Experimental Investigations of a New Approach to Unsteady Separation

Thomas Peacock, Raul Coral & George Haller
Department of Mechanical Engineering
Massachusetts Institute of Technology

1 Introduction

Unsteady flow separation plays a pivotal role in the stability, maneuverability and efficiency of aerospace technology. For example, dynamic stall at high Reynolds numbers, initiated by boundary-layer separation near the leading edge of an airfoil, limits the performance of modern air vehicles. The ability to monitor and control unsteady separation is therefore crucial for the next generation of helicopters, aircraft and projectiles.

The goal of this research is to implement a new theory of separation that identifies both the location and geometry of two-dimensional unsteady separation based on distributed surface shear-stress and pressure measurements.

1.1 Background

In his classic paper, Prandtl [7] derived a separation criterion for two-dimensional steady incompressible velocity fields \( \mathbf{v}(x) = (u(x, y), v(x, y)) \) that satisfy the no-slip boundary condition \( u(x, 0) = v(x, 0) = 0 \) on the \( y = 0 \) boundary. Prandtl’s criterion states that separation takes place at a boundary point \( p = (\gamma, 0) \) whenever

\[
\begin{align*}
  u_y(\gamma, 0) &= 0, \\
  u_{xy}(\gamma, 0) &< 0.
\end{align*}
\]

These conditions are necessary for separation on all length scales, including both boundary-layer separation and small-scale separation within a boundary layer. The zero-skin-friction principle \( (1) \) has become the most broadly used indicator of separation, even though numerical work in the 1970’s by Sears and Tellionis [10][11] and others showed that the principle fails for unsteady flows. Furthermore, it is now widely understood that even in steady three-dimensional flows, separation is rarely associated with the vanishing of wall shear-stress [1].

Concluding that vanishing skin friction “does not denote separation in any meaningful sense in unsteady flow", Sears and Tellionis [11] proposed a separation criterion known as the Moore-Rott-Sears (MRS) principle. This principle locates separation at off-boundary points where the wall-tangential shear component vanishes, and the local streamwise velocity equals the velocity of the moving separation structure. This postulate, however, requires the \textit{a priori} knowledge of the separation speed, making the MRS principle practically inapplicable. Indeed, the principle relies on information that is not foreknown for rapidly maneuvering air vehicles [15][19].

As an alternative, Van Dommelen [15] and Van Dommelen and Shen [16] proposed that separation takes place at singularities in the solution of the boundary layer equations. Analytic results show, however, that separation in the boundary layer equations has no direct connection with velocity singularities [6]. In addition, physical velocity fields display no singularities, making Van Dommelen’s principle inapplicable to Navier-Stokes flows.

Despite its shortcomings, Van Dommelen’s \textit{Lagrangian} view – that unsteady separation is to be understood as the formation of a material spike – was a major advance. The so-called Van-Dommelen-Shen process [17] is consistent with the narrow eruptive spires readily observable in careful separation experiments at both high and low Reynolds numbers [2]. On pitching airfoils, the erupting shear layer is unstable, and the subsequent roll-up leads to the formation of the dynamic stall vortex.
Studying time-periodic incompressible flows, Shariff, Pulliam and Ottino [12] realized that fixed separation (i.e., separation at a constant location) in unsteady flows can indeed be viewed as material ejection from the boundary along a time-dependent unstable manifold, which is a material line that shrinks to the separation point in backward time. A schematic of fixed separation along an unstable manifold is shown in figure 1. Due to the no-slip boundary condition, the unstable manifold of figure 1 is nonhyperbolic: it cannot be located from a linear analysis of the flow at the wall. For this reason, locating such manifolds in compressible flows with general time-dependence has remained a formidable technical challenge. As a further difficulty, moving separation (i.e., separation at time-varying locations) cannot be explained by unstable manifolds, because material lines emanating from a no-slip boundary remain anchored at the same boundary point for all times.

Figure 1: Fluid separation from a point \((\gamma, 0)\) along a time-dependent unstable manifold \(M(t)\).

In the absence of a theoretically-sound and practical separation criterion for unsteady two- and three-dimensional flows, experimentalists have been forced to approach separation empirically. Examples include using oil films or directionally sensitive hot-films to locate converging skin-friction lines [18], and determining local minima in the skin friction magnitudes accompanied by a 180° phase relationship between adjacent shear-stress sensors [14]. Despite such approaches, the experimental detection and control of unsteady separation has remained ambiguous [9].

1.2 An exact two-dimensional theory

Recently, Haller and co-workers have derived an analytic criterion for the location of wall-based unstable manifolds in a two-dimensional velocity field \(v(x, t) = (u(x, y, t), v(x, y, t))\) [4][5]. These wall-based unstable manifolds are the eruptive material spires reported in both low and high Reynolds number separation [17]. According to the new criterion, any fixed separation point \(p = (\gamma, 0)\), satisfies the following conditions:

\[
\lim_{T\to\infty} \frac{1}{T} \int_{t_0-T}^{t_0} e^{\int_{t_0}^{t} v_y(\gamma, 0, s) \, ds} u_y(\gamma, 0, t) \, dt = 0, \\
\lim_{T\to\infty} \frac{1}{T} \int_{t_0-T}^{t_0} e^{\int_{t_0}^{t} v_y(\gamma, 0, s) \, ds} \left[ u_y(\gamma, 0, t) + u_y(\gamma, 0, t) \int_{t_0}^{t} v_{xy}(\gamma, 0, s) \, ds \right] \, dt < 0, \\
\lim_{T\to\infty} \frac{1}{T} \int_{t_0-T}^{t_0} e^{\int_{t_0}^{t} v_y(\gamma, 0, s) \, ds} v_{yy}(\gamma, 0, t) \, dt > 0. \tag{2}
\]

The conditions in (2) are valid for any mass-conserving fluid flow, compressible or incompressible, subsonic, supersonic or transonic. They require a knowledge of the time history of kinematic quantities along the wall, such as velocity gradients \(u_y(\gamma, 0, t)\), up to the present time \(t_0\). An example in which such a fixed separation point has been identified is separating flow past a cylinder [12].

The first condition in (2) is the most important one: it is a necessary condition for the existence of a material separation or reattachment profile at \(x = \gamma\). The second inequality in (2) – a nondegeneracy condition – guarantees a unique separation profile by ensuring that all material lines, other than the
unstable manifold, align with the wall in backward time. The third condition in (2) is to distinguish \( p \) from a reattachment point, ensuring material ejection from \( p \) into the mean flow. This condition is intimately related to the strength of separation, and can be used to distinguish between small scale separation and boundary-layer separation in a physical situation. Together, the three conditions in (2) yield a sufficient criterion for unsteady separation in general two-dimensional flows.

As an example of the application of criterion (2), consider incompressible flows, for which

\[
\frac{\partial v_y}{\partial y} = 0, \quad \text{and the no-slip boundary condition implies} \quad u_x(x, 0, t) = v_y(\gamma, 0, t) = 0. \]

Thus, the separation criterion (2) simplifies to

\[
\lim_{T \to -\infty} \frac{1}{T} \int_{t_0}^{t_0 + T} u_y(\gamma, 0, t) \, dt = 0,
\]

\[
\lim_{T \to -\infty} \frac{1}{T} \int_{t_0}^{t_0 + T} u_{xy}(\gamma, 0, t) \, dt < 0. \tag{3}
\]

For steady incompressible flows, (3) further simplifies to the original Prandtl criterion (1).

All of the conditions in (2) can be experimentally verified by measuring the skin friction \( \tau_w(x, t) \), the fluid density at the wall \( \rho(x, 0, t) \), and the viscosity \( \nu \), since it can readily be shown that

\[
u_y(x, 0, t) = \tau_w(x, t)/(\nu \rho(x, 0, t)), \quad e^{-\int_{t_0}^{t} v_y(x, 0, s) \, ds} = \rho(x, 0, t)/\rho(x, 0, t_0). \]

For incompressible flow, (3) show that a knowledge of the time history of wall shear-stress is all that is required to identify the location of fixed unsteady separation. Condition (3) applies more generally, as the effects of compressibility do not become important until the freestream Mach number reaches approximately 4 (and the formation of shocklets is not seen in boundary layer turbulence until the freestream Mach number of the flow reaches approximately 7) [13].

In addition to monitoring the location of a separation point in a two-dimensional flow, Haller also derived analytic formulae for the time-dependent unstable manifold (separation profile) along which particles are ejected from the wall [4]. Specifically, the time-dependent separation angle can be calculated from distributed skin-friction and pressure measurements along using the relation

\[
\tan \alpha(t_0) = \lim_{T \to -\infty} \frac{3 \int_{t_0}^{t_0 + T} \tau_{w,x}(\gamma, s) \, ds}{\int_{t_0}^{t_0 + T} p_x(\gamma, 0, s) + 3\tau_{w,x}(\gamma, s) \int_{t_0}^{s} \tau_w(\gamma, 0, r) \, dr \, ds}, \tag{4}
\]

where \( \alpha \) is the angle the separation profile makes with the boundary and \( p(x, y, t) \) is the surface pressure.

### 1.3 The rotor-oscillator flow

Our experiment seeks to implement criteria (3) in a physical setting for the first time. For practical reasons we choose a low Reynolds number flow, for which the time- and length scales of unsteady separation are amenable to experimental investigation. In making this choice, however, we emphasise an important point. The theoretical basis for the approach of Haller [4] is kinematical, and thus independent of Reynolds number. Thus, these experiments are a logical first step towards implementation of the method in more practically relevant geometries.

The experimental flow we investigate, the so-called rotor-oscillator flow, is a two-dimensional flow generated by a rotlet (a local source of angular momentum) adjacent to a rigid boundary. This flow has been investigated in detail by Hackborne [3]. To the best of our knowledge, this arrangement is the simplest geometry in which one can establish and influence a low Reynolds number separating flow. Other possibilities, such as flow past blunt bodies and flow past a backward facing step, are far less amenable to experimental investigation.

A sketch of the steady flow field generated by a rotlet near a flat boundary is presented in figure 2. Flow separates from the boundary at point A and reattaches to the boundary at point B, both of which are
locations of zero shear-stress. Topologically, the rotlet generates a recirculation bubble on the boundary. Unsteadiness can be introduced into the system by moving the position of the rotlet from side-to-side. In this case it is straightforward to determine the unsteady flow field, as time-dependence only enters the problem through the boundary conditions. Thus, the velocity field associated with the steady rotlet translates along with the source [3].

![Sketch of a rotlet](image)

**Figure 2:** Sketch of a rotlet (point source of angular momentum) adjacent to a wall. Separation occurs at point A and reattachment at point B.

## 2 Experimental Apparatus

An image of the experiment is shown in figure 3. The experiment comprises a 40cm x 10cm x 10cm acrylic tank filled with a viscous fluid (corn syrup). The tank sits inside an aluminum support stand. A cylinder 10cm long and 8 mm in diameter is positioned vertical in the tank, parallel to one of the sidewalls. The cylinder can be seen in the center of the image in figure 3.

The rotation of the cylinder is driven by a micro-stepper motor, which is in turn mounted on a linear traverse. The rotation generates a nominally two-dimensional flow, uniform along the length of the cylinder. In a cross-section of the experiment the rotating cylinder acts as a rotlet, generating a recirculation bubble that separates and reattaches to the vertical sidewall (see figure 9). As the cylinder rotates, it can be translated back and forth with a desired time dependence (e.g., periodic, quasi-periodic or random) using a National Instruments motion control system.

Visualization of flow within the tank is achieved through the mechanical injection of dye adjacent to the boundary. When unsteady-separation occurs dyed fluid is drawn away from the wall and into the bulk. The images of this are captured using a CCD camera looking up through the tank, via a 45 degree mirror placed beneath the experiment.

The shear-stress sensors are flush-mounted hot wires running in constant temperature mode. A sensor comprises a 1mm piece of 15 micron diameter platinum wire, mounted between electrode connections on a circuitboard substrate. The sensors, which have been successfully utilized in active control of turbulent boundary layers [8], are fabricated and calibrated ‘in-house’. The calibration is achieved by positioning the sensors in the experimental tank, under known flow conditions driven by the rotting cylinder.

Initially, we implement an array of 10 shear-stress sensors. The sensor arrangement has the sensors separated by 2mm; the sensor array therefore covers 1.8cm. The directionality of shear-stress for the two-dimensional experiments, which is required by criterion (3), is readily inferred from a knowledge of the magnitude of the shear-stress field. Consequently, there is no need to use angled hotwire pairs to achieve directionally sensitive shear-stress sensors in this experiment. To obtain more spatially extended shear-stress data, we re-run the same experiment with the relative position of the cylinder and the shear-stress sensors altered. The data from these sensors is recorded using a 16-bit National Instruments data-acquisition system.
Figure 3: Experimental apparatus for investigating two-dimensional unsteady separation. The acrylic tank sits in an aluminum support frame. A cylinder is mounted on a micro-stepper motor, which is in turn mounted on a linear traverse. The cylinder (seen center image) may be rotated and translated in the fluid. The length of the acrylic tank is 40cm.

Figure 4: An array of four shear stress sensors. A 1mm long platinum wire of diameter 15 microns is connected between each electrode pair on the left of the image.

In addition, we shall also incorporate a commercial array of pressure sensors into the experiment, in order to determine the angle of unsteady separation using relation 4. We note that since the flow in the experiment is nominally two-dimensional, we do not have to develop an integrated array of shear-stress and pressure sensors. We need only position our two sensor arrays at different locations along the length of the cylinder, since they will both experience the same two-dimensional flow.

3 Results

Prior to the experiments, we have performed a detailed set of numerical simulations, to determine the effectiveness of criteria (2) and (4) for the rotor-oscillator flow. In figure 5 we show the results of a numerical simulation of the experimental arrangement using FLUENT. For this steady flow, separation takes place near the 3cm marker on the sidewall and reattachment near the -3cm marker. Upon inspection, one can determine a recirculation bubble, driven by the anticlockwise rotation of the cylinder.
Figure 5: Numerical simulation of experimental geometry using FLUENT. The rotating cylinder generates separation near the 3cm marker on the lower boundary.

The surface shear-stress and pressure profiles near the steady separation point are shown in figures 6 and 7. In accordance with Prandtl [7], we see that for the steady flow the shear-stress goes to zero at the point of separation. The characteristic variation of the shear stress and pressure is on the order of several Pascals. At low Reynolds number, the unsteady shear-stress and pressure fields associated with side-to-side motion of the cylinder are simply translations of the profiles presented in figure 6 and 7.

Figure 6: Shear stress along the wall, with cylinder rotating.

When the cylinder is periodically translated, fluid does not separate from the boundary at a point of instantaneous zero shear stress. Rather, separation occurs as a sharp material spike whose location can be identified using criteria (2) and (4). Using the shear stress and pressure information, we are able to determine both the location and angle of separation for this unsteady flow. As an example, the results of an unsteady simulation can be seen in figure 8, in which material separation is visualised by red tracer particles placed initially adjacent to the sidewall. The instantaneous point of zero shear does not coincide with the point of separation in this case. The separation criteria (2) and (4), however, do an excellent job of determining both the location and orientation of the separation profile.

3.1 Experimental results

The rotation of the cylinder establishes a nominally two-dimensional separation bubble in a cross section of the experiment. This is evident in figure 9, in which we present an image of flow driven by the rotating
cylinder. A small amount of dye has been introduced along the sidewall. The anticlockwise rotation of the cylinder draws fluid away from the wall, creating a separation point that is evidenced by a black material spike.

In our experiments thus far we have investigated the two-dimensional nature of the flow in our experiment. This we have done by locating measuring the location of separation for several different positions along the length of the cylinder. The results of these investigations are shown in figure 10. Within experimental error, it was not possible to detect any change in the location of separation along the central length of the cylinder, and the flow may therefore reasonably be considered as being two-dimensional.

We have also made direct comparisons between the experiment and the numerical simulations. Specifically, we have identified the location of the point of separation as a function of the location of the cylinder relative to the wall. The results of these studies are shown in figure 11, in which there is very good agreement between experiment and numerics. The maximum error is 5%, which corresponds to a physical accuracy of 1mm in locating the material spike (which is of the order of the width of the material spike).

In addition to these necessary initial experiments, we have investigated qualitatively the effect of unsteadiness on separation in the experiment. For a time periodic flow, separation occurs at a point of zero mean shear stress, consistent with criterion (3). Furthermore, a random unsteady flow field with zero time-average (generated by random motion of the cylinder in the experiment), also produced sharp separation at a fixed location, consistent with the theory of Haller [4].

We have successfully tested the shear-stress sensors in the working fluid, and are currently undertaking experiments to measure the time-dependent shear stress field on the surface. From these measurements we will determine the location of unsteady separation for the experimental flow, and will report the results of these investigations at the AIAA Fluid Dynamics meeting in June 2005. This will be a significant breakthrough, as it will demonstrate the ability of this approach to identify the geometry of unsteady separation in a physical fluid flow, based solely on surface measurements.

References


Figure 8: Simulation of unsteady separation. The red line is a material line drawn away from the wall by the unsteady separating flow. The green lines are streaklines. The blue line is the separation profile calculated from surface shear-stress and pressure measurements using the new separation criteria.


Figure 9: Unsteady separation generated by a rotating cylinder being translated in a random manner from side-to-side. The fixed separation point exists where the dye streak emanates from the wall.

Figure 10: The location of a separation point plotted as a function of height in the tank. There is little variation, indicating a two-dimensional flow.
Figure 11: The ratio (experimental separation location)/(numerical separation location) plotted as a function of the Reynolds number. The different symbols correspond to different positions of the cylinder relative to the wall.