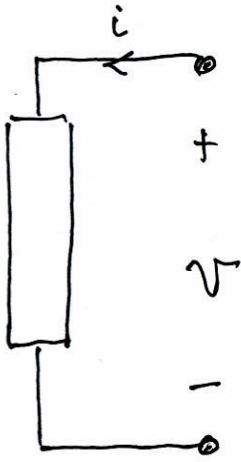


6.002 - Lecture 2

Circuit Analysis Simplifications

- Parallel & Series Reductions
- Node Analysis

Device Laws



Voltage Source $\Rightarrow$ Known v

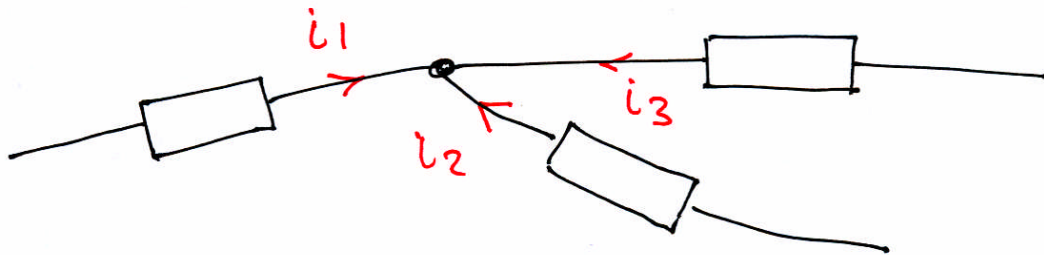
Current Source $\Rightarrow$ known i

Linear Resistor $\Rightarrow v = Ri$

Power $P_{in} = vi$

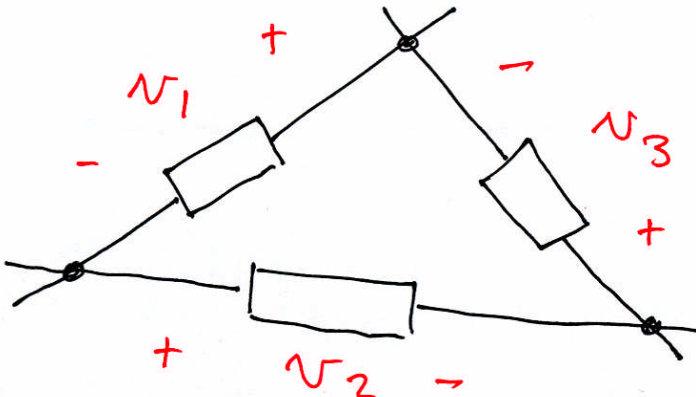
KCL

$$i_1 + i_2 + i_3 = 0$$



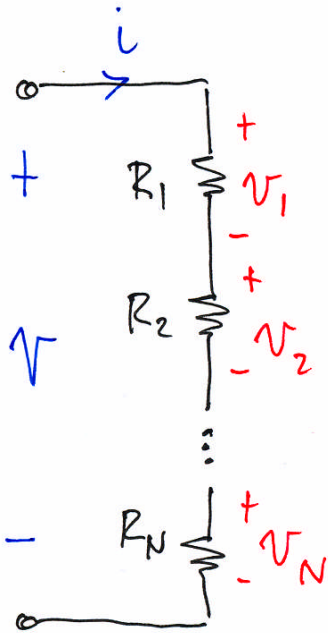
KVL

$$v_1 + v_2 + v_3 = 0$$



Series Resistors

Common Current i

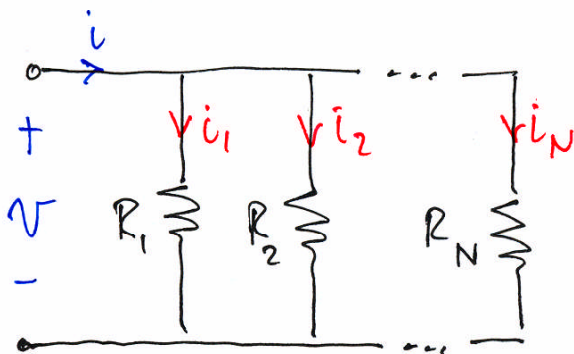


$$\begin{aligned} V &= V_1 + V_2 + \dots + V_N \\ &= R_1 i + R_2 i + \dots + R_N i \\ &= (R_1 + R_2 + \dots + R_N) i \\ &\equiv R i \end{aligned}$$

$$R = R_1 + R_2 + \dots + R_N$$

Parallel Resistors

Common Voltage V

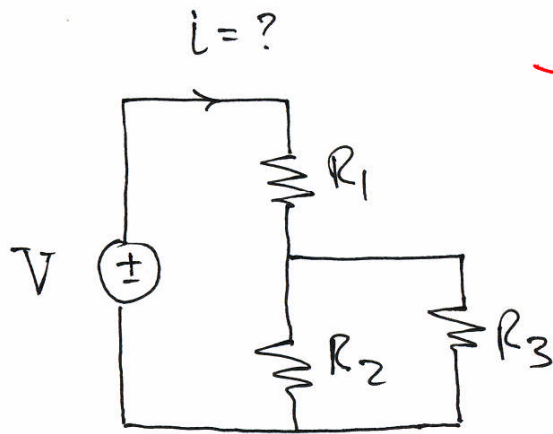


$$\begin{aligned} i &= i_1 + i_2 + \dots + i_N \\ &= G_1 V + G_2 V + \dots + G_N V \\ &= (G_1 + G_2 + \dots + G_N) V \\ &\equiv G V \end{aligned}$$

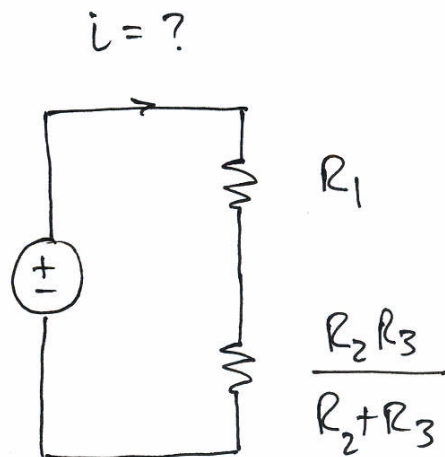
$$G = G_1 + G_2 + \dots + G_N$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}$$

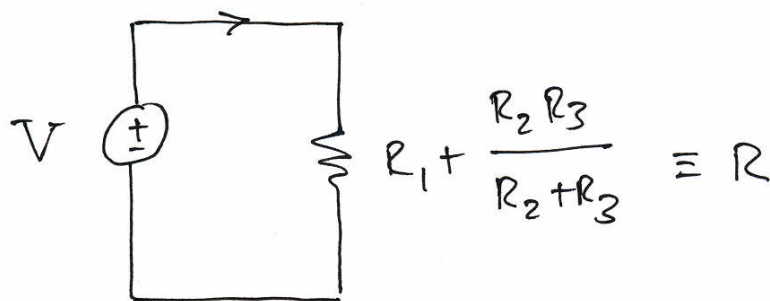
Simplification Process



Parallel Reduction



$$i = V/R$$



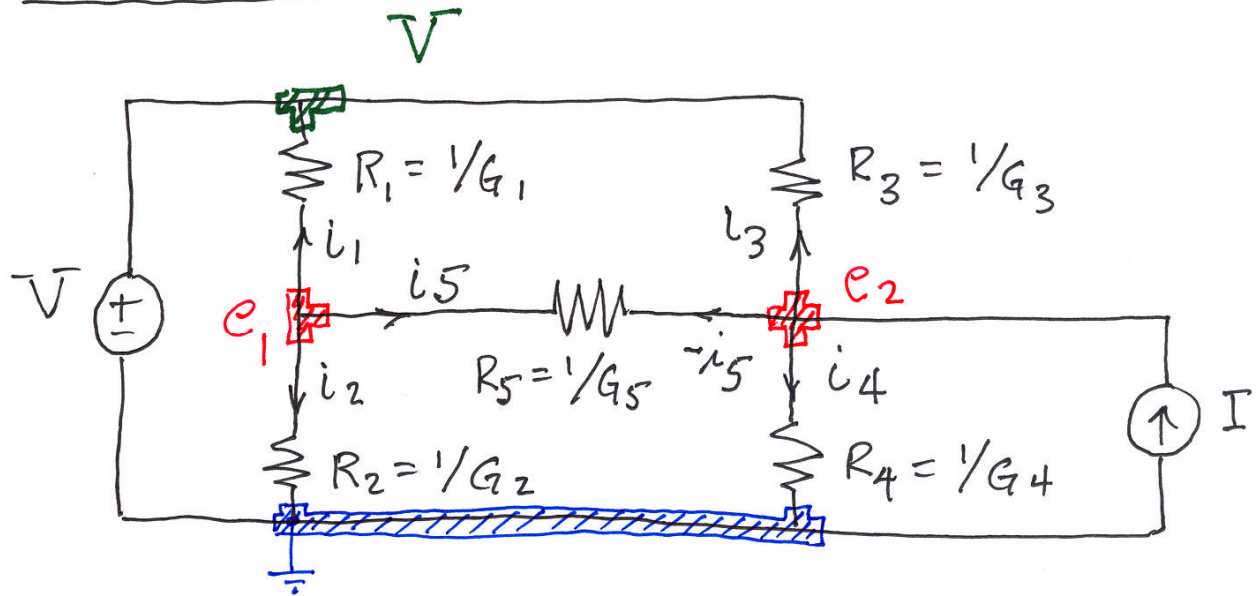
Series Reduction

Use voltage and current dividers, and device laws to determine remaining branch variables.

(Simplified) Node Analysis

- (0) Draw the circuit neatly.
- (1) Select a reference node from which all other node voltages are measured. Define its voltage to be zero. This node is "ground" ($\perp \rightarrow \perp$).
- (2) Label all remaining un-sourced node voltages with respect to ground. These are the primary analysis variables.
- (3) Write KCL for all nodes with an unknown voltage, and immediately substitute the device laws and KVL. The resulting equations are now in terms of the node voltages.
- (4) Solve the KCL equations for the node voltages.
- (5) Back solve for branch variables.

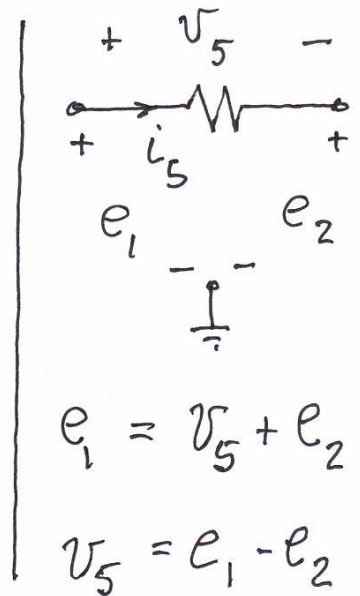
Example



e_1 Node: $i_1 + i_2 + i_5 = 0$

$$G_1(e_1 - V) + G_2(e_1 - 0) + G_5(e_1 - e_2) = 0$$

$$e_1(G_1 + G_2 + G_5) + e_2(-G_5) = V(G_1)$$



e_2 Node: $i_3 + i_4 + (-i_5) - I = 0$

$$G_3(e_2 - V) + G_4(e_2 - 0) + G_5(e_2 - e_1) - I = 0$$

$$e_1(-G_5) + e_2(G_3 + G_4 + G_5) = V(G_3) + I$$

Solution

$$\begin{bmatrix} G_1 + G_2 + G_5 & -G_5 \\ -G_5 & G_3 + G_4 + G_5 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} G_1 V \\ G_3 V + I \end{bmatrix}$$

$$\begin{bmatrix} G_3 + G_4 + G_5 & G_5 \\ G_5 & G_1 + G_2 + G_5 \end{bmatrix} \begin{bmatrix} G_1 V \\ G_3 V + I \end{bmatrix} = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

$$(G_1 + G_2 + G_5)(G_3 + G_4 + G_5) - G_5^2$$

$$e_1 = \frac{(G_3 + G_4 + G_5) G_1 V + G_5 (G_3 V + I)}{G_1 G_3 + G_1 G_4 + G_1 G_5 + G_2 G_3 + G_2 G_4 + G_2 G_5 + G_5 G_3 + G_5 G_4}$$

$$e_2 = \frac{G_5 G_1 V + (G_1 + G_2 + G_5) (G_3 V + I)}{(\text{Same Denominator})}$$

Demo

$$R_1 = R_4 = 8.2 \text{ k}\Omega$$

$$R_2 = R_3 = 3.9 \text{ k}\Omega$$

$$R_5 = 1.5 \text{ k}\Omega$$

$$V = 3 \text{ V}$$

$$I = 0 \text{ A}$$

$$e_1 = 1.382 \text{ V}$$

$$e_2 = 1.618 \text{ V}$$

Warning

The node method as presented does not handle floating voltage sources. Next lecture!