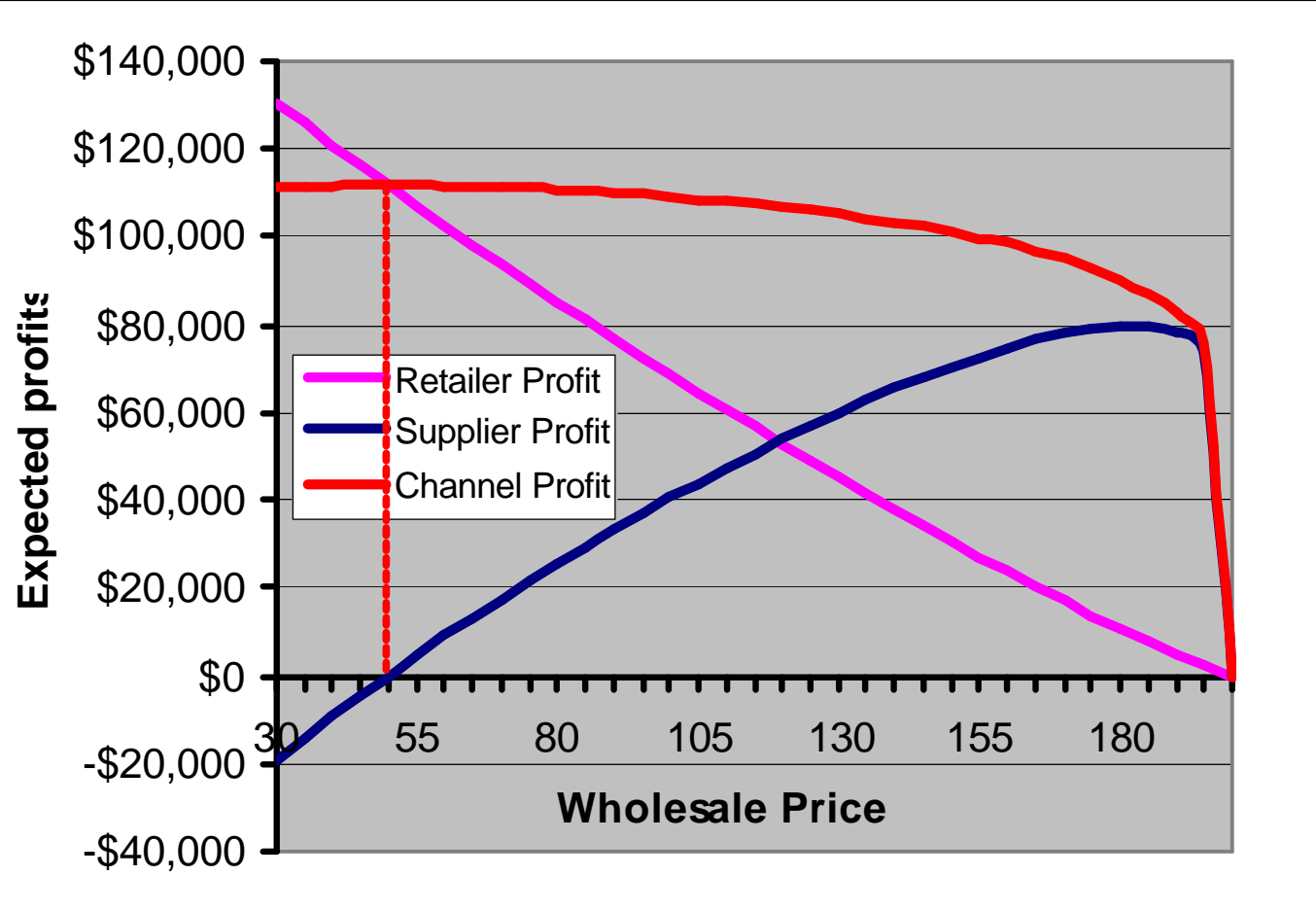
Optimal Retailer order =

Optimal channel order =

$$F^{*1} \frac{R - W}{R - S} \text{ NORMINV} \left(\frac{200 - 135}{200 - 10}, 811,154 \right) = 749$$

$$F^{*1} \frac{R - C}{R - S} \text{ NORMINV} \left(\frac{200 - 50}{200 - 10}, 811,154 \right) = 936$$

Effects of Wholesale Price on Profits



Wholesale Price Contract Coordination the Channel

$$Q_{Retailer}^* \quad ? \quad Q_{Channel}^*$$

$$F^{?1} \frac{?R?W?}{?R?S?} \quad ? \quad F^{?1} \frac{?R?C?}{?R?S?}$$

$$\frac{?R?W?}{?R?S?} \quad ? \quad \frac{?R?C?}{?R?S?}$$

$$W \quad ? \quad C$$

Buyback Contract

- ✎ The problem: how can the supplier convince the retailer to move towards the optimal order size?
- ✎ The supplier offer to the retailer to buy back all unsold items ($\$B/\text{item}$).
- ✎ For the retailer – this is like a higher salvage value, so he will order more.
- ✎ The supplier now shares in the overage risk (he can still salvage, though, at the same price).
- ✎ Note: supplier may simply pay $(\$B - \$S)$ rather than actually buy back (unless he has a better use for it)

R=\$200
W=\$135
C=\$50
S=\$10
B=\$80

Buyback Contract Calculations

Retailer:

Order:		200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600
Demand	Prob															
400	0.00	\$13,000	\$19,500	\$26,000	\$20,500	\$15,000	\$9,500	\$4,000	-\$1,500	-\$7,000	-\$12,500	-\$18,000	-\$23,500	-\$29,000	-\$34,500	-\$40,000
500	0.04	\$13,000	\$19,500	\$26,000	\$32,500	\$27,000	\$21,500	\$16,000	\$10,500	\$5,000	-\$500	-\$6,000	-\$11,500	-\$17,000	-\$22,500	-\$28,000
600	0.10	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$33,500	\$28,000	\$22,500	\$17,000	\$11,500	\$6,000	\$500	-\$5,000	-\$10,500	-\$16,000
700	0.21	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$40,000	\$34,500	\$29,000	\$23,500	\$18,000	\$12,500	\$7,000	\$1,500	-\$4,000
800	0.29	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$46,500	\$41,000	\$35,500	\$30,000	\$24,500	\$19,000	\$13,500	\$8,000
900	0.19	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$53,000	\$47,500	\$42,000	\$36,500	\$31,000	\$25,500	\$20,000
1,000	0.10	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$59,500	\$54,000	\$48,500	\$43,000	\$37,500	\$32,000
1,100	0.06	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$71,500	\$66,000	\$60,500	\$55,000	\$49,500	\$44,000
1,200	0.02	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$71,500	\$78,000	\$72,500	\$67,000	\$61,500	\$56,000
1,300	0.00	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$71,500	\$78,000	\$84,500	\$79,000	\$73,500	\$68,000
Expected Profit:		\$13,000	\$19,500	\$26,000	\$32,500	\$38,538	\$43,423	\$45,769	\$44,654	\$41,231	\$36,654	\$31,385	\$25,885	\$20,385	\$14,885	\$9,385
Maximum Profit:								1								

Supplier:

Order:		200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600
Demand	Prob															
400	0.00	\$17,000	\$25,500	\$34,000	\$35,500	\$37,000	\$38,500	\$40,000	\$41,500	\$43,000	\$44,500	\$46,000	\$47,500	\$49,000	\$50,500	\$52,000
500	0.04	\$17,000	\$25,500	\$34,000	\$42,500	\$44,000	\$45,500	\$47,000	\$48,500	\$50,000	\$51,500	\$53,000	\$54,500	\$56,000	\$57,500	\$59,000
600	0.10	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$52,500	\$54,000	\$55,500	\$57,000	\$58,500	\$60,000	\$61,500	\$63,000	\$64,500	\$66,000
700	0.21	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$61,000	\$62,500	\$64,000	\$65,500	\$67,000	\$68,500	\$70,000	\$71,500	\$73,000
800	0.29	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$69,500	\$71,000	\$72,500	\$74,000	\$75,500	\$77,000	\$78,500	\$80,000
900	0.19	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$78,000	\$79,500	\$81,000	\$82,500	\$84,000	\$85,500	\$87,000
1,000	0.10	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$86,500	\$88,000	\$89,500	\$91,000	\$92,500	\$94,000
1,100	0.06	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$95,000	\$96,500	\$98,000	\$99,500	\$101,000
1,200	0.02	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$103,500	\$105,000	\$106,500	\$108,000
1,300	0.00	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$112,000	\$113,500	\$115,000
Expected Profit:		\$17,000	\$25,500	\$34,000	\$42,500	\$50,731	\$58,288	\$64,365	\$68,423	\$71,135	\$73,173	\$74,808	\$76,308	\$77,808	\$79,308	\$80,808
Maximum Profit:																!!!
Total Profit:		\$30,000	\$45,000	\$60,000	\$75,000	\$89,269	\$101,712	\$110,135	\$113,077	\$112,365	\$109,827	\$106,192	\$102,192	\$98,192	\$94,192	\$90,192
Maximum Profit									!!!							

Buyback Contract Calculations

R=\$200
W=\$135
C=\$50
S=\$10
B=\$80

Retailer:

Order:	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	
400	0.00	\$13,000	\$19,500	\$26,000	\$20,500	\$15,000	\$9,500	\$4,000	-\$1,500	-\$7,000	-\$12,500	-\$18,000	-\$23,500	-\$29,000	-\$34,500	-\$40,000
500	0.04	\$13,000	\$19,500	\$26,000	\$32,500	\$27,000	\$21,500	\$16,000	\$10,500	\$5,000	-\$500	-\$6,000	-\$11,500	-\$17,000	-\$22,500	-\$28,000
600	0.10	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$33,500	\$28,000	\$22,500	\$17,000	\$11,500	\$6,000	\$500	-\$5,000	-\$10,500	-\$16,000
700	0.21	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$40,000	\$34,500	\$29,000	\$23,500	\$18,000	\$12,500	\$7,000	\$1,500	-\$4,000
800	0.29	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$47,500	\$41,000	\$35,500	\$30,000	\$24,500	\$19,000	\$13,500	\$8,000
900	0.19	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$52,000	\$47,500	\$42,000	\$36,500	\$31,000	\$25,500	\$20,000
1,000	0.10	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$59,500	\$54,000	\$48,500	\$43,000	\$37,500	\$32,000
1,100	0.06	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$71,500	\$66,000	\$60,500	\$55,000	\$49,500	\$44,000
1,200	0.02	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$71,500	\$77,500	\$83,500	\$89,500	\$95,500	\$101,500
1,300	0.00	\$13,000	\$19,500	\$26,000	\$32,500	\$39,000	\$45,500	\$52,000	\$58,500	\$65,000	\$71,500	\$77,500	\$83,500	\$89,500	\$95,500	\$101,500
Expected Profit:		\$13,000	\$19,500	\$26,000	\$32,500	\$39,423	\$45,769									
Maximum Profit:							1									

800 ordered and 600 demanded
 $\text{Profit} = 600 \times (\$200 - \$135) + (800 - 600) \times (\$80 - \$135) = \$28,000$

800 ordered and 1000 demanded
 $\text{Profit} = 800 \times (\$200 - \$135) = \$52,000$

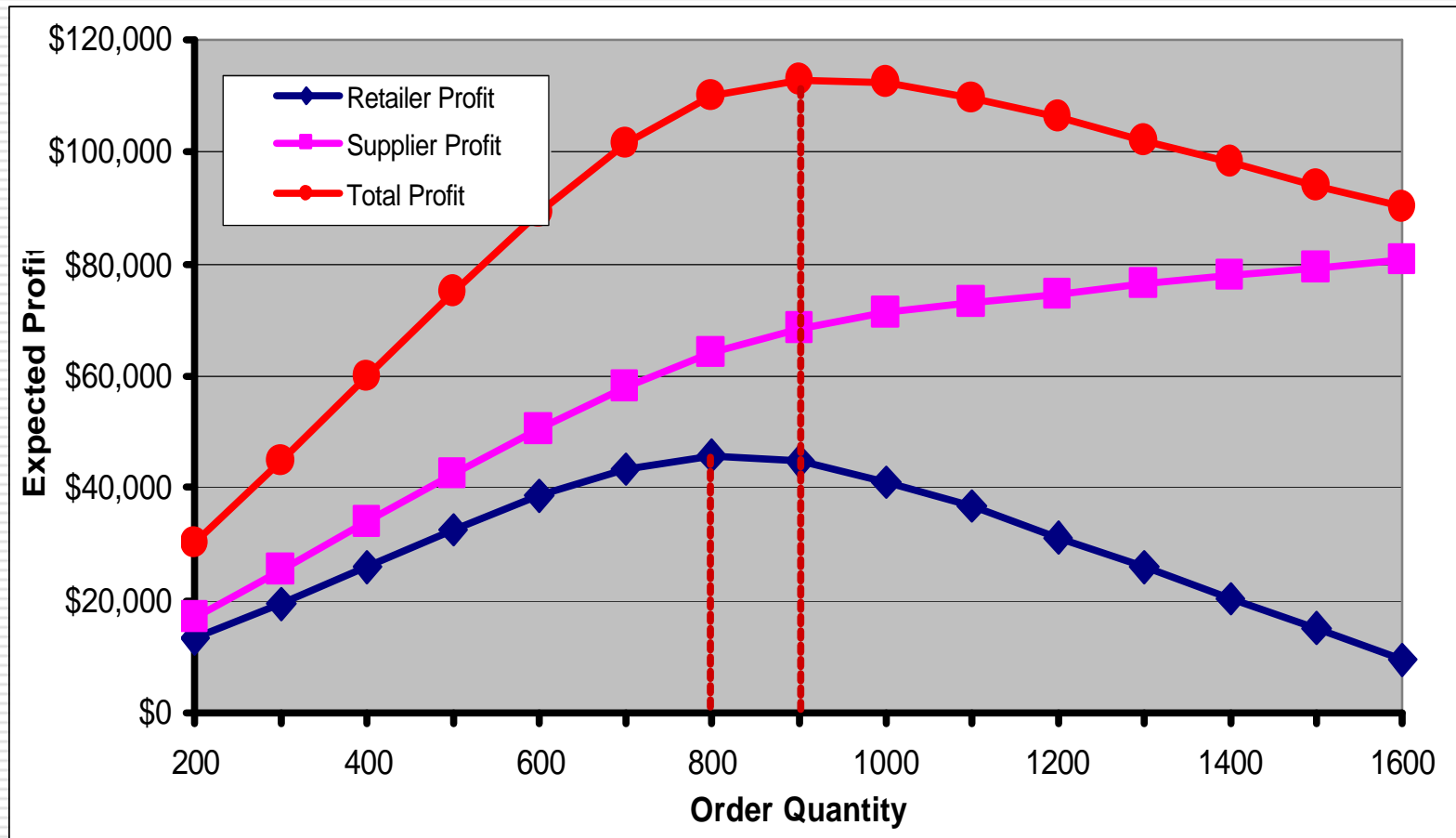
800	0.29	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
900	0.19	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,000	0.10	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,100	0.06	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,200	0.02	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
1,300	0.00	\$17,000	\$25,500	\$34,000	\$42,500	\$51,000	\$59,500	\$68,000	\$76,500	\$85,000	\$93,500	\$102,000	\$110,500	\$119,000	\$127,500	\$136,000
Expected Profit:		\$17,000	\$25,500	\$34,000	\$42,500	\$51,288	\$59,888	\$68,365								
Maximum Profit:								35								

800 ordered and 600 demanded
 $\text{Profit} = 800 \times (\$135 - \$50) + (800 - 600) \times (\$10 - \$80) = \$54,000$

800 ordered 1000 demanded
 $\text{Profit} = 800 \times (\$135 - \$50) = \$68,000$

Channel profit with Buyback

R=\$200
W=\$135
C=\$50
S=\$10
B=\$80

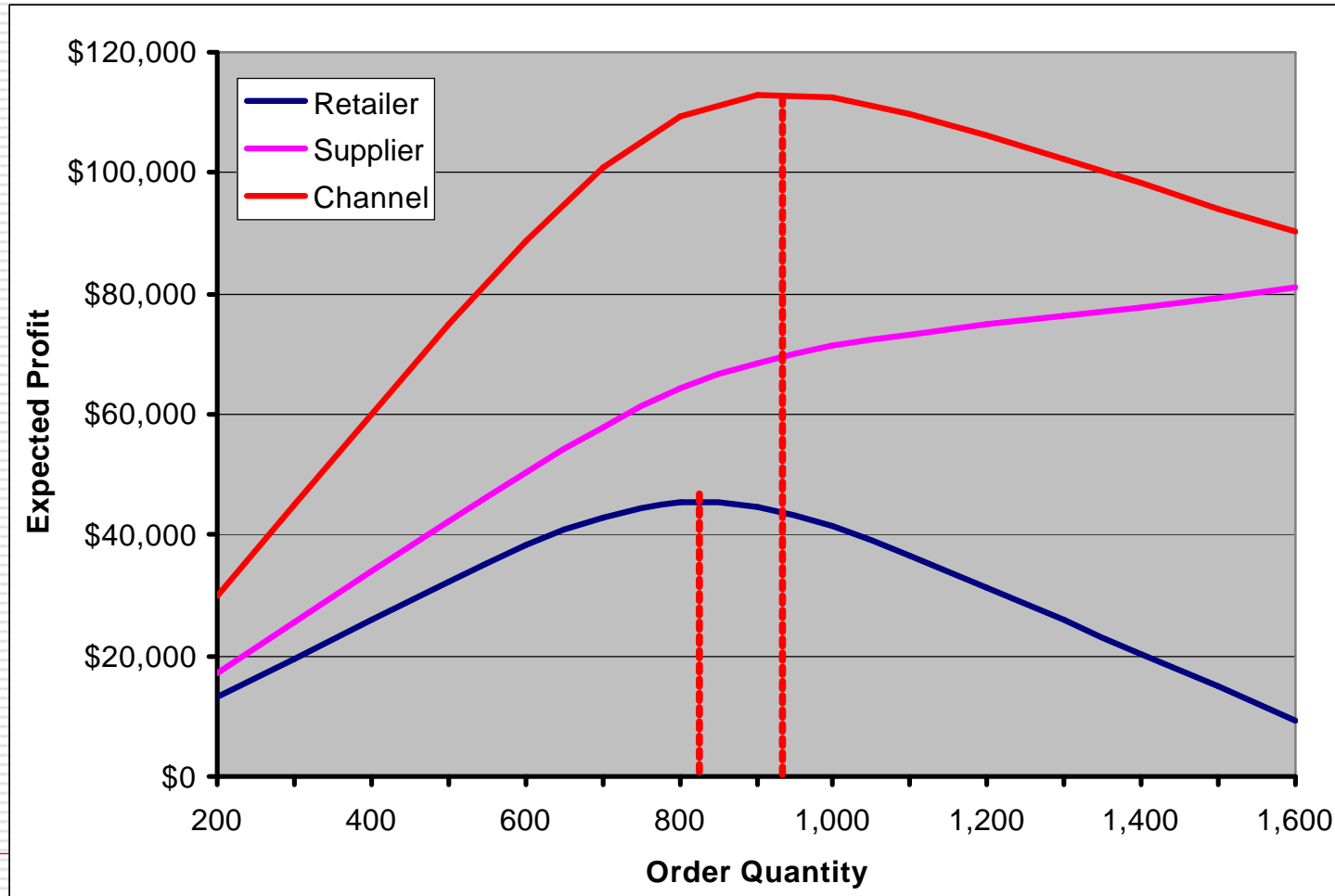


Optimal Buyback and Wholesale price

- ✍ Higher wholesale price requires a higher buyback rate
- ✍ As the wholesale price (and the buyback rate) grows the supplier's share of the profit increases
- ✍ Wholesale price ranges from \$50 (supplier's cost) to \$200 (retail price)
- ✍ Buyback rate ranges from \$10 (salvage value) to \$200.

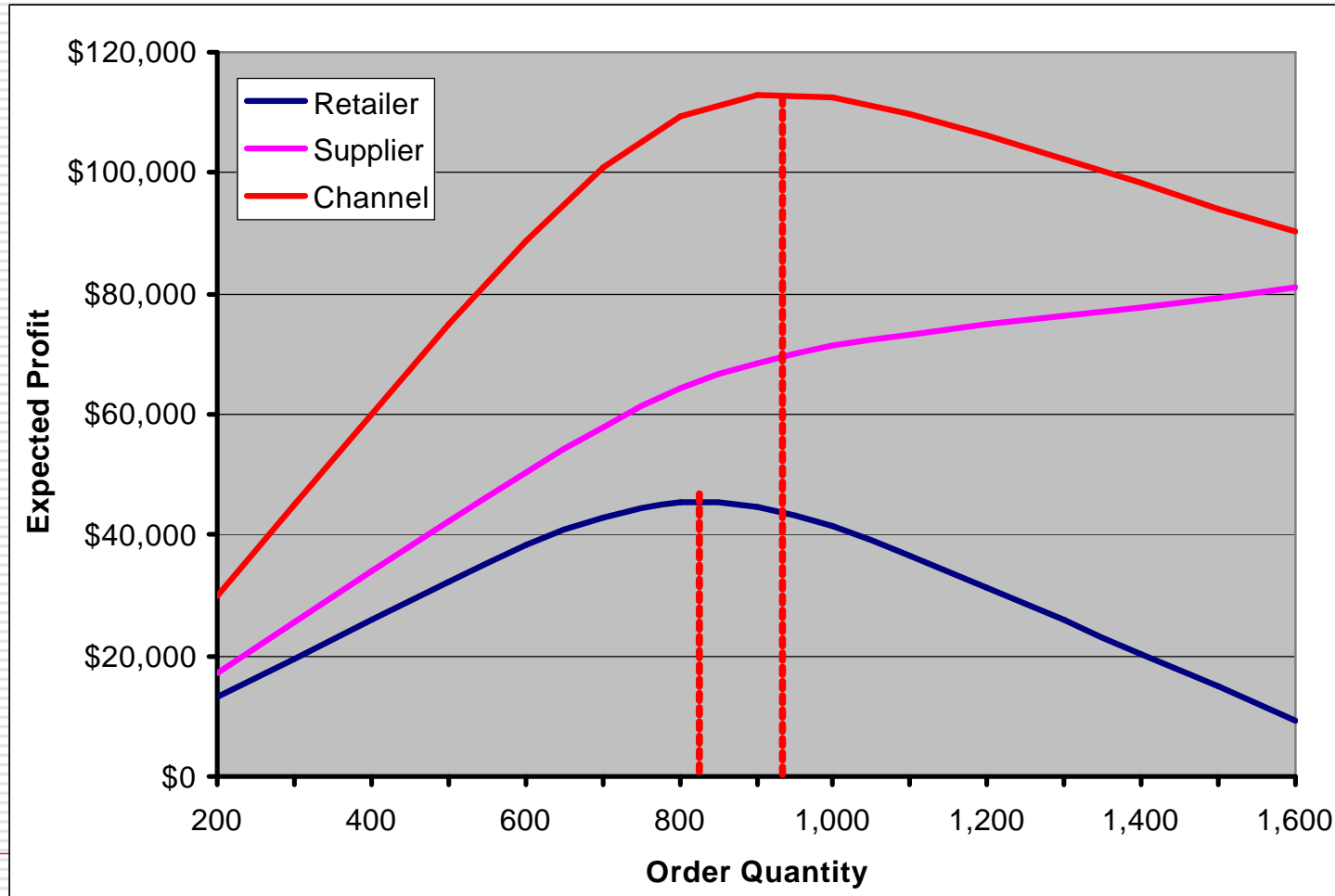
Expected Profits with Buyback Contract

$R = \$200$
 $W = \$135$
 $C = \$50$
 $S = \$10$
 $B = \$80$



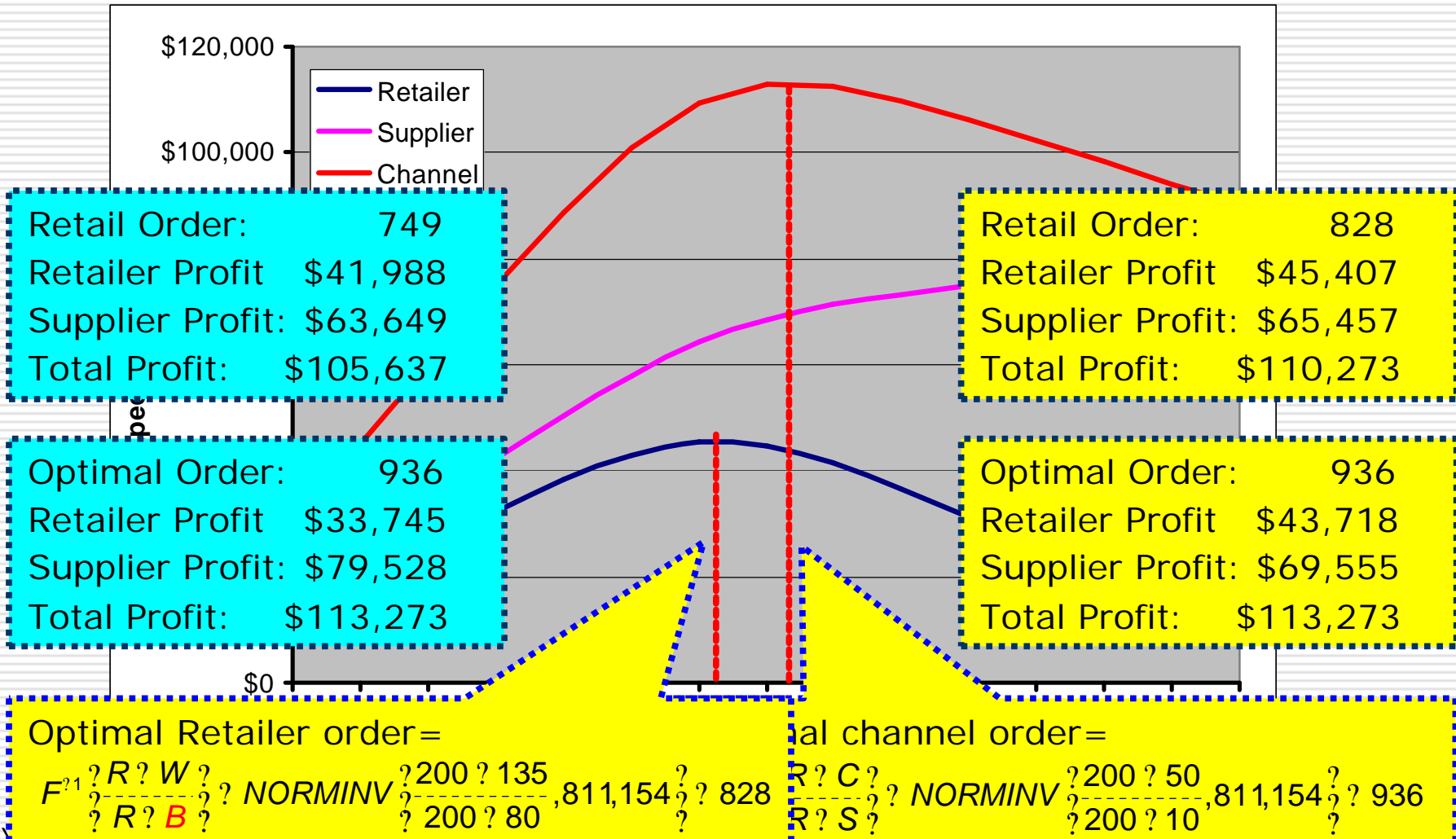
Expected Profits with Buyback Contract

$R = \$200$
 $W = \$135$
 $C = \$50$
 $S = \$10$
 $B = \$80$



Expected Profits with Buyback Contract

R=\$200
 W=\$135
 C=\$50
 S=\$10
 B=\$80



Optimal Buyback Price

$$Q_{Retailer}^* \quad ? \quad Q_{Channel}^*$$

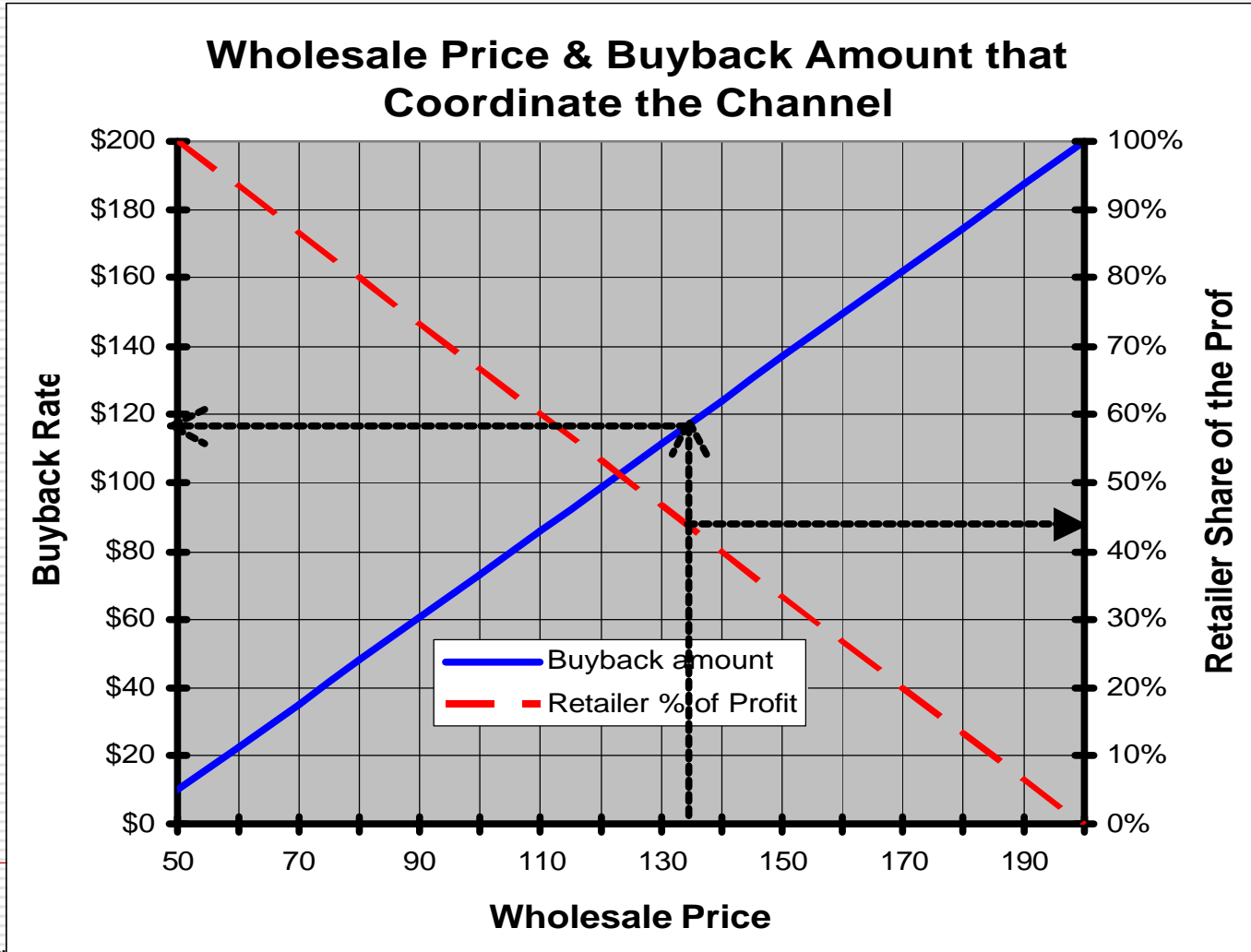
$$F^{?1} \frac{?R?W?}{?R?B?} \quad ? \quad F^{?1} \frac{?R?C?}{?R?S?}$$

$$\frac{?R?W?}{?R?B?} \quad ? \quad \frac{?R?C?}{?R?S?}$$

$$B? \frac{?R?S?}{?R?C?} \quad ? \quad W? \frac{?R?(C?S)?}{?R?C?}$$

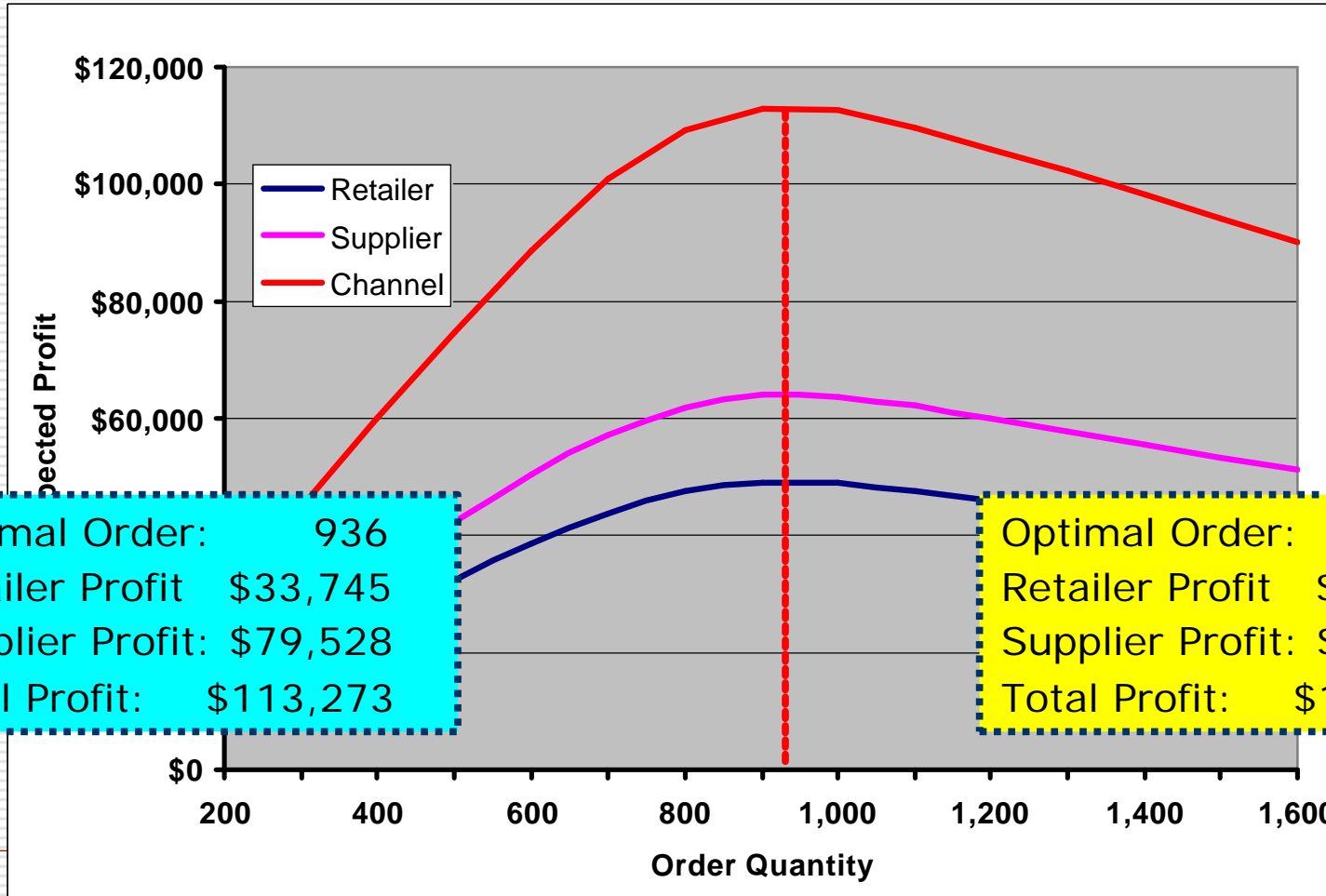
Channel Coordination with Buyback

$R = \$200$
 $W = \$135$
 $C = \$50$
 $S = \$10$
 $B = \$80$



Expected Profit with Coordinating Buyback Rate

$R = \$200$
 $W = \$135$
 $C = \$50$
 $S = \$10$
 $B^* = \$118$



Optimal Order: 936
 Retailer Profit: \$33,745
 Supplier Profit: \$79,528
 Total Profit: \$113,273

Optimal Order: 936
 Retailer Profit: \$49,086
 Supplier Profit: \$64,188
 Total Profit: \$113,273

Buyback Contracts in Practice

- ✍ Book publishing
- ✍ Periodicals/newspapers
- ✍ Price support in consumer electronics

Revenue Sharing

- ✍ Supplier still needs to get the retailer to order more
 - ✍ Another risk-sharing scheme: supplier lowers the wholesale price but takes a percentage $(1-p)$ of the revenue
 - ✍ Question: how to choose W and p so the retailer will order the optimal amount
 - ✍ Note: wholesale price has to be lower than the supplier's cost.
-

The Players

Paramount



Blockbuster



The Economics of Revenue Sharing

In the video industry, revenue sharing may increase profits for the supplier and the retailer alike, as this hypothetical example reveals.

FOR THE RETAILER

	Traditional Pricing	Revenue Sharing
A. Number of tapes purchased	10	30
B. Price per tape	\$60	\$9
C. Purchase cost	\$600	\$270
D. Number of rentals	300	500
E. Total rental revenue ($D \times \$3/\text{rental}$)	\$900	\$1,500
F. Retailer's share of rental revenue	\$900 (100%)	\$750 (50%)
G. Retailer's profit	\$300	\$480
H. Profit per dollar of inventory	\$0.50	\$1.78

FOR THE SUPPLIER

	Traditional Pricing	Revenue Sharing
I. Number of tapes purchased	10	30
J. Price per tape	\$60	\$9
K. Revenue from selling tapes	\$600	\$270
L. Number of rentals	300	500
M. Total rental revenue ($L \times \$3/\text{rental}$)	\$900	\$1,500
N. Supplier's share of rental revenue	\$0 (0%)	\$750 (50%)
O. Supplier's total revenues	\$600	\$1,020
P. Supplier's production and distribution cost ($I \times \$10/\text{tape}$)	\$100	\$300
Q. Supplier's profit	\$500	\$720

R = \$200
W = \$40
C = \$50
S = \$10
p = 0.40

Revenue Sharing

Retailer:

Order:	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	
Demand																
Prob																
400	0.00	\$8,000	\$12,000	\$16,000	\$13,000	\$10,000	\$7,000	\$4,000	\$1,000	-\$2,000	-\$5,000	-\$8,000	-\$11,000	-\$14,000	-\$17,000	-\$20,000
500	0.04	\$8,000	\$12,000	\$16,000	\$20,000	\$17,000	\$14,000	\$11,000	\$8,000	\$5,000	\$2,000	-\$1,000	-\$4,000	-\$7,000	-\$10,000	-\$13,000
600	0.10	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$21,000	\$18,000	\$15,000	\$12,000	\$9,000	\$6,000	\$3,000	\$0	-\$3,000	-\$6,000
700	0.21	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$25,000	\$22,000	\$19,000	\$16,000	\$13,000	\$10,000	\$7,000	\$4,000	\$1,000
800	0.29	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$29,000	\$26,000	\$23,000	\$20,000	\$17,000	\$14,000	\$11,000	\$8,000
900	0.19	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$36,000	\$33,000	\$30,000	\$27,000	\$24,000	\$21,000	\$18,000	\$15,000
1,000	0.10	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$36,000	\$40,000	\$37,000	\$34,000	\$31,000	\$28,000	\$25,000	\$22,000
1,100	0.06	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$36,000	\$40,000	\$44,000	\$41,000	\$38,000	\$35,000	\$32,000	\$29,000
1,200	0.02	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$36,000	\$40,000	\$44,000	\$48,000	\$45,000	\$42,000	\$39,000	\$36,000
1,300	0.00	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$36,000	\$40,000	\$44,000	\$48,000	\$52,000	\$49,000	\$46,000	\$43,000
Expected Profit:		\$8,000	\$12,000	\$16,000	\$20,000	\$23,731	\$26,788	\$28,365	\$27,923	\$26,135	\$23,673	\$20,808	\$17,808	\$14,808	\$11,808	\$8,808
Maximum Profit:		1														

Supplier:

Order:	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	
Demand																
Prob																
400	0.00	\$22,000	\$33,000	\$44,000	\$43,000	\$42,000	\$41,000	\$40,000	\$39,000	\$38,000	\$37,000	\$36,000	\$35,000	\$34,000	\$33,000	\$32,000
500	0.04	\$22,000	\$33,000	\$44,000	\$55,000	\$54,000	\$53,000	\$52,000	\$51,000	\$50,000	\$49,000	\$48,000	\$47,000	\$46,000	\$45,000	\$44,000
600	0.10	\$22,000	\$33,000	\$44,000	\$55,000	\$66,000	\$65,000	\$64,000	\$63,000	\$62,000	\$61,000	\$60,000	\$59,000	\$58,000	\$57,000	\$56,000
700	0.21	\$22,000	\$33,000	\$44,000	\$55,000	\$66,000	\$77,000	\$76,000	\$75,000	\$74,000	\$73,000	\$72,000	\$71,000	\$70,000	\$69,000	\$68,000
800	0.29	\$22,000	\$33,000	\$44,000	\$55,000	\$66,000	\$77,000	\$88,000	\$87,000	\$86,000	\$85,000	\$84,000	\$83,000	\$82,000	\$81,000	\$80,000
900	0.19	\$22,000	\$33,000	\$44,000	\$55,000	\$66,000	\$77,000	\$88,000	\$99,000	\$98,000	\$97,000	\$96,000	\$95,000	\$94,000	\$93,000	\$92,000
1,000	0.10	\$22,000	\$33,000	\$44,000	\$55,000	\$66,000	\$77,000	\$88,000	\$99,000	\$110,000	\$109,000	\$108,000	\$107,000	\$106,000	\$105,000	\$104,000
1,100	0.06	\$22,000	\$33,000	\$44,000	\$55,000	\$66,000	\$77,000	\$88,000	\$99,000	\$110,000	\$121,000	\$120,000	\$119,000	\$118,000	\$117,000	\$116,000
1,200	0.02	\$22,000	\$33,000	\$44,000	\$55,000	\$66,000	\$77,000	\$88,000	\$99,000	\$110,000	\$121,000	\$132,000	\$131,000	\$130,000	\$129,000	\$128,000
1,300	0.00	\$22,000	\$33,000	\$44,000	\$55,000	\$66,000	\$77,000	\$88,000	\$99,000	\$110,000	\$121,000	\$132,000	\$143,000	\$142,000	\$141,000	\$140,000
Expected Profit:		\$22,000	\$33,000	\$44,000	\$55,000	\$65,538	\$74,923	\$81,769	\$85,154	\$86,231	\$86,154	\$85,385	\$84,385	\$83,385	\$82,385	\$81,385
Maximum Profit:		!!!														
Total Profit:		\$30,000	\$45,000	\$60,000	\$75,000	\$89,269	\$101,712	\$110,135	\$113,077	\$112,365	\$109,827	\$106,192	\$102,192	\$98,192	\$94,192	\$90,192
Maximum Profit		!!!														

Revenue Sharing

R = \$200
 W = \$40
 C = \$50
 S = \$10
 p = 0.40

Retailer:

Order:	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600
400 0.00	\$8,000	\$12,000	\$16,000	\$13,000	\$10,000	\$7,000	\$4,000	\$1,000	-\$2,000	-\$5,000	-\$8,000	-\$11,000	-\$14,000	-\$17,000	-\$20,000
500 0.04	\$8,000	\$12,000	\$16,000	\$20,000	\$17,000	\$14,000	\$11,000	\$8,000	\$5,000	\$2,000	-\$1,000	-\$4,000	-\$7,000	-\$10,000	-\$13,000
600 0.10	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$21,000	\$18,000	\$15,000	\$12,000	\$9,000	\$6,000	\$3,000	\$0	-\$3,000	-\$6,000
700 0.21	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$25,000	\$22,000	\$19,000	\$16,000	\$13,000	\$10,000	\$7,000	\$4,000	\$1,000
800 0.29	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$28,000	\$26,000	\$23,000	\$20,000	\$17,000	\$14,000	\$11,000	\$8,000
900 0.19	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$36,000	\$32,000	\$30,000	\$27,000	\$24,000	\$21,000	\$18,000	\$15,000
1,000 0.10	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$36,000	\$40,000	\$38,000	\$34,000	\$31,000	\$28,000	\$25,000	\$22,000
1,100 0.06	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$36,000	\$40,000	\$44,000	\$42,000	\$38,000	\$35,000	\$32,000	\$29,000
1,200 0.02	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$36,000	\$40,000	\$44,000	\$48,000	\$46,000	\$42,000	\$39,000	\$36,000
1,300 0.00	\$8,000	\$12,000	\$16,000	\$20,000	\$24,000	\$28,000	\$32,000	\$36,000	\$40,000	\$44,000	\$48,000	\$52,000	\$50,000	\$46,000	\$43,000
Expected Profit:	\$8,000	\$12,000	\$16,000	\$20,000	\$23,000	\$26,000	\$28,365	\$29,000	\$30,000	\$31,000	\$32,000	\$33,000	\$34,000	\$35,000	\$36,000
Maximum Profit:							1								

800 ordered and 600 demanded
 Profit = $600 \times \$200 \times 0.4 - 600 \times \$40 + (800 - 600) \times (\$10 - \$40) = \$18,000$

800 ordered and 1000 demanded
 Profit = $800 \times \$200 \times 0.4 - 800 \times \$40 = \$32,000$

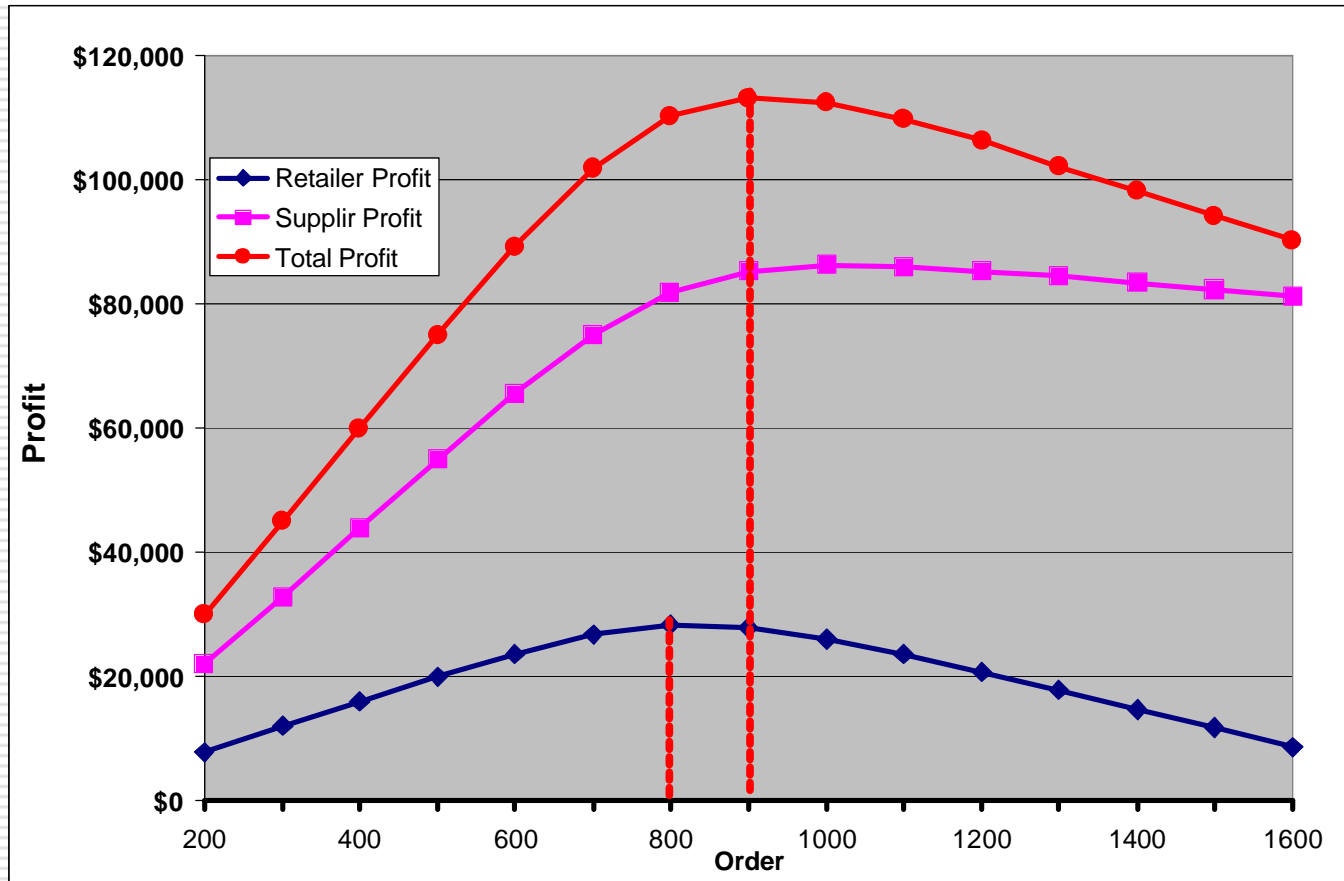
900	1000	1100	1200	1300	1400	1500	1600
\$39,000	\$38,000	\$37,000	\$36,000	\$35,000	\$34,000	\$33,000	\$32,000
\$51,000	\$50,000	\$49,000	\$48,000	\$47,000	\$46,000	\$45,000	\$44,000
\$63,000	\$62,000	\$61,000	\$60,000	\$59,000	\$58,000	\$57,000	\$56,000
\$75,000	\$74,000	\$73,000	\$72,000	\$71,000	\$70,000	\$69,000	\$68,000
\$86,000	\$85,000	\$84,000	\$83,000	\$82,000	\$81,000	\$80,000	\$79,000
\$99,000	\$98,000	\$97,000	\$96,000	\$95,000	\$94,000	\$93,000	\$92,000
\$99,000	\$99,000	\$98,000	\$97,000	\$96,000	\$95,000	\$94,000	\$93,000
\$109,000	\$108,000	\$107,000	\$106,000	\$105,000	\$104,000	\$103,000	\$102,000
\$119,000	\$118,000	\$117,000	\$116,000	\$115,000	\$114,000	\$113,000	\$112,000
\$129,000	\$128,000	\$127,000	\$126,000	\$125,000	\$124,000	\$123,000	\$122,000

800 ordered and 600 demanded
 Profit = $800 \times (\$40 - \$50) + 600 \times \$200 \times 0.6 = \$64,000$

800 ordered 1000 demanded
 Profit = $800 \times (\$40 - \$50) + 800 \times \$200 \times 0.6 = \$88,000$

Expected Profit with Revenue Sharing

$R = \$200$
 $W = \$40$
 $C = \$50$
 $S = \$10$
 $p = 0.40$



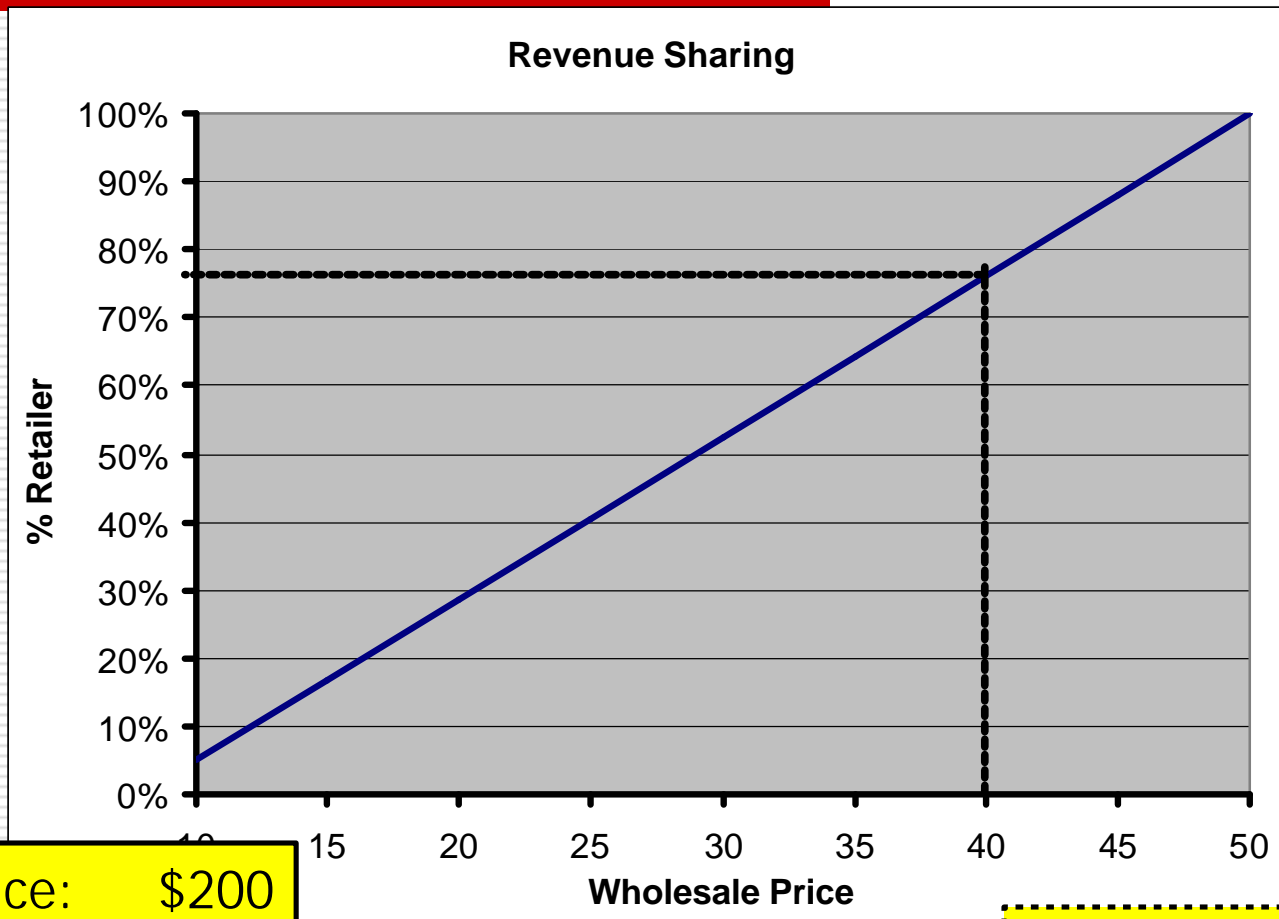
Optimal Revenue Share

$$Q_R^* = F^{-1} \left(\frac{p_R W}{p_R S} \right)$$

$$\frac{p_R W}{p_R S} = \frac{R - C}{R - S}$$

$$p_R W = \frac{(R - C)}{R - S} \cdot \frac{S(R - C)}{R - S}$$

Coordination with Rev. Sharing

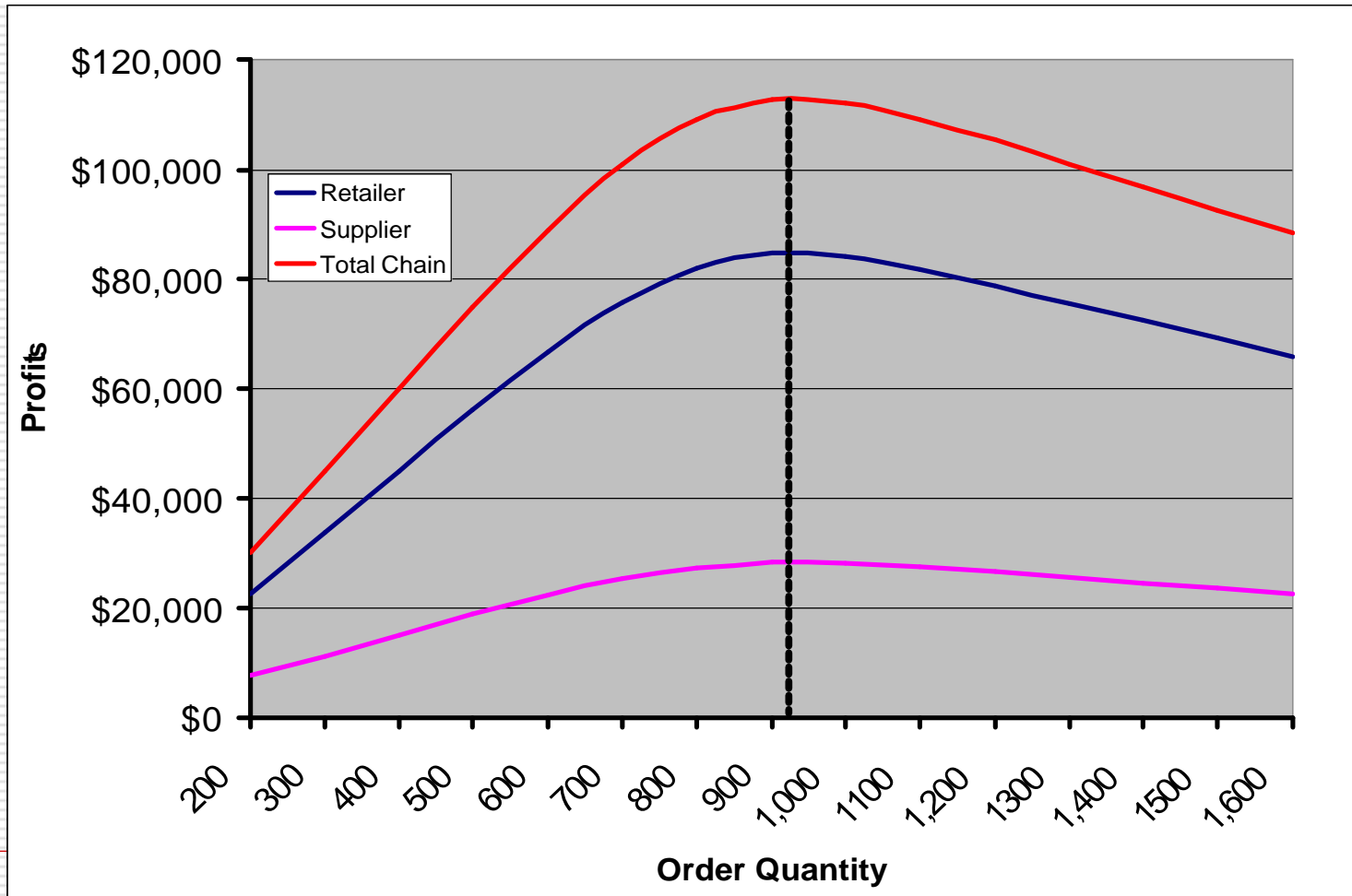


Retail Price: \$200
Supplier Costs: \$50
Salvage value: \$10

Wholesale price: \$40
Retail Rev Share: 76%

Revenue Sharing (Normal Approx)

$R = \$200$
 $W = \$40$
 $C = \$50$
 $S = \$10$
 $p = 0.76$



Summary

- ✍ Wholesale contracts give too much risk and not enough expected reward to the retailer
 - ✍ To make the retailer order more (and increase the channel's profit) the supplier has to take on part of the risk
 - ✍ Risk sharing mechanisms covered include:
 - ✍ Buybacks; revenue sharing
 - ✍ Other mechanisms
 - ✍ Options contracts
 - ✍ Quantity-flexibility (refund on a portion of the unsold units)
 - ✍ Sales-rebate (rebate on unsold units above a threshold)
 - ✍ There are many other mechanisms
 - ✍ Each mechanism can coordinate the channel with various allocations of the profit between the retailer and the supplier.
-

Any Questions?



Yossi Sheffi
sheffi@mit.edu