How Networked Agents Make Decisions:

Coordination with Local Information & The Value of Temporal Data for Learning Influence Networks

Ph.D. Thesis Defense

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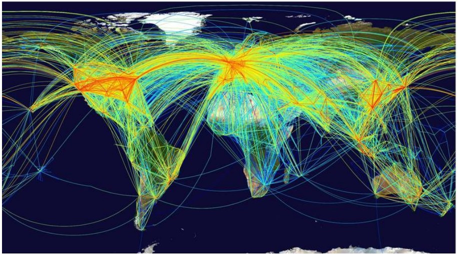
April 17, 2014

Motivation



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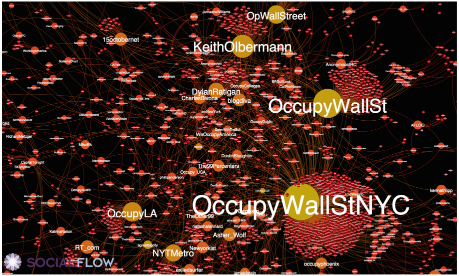
OpWallStreet
KeithOlbermann
15octobernet
DylanRatigan
CharlesBivona
blogdiva
OccupyWallSt
WeOccupyAmerica
DustinSlaughter
The99Percenters
OccupyWallStNYC
OccupyLA
TheOther99
Asher_Wolf
RT_com
NYTMetro
Newyorkist
exiledsurfer
occupyphoenix
SOCIALFLOW

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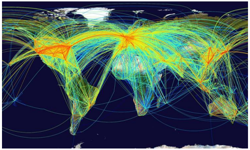

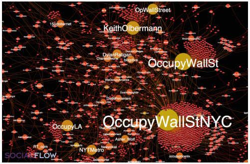

- Networks shape decisions/outcomes

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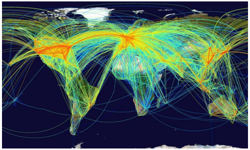

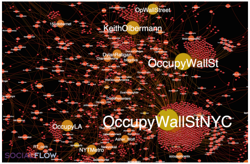

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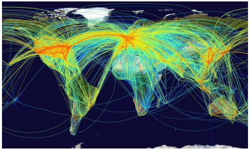

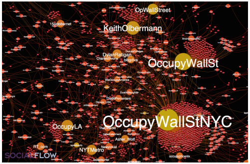

- Networks shape decisions/outcomes
- How do networked entities make decisions?
    - What does knowledge of the network tell us about decision making of rational agents?
    - What do decisions/outcomes tell us about the network?

From Networks to Outcomes; from Outcomes to Networks

- With knowledge of the *network*, characterize the *outcomes*

- With knowledge of the *outcomes*, infer the *network*

Theme: Locality

- With knowledge of the *network*, characterize the *outcomes*

  - How do the *equilibria* of *coordination games* depend on *local* information sharing?

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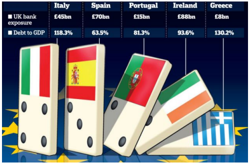
  - How can we *learn* the network of *local influence* between agents from records of their behavior?

Coordination with Local Information



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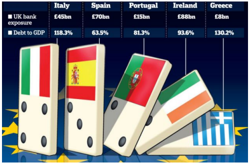

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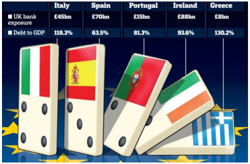

- Self-fulfilling crises
    - Debt crises, bank runs, currency attacks, social upheavals, ...
- *Local* information sharing enables coordination. How do outcomes depend on local information sharing?

Model — Agents and Payoffs

- Agents $1, \ldots, n$
- Actions: *risky* ($\alpha_i = 1$), *safe* ($\alpha_i = 0$)
- Payoffs:

$$u_i(a_i, a_{-i}, \theta) = \begin{cases} \pi(k, \theta) & \text{if } a_i = 1 \\ 0 & \text{if } a_i = 0, \end{cases}$$

where $k = \sum_{j=1}^{n} a_j$, $\theta \in \mathbb{R}$ *the fundamentals*.

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- Assumptions on π
  1. Strategic complementarities: $\pi(k, \theta) - \pi(k-1, \theta) > \rho > 0$
  2. State monotonicity: π strictly decreasing in θ
  3. Strict dominance regions: for sufficiently low (high) fundamentals, risky (safe) is strictly dominant

Model — Information

- θ is realized, agents hold improper prior over $\mathbb{R}$
- Conditional on θ, signals $(x_1, \ldots, x_m)$ generated: $x_r = \theta + \xi_r$
    - $(\xi_1, \ldots, \xi_m)$ independent of θ, drawn from continuous density with full support over $\mathbb{R}^m$

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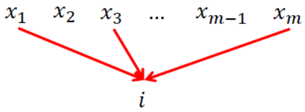
- Agent i: *observation set* $I_i \subseteq \{x_1, \ldots, x_m\}$
- $\{I_i\}_{i=1}^n$: *information structure*
- Strategy: $s_i : \mathbb{R}^{|I_i|} \to \{0, 1\}$



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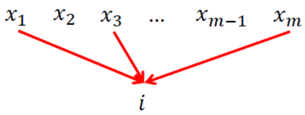

- We are interested in *information locality*
  - $x_r \in I_i \forall i$: public signal
  - $x_r \in I_i$ for only one i: private signal
  - all other cases: *local* signal

Problem and Contributions

- Positioning
  - Common knowledge of the fundamentals leads to multiple equilibria
  - Global-games framework leads to unique equilibrium selection
  - Endogeneity of information can restore multiplicity
  - Other natural, unstudied mechanism through which multiplicity can reemerge: *exogenous* information structure per se

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  - Other natural, unstudied mechanism through which multiplicity can reemerge: *exogenous* information structure per se
- Problem: How does the number of equilibria depend on the exogenous information structure?
- Contribution: The number of equilibria is highly sensitive to the details of information locality
  - Conditions for uniqueness vs. multiplicity that pertain solely to the details of information sharing
  - Characterization of width of multiplicity as a function of the information structure

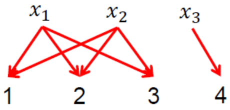
Sufficient Conditions for Multiplicity

Proposition

Information structure with $n \geq 2$ and collection C, $|C| \geq 2$, such that

- *all agents in C have the same observation set I and,*
- *for $i \notin C$, $I_i \cap I = \emptyset$.*

Then multiple Bayesian Nash equilibria.



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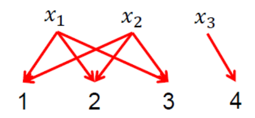

Proposition

Payoffs $\pi(k, \theta) = \frac{k-1}{n-1} - \theta$, noise $\xi_r \sim \mathcal{N}(0, \sigma^2)$ i.i.d.
Information structure with $n \geq 2$, $\ell \geq 2$, such that

- $I_1 \subset I_2 \subset \ldots \subset I_\ell$
- *for $i > \ell$, $I_i \cap I_\ell = \emptyset$.*

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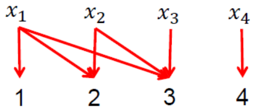

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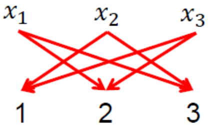

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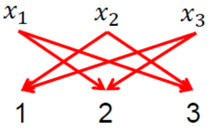

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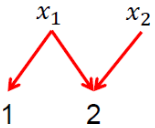

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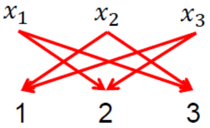

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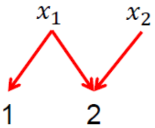

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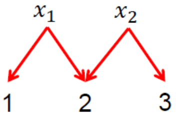

Uniqueness
(surprising)

Local Information and the Set of Equilibria

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Proposition

As $\sigma \to 0$, the strategy s_i of agent i is rationalizable if and only if

$$s_i(x) = \begin{cases} 1 & \text{if } x < \underline{\tau} \\ 0 & \text{if } x > \bar{\tau}, \end{cases}$$

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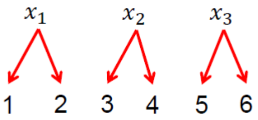
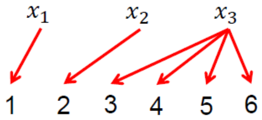


Narrower multiplicity



Wider multiplicity

Large Coordination Games

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- Application: networks of information exchange
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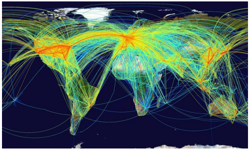
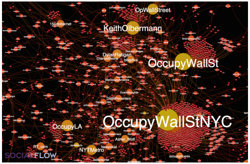
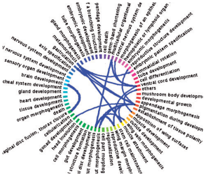
Proposition

Assume network is union of equally sized disconnected cliques. As $n \to \infty$, uniqueness if and only if cliques grow **sublinearly** *with n.*

The Value of Temporal Data for Learning Influence Networks







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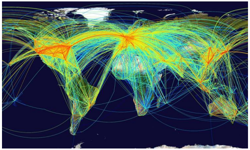

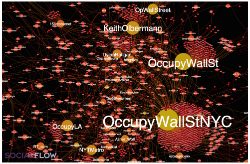

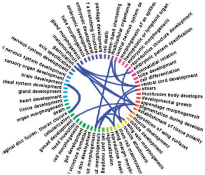

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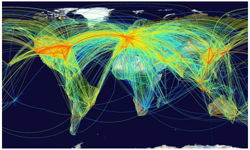

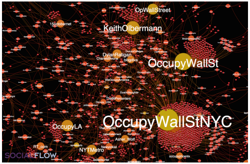

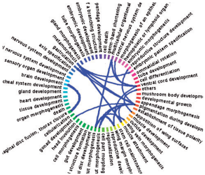

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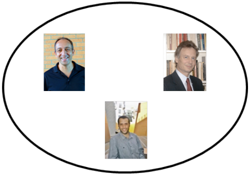
- Can influences be untangled based on the outcomes in a principled manner?
- How much faster can we learn influences with access to increasingly informative temporal data?

Three modes of data

- Learning with **sets** of decisions



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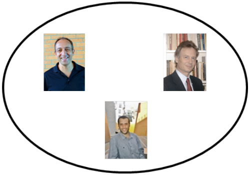

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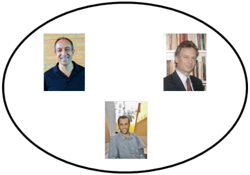

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- Agents $1, \dots, 5$.

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- Richer data modes allow faster/better learning, but may require more effort/cost

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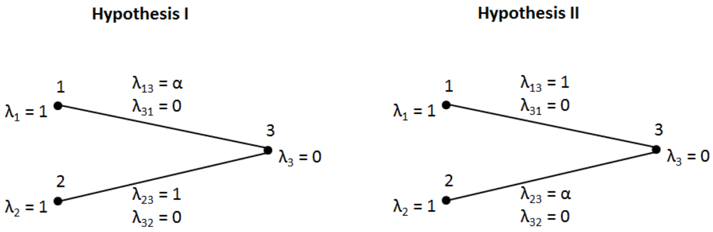

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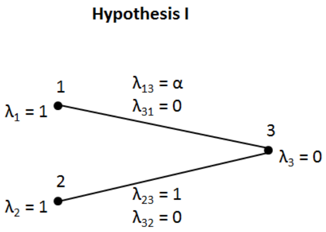

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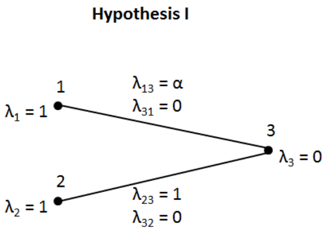

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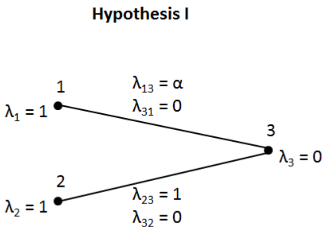

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- Focus on KL divergence $D(P_1||P_2) \equiv \mathbb{E}_{P_1}\left[\log \frac{dP_1}{dP_2}\right]$ for cases of outcomes: sets, sequences, times.

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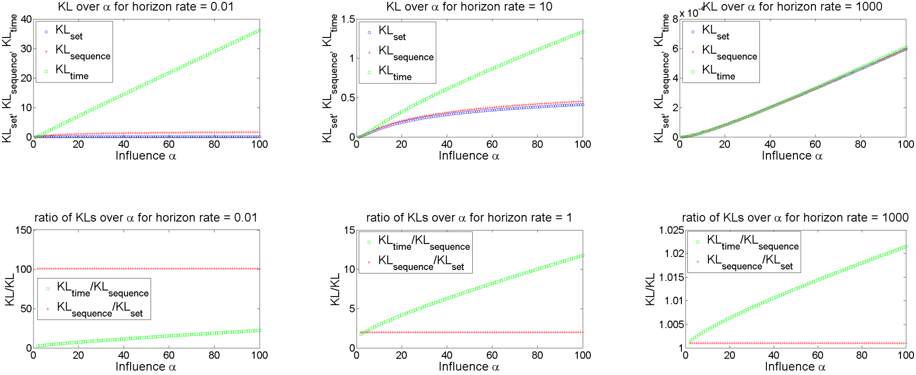

Large horizon: sequences large gain over sets, times have gain but smaller for small α

Small horizon: sets provide almost all the information, times have value only for large α

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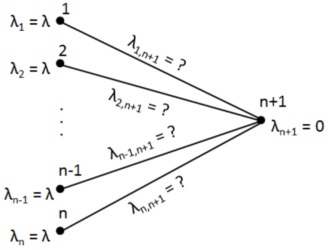

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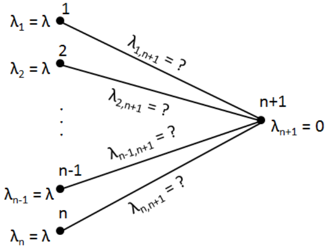

- Consider $\lambda_{ij} = 0$ or ∞

Sample Complexity Results

- Bayesian setting, $p = \frac{1}{2}$ of ∞ for each edge

	$\lambda_{hor} = \lambda$		$\lambda_{hor} = n\lambda$	
	Sets	Sequences	Sets	Sequences
Learn one	$\Theta(n^2)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Learn all	$\Theta(n^2 \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$

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- Known scaling ℓ of agents with influence rate ∞ to $n+1$

	$\lambda_{hor} = \lambda$		$\lambda_{hor} = n\lambda$	
	Sets	Sequences	Sets	Sequences
$\ell = 1$	$\Theta(\log n)$	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
$\ell = \alpha n, \alpha \in (0,1)$	$\Theta(n^2 \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$	$\Theta(n \log n)$
$\ell = n-1$	$\Theta(n^2)$	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$

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- Mobile applications dataset (courtesy of Sandy Pentland, MIT Media Lab)
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Out of top	10	joint influence edges, friendship exists in	70%
	20		65%
	50		54%
	100		38%

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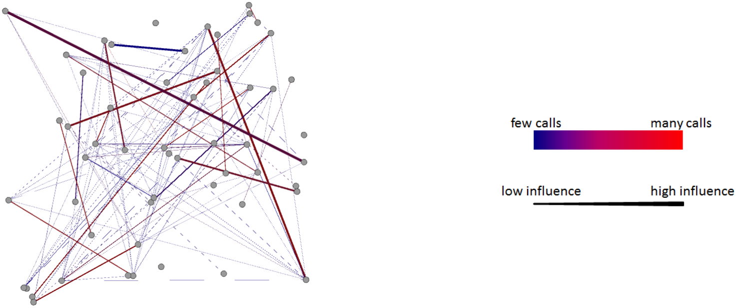

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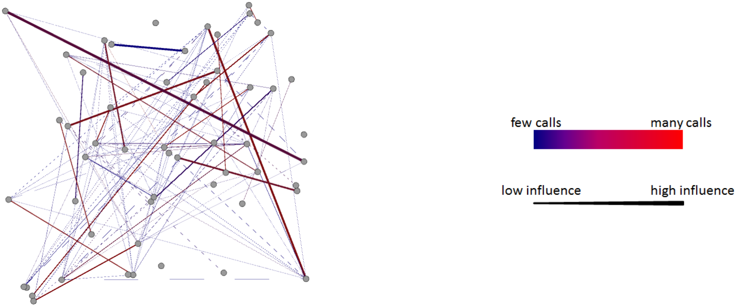
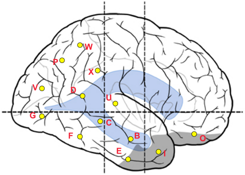


- Correlation only slightly lower when learning with *sets*

Real Data — Epileptic Seizures

- EEG data obtained from epileptic patients
  (courtesy of Sri Sarma, Johns Hopkins Inst. for Comp. Medicine)
  - 10-15 electrodes per patient, 10 channels per electrode
  - Different electrodes/channels monitor different regions of the brain
  - 3-6 seizure events per patient
  - For each seizure event, data is voltage measurements per millisecond across all channels, over 4-7 minutes



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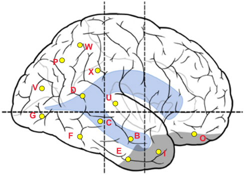
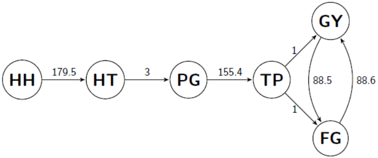


- Infer influence network from data on seizures; generate hypotheses:
  - What regions are more likely to start the seizure?
  - How likely is it that a region that is under seizure infects another region?

Real Data — Epileptic Seizures

- Unconstrained optimization of the likelihood, ℓ_1-regularized. Sparsest inferred network that is connected:



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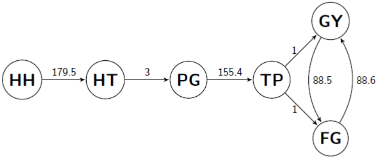
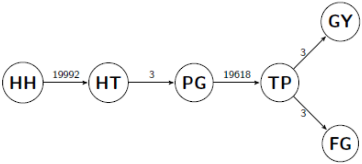


- Constrained optimization of the likelihood over trees whose nodes have at most two children



Summary

Networks

Game theory

Estimation
& Learning

Outcomes

Current and Future Work, Open Questions

- Coordination with local information
  - What happens with information asymmetry about the information structure?
  - Prove sufficient condition for multiplicity is not necessary
  - Open: complete characterization of sufficient and necessary conditions
  - Open: dependence of width of multiplicity on the information structure in the general case

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  - Prove sufficient condition for multiplicity is not necessary
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- The value of temporal data for learning influence networks
  - Sample complexity results for broader families of networks
  - Reconstructing trees with latent nodes
  - Open: other influence/observation window models

Thank You!

- John, Munzer

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- Devavrat, Marios

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- LIDS

Thank You!

- John, Munzer
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- Come visit in Paris!