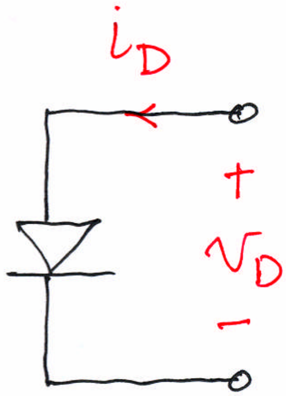
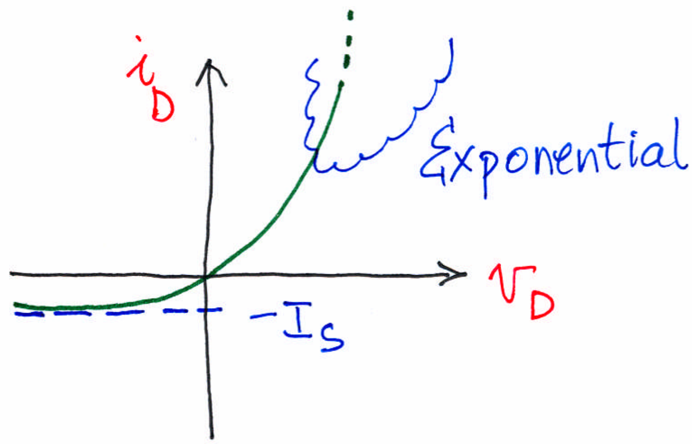
6.002 - Lecture 4

Circuits With Nonlinear Resistors

- Nonlinear Resistor Example ⇒ Diode
- Circuits With Nonlinear Resistors
- Analysis Options:
  - Closed-Form (Difficult)
  - Numerical (Mention Only)
  - Graphical (Provides Intuition)
  - Piecewise Linear Approximation (Later)
  - Linearized/Incremental (Mainstay)

Exponential Diode





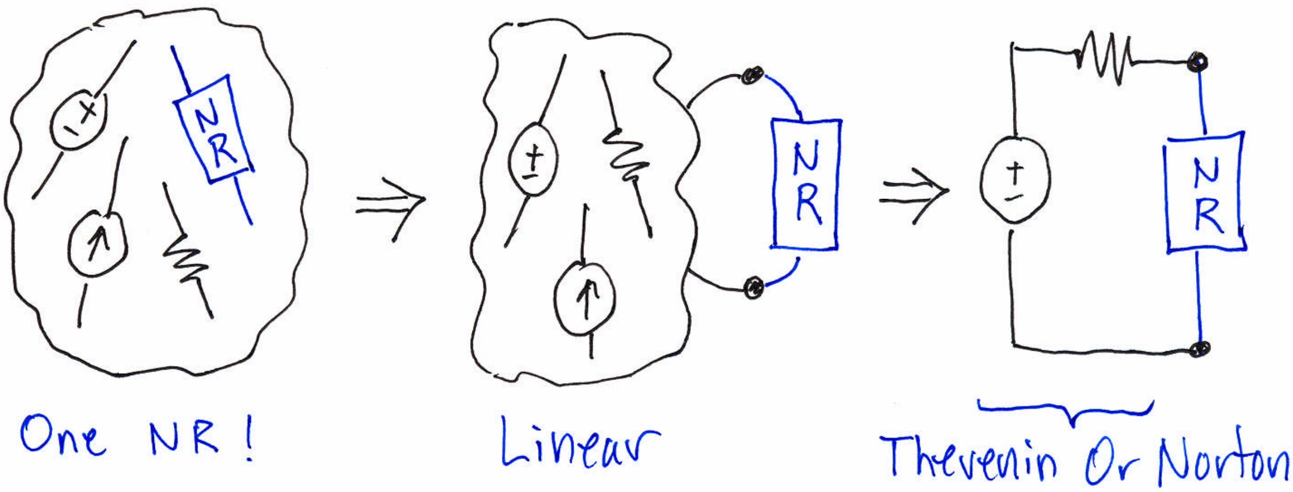
$$i_D = I_S\left(e^{\frac{v_D}{KT/q}} - 1\right)$$

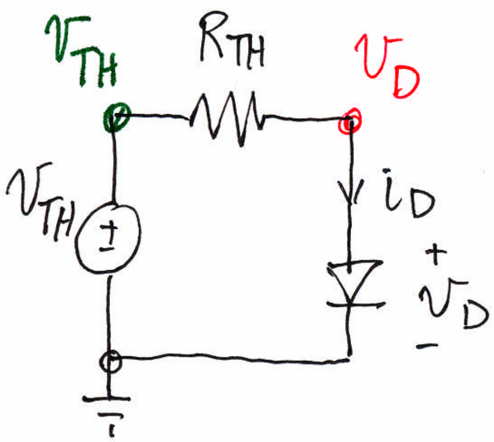
$\frac{KT}{q} \approx 26\text{mV} @ 300C$

$I_S \approx 10^{-13}\text{A} \ldots 10^{-9}\text{A}$
(Signal) (Power)

- Common in communication, power and lighting applications.
- Solar cells, LEDs, and many rectifiers and photodetectors are diodes.

Circuits With Nonlinear Resistors





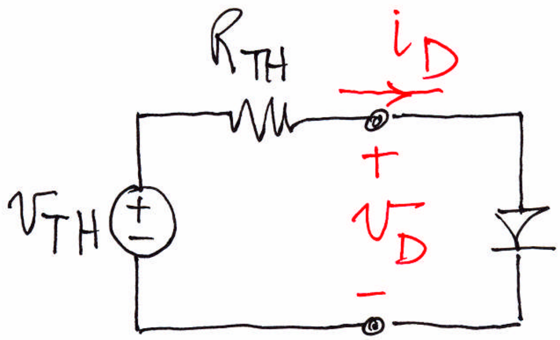
Node method still works!

$$\frac{v_D - v_{TH}}{R_{TH}} + I_S \left(e^{\frac{v_D}{KT/q}} - 1 \right) = 0$$

No closed-form solution, so:

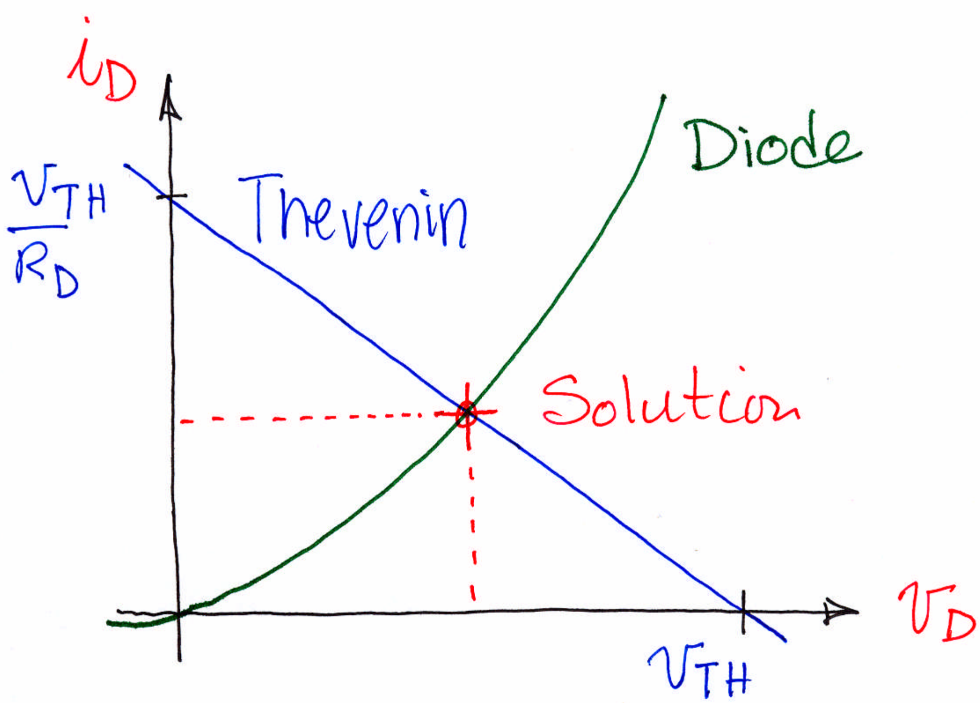
- numerical analysis;
- graphical analysis;
- approximate analysis;
- linearized/incremental analysis.

Graphical (Load-Line) Analysis

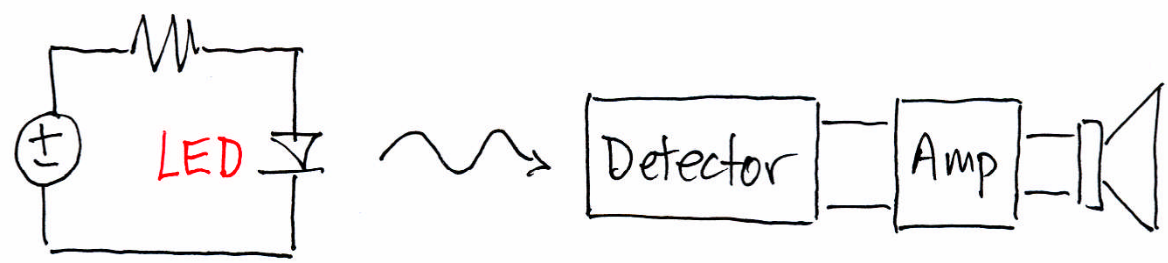


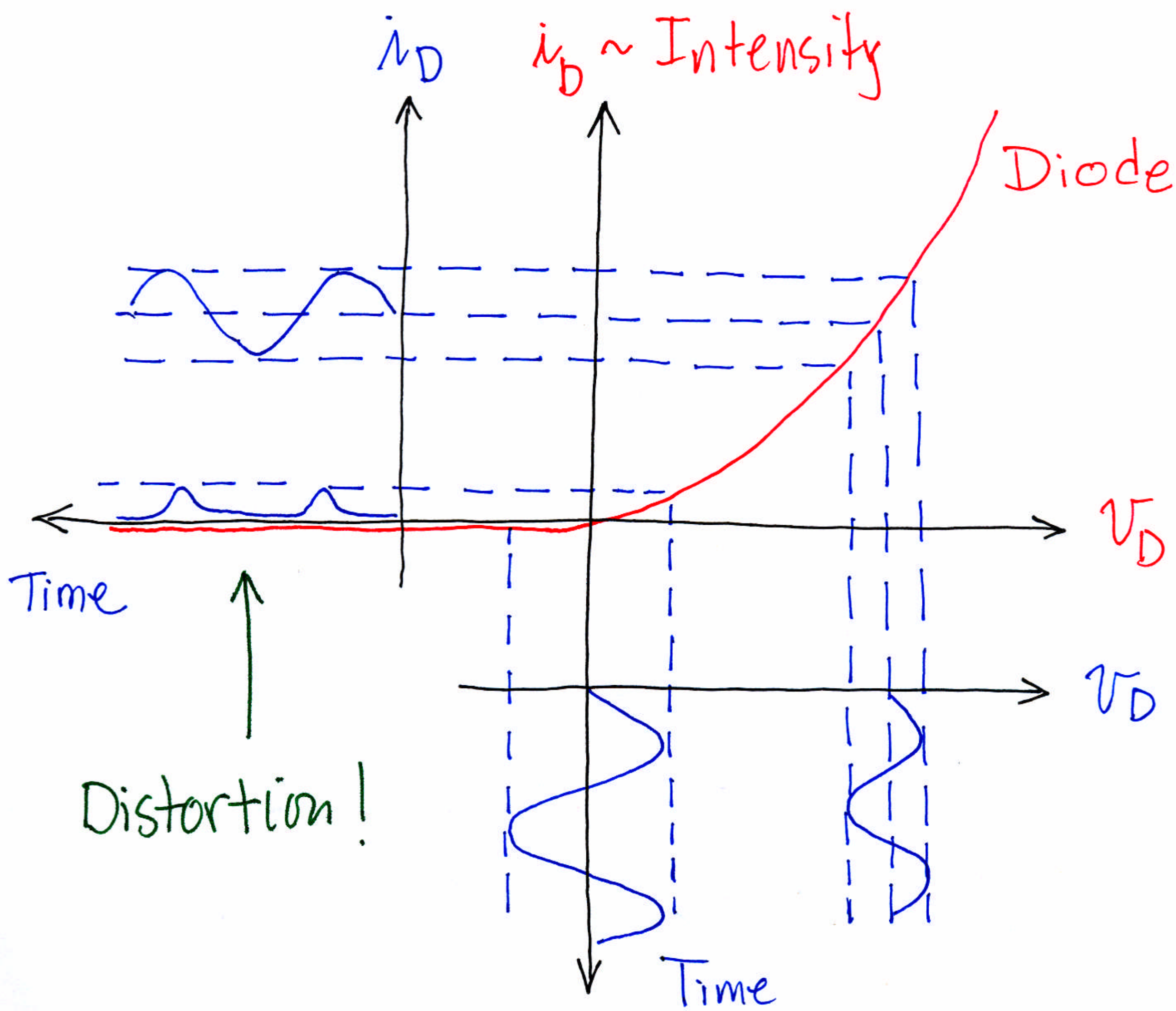
Thevenin: $v_D = v_{TH} - R_D i_D$

Diode: $i_D = I_S\left(e^{\frac{v_D}{kT/q}} - 1\right)$

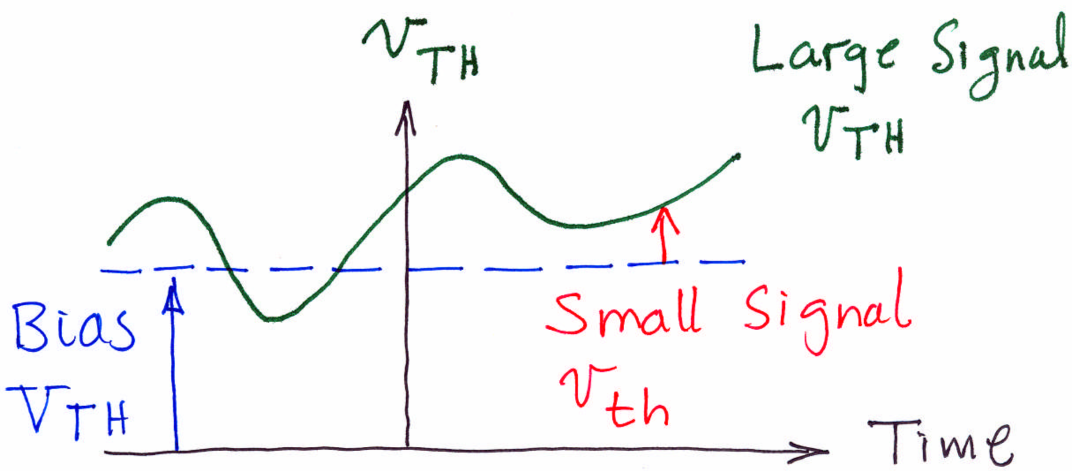


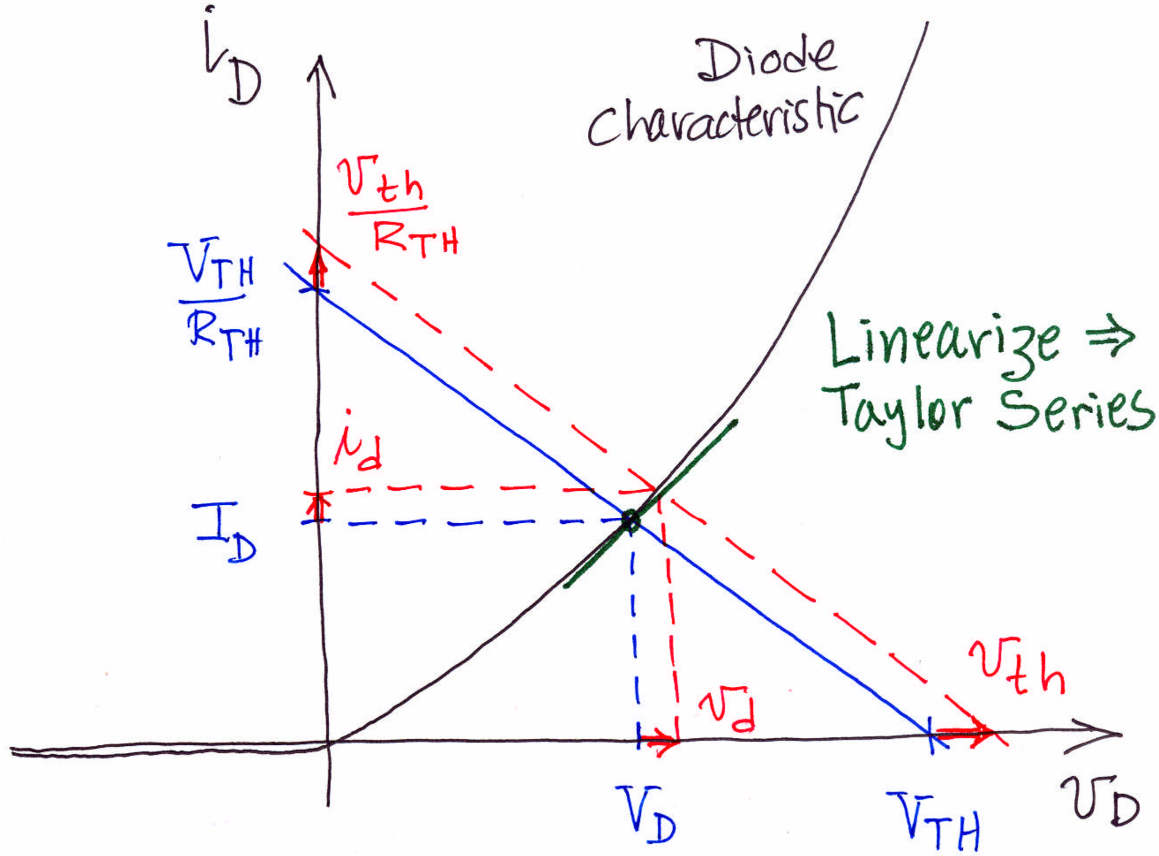
Motivating Small Signals





Small-Signal Analysis (I)





Small-Signal Analysis (II)

Signal Components:

$$v_{TH} = V_{TH} + v_{th}$$
$$v_D = V_D + v_d$$
$$i_D = I_D + i_d$$

$$\underbrace{\qquad}_{\text{Totals}} \quad \underbrace{\qquad}_{\text{Biases}} \quad \underbrace{\qquad}_{\text{Increments}}$$

Biases:

$$\frac{V_D - V_{TH}}{R_{TH}} + \underbrace{I_S\left(e^{\frac{V_D}{KT/q}} - 1\right)}_{I_D} = 0$$

Totals:

$$\frac{V_D + v_d - V_{TH} - v_{th}}{R_{TH}} + \underbrace{I_S\left(e^{\frac{\overbrace{V_D + v_d}^{v_D}}{KT/q}} - 1\right)}_{i_D = I_D + i_d} = 0$$

Small-Signal Analysis (III)

Linearize: $$f(v_D = \bar{V}_D + v_d) \approx f(\bar{V}_D) + \frac{df}{dv_D}(\bar{V}_D) \cdot v_d$$

Biases Cancel

Linearized Totals: $$\frac{\bar{V}_D + v_d - V_{TH} - v_{th}}{R_{TH}} + I_D + \underbrace{\frac{\overbrace{I_S \, e^{\frac{V_D}{KT/q}}}^{\approx I_D}}{KT/q} v_d}_{i_d} \approx 0$$

Small Signals: $$\frac{v_d - v_{TH}}{R_{TH}} + \frac{I_D}{KT/q} v_d \approx 0$$

$$v_d \approx \frac{1}{1 + R_{TH} I_D / \frac{KT}{q}} v_{th}$$

$$i_d \approx \frac{d i_D}{d v_D}(\bar{V}_D) \, v_d \approx \frac{I_D}{KT/q} v_d$$