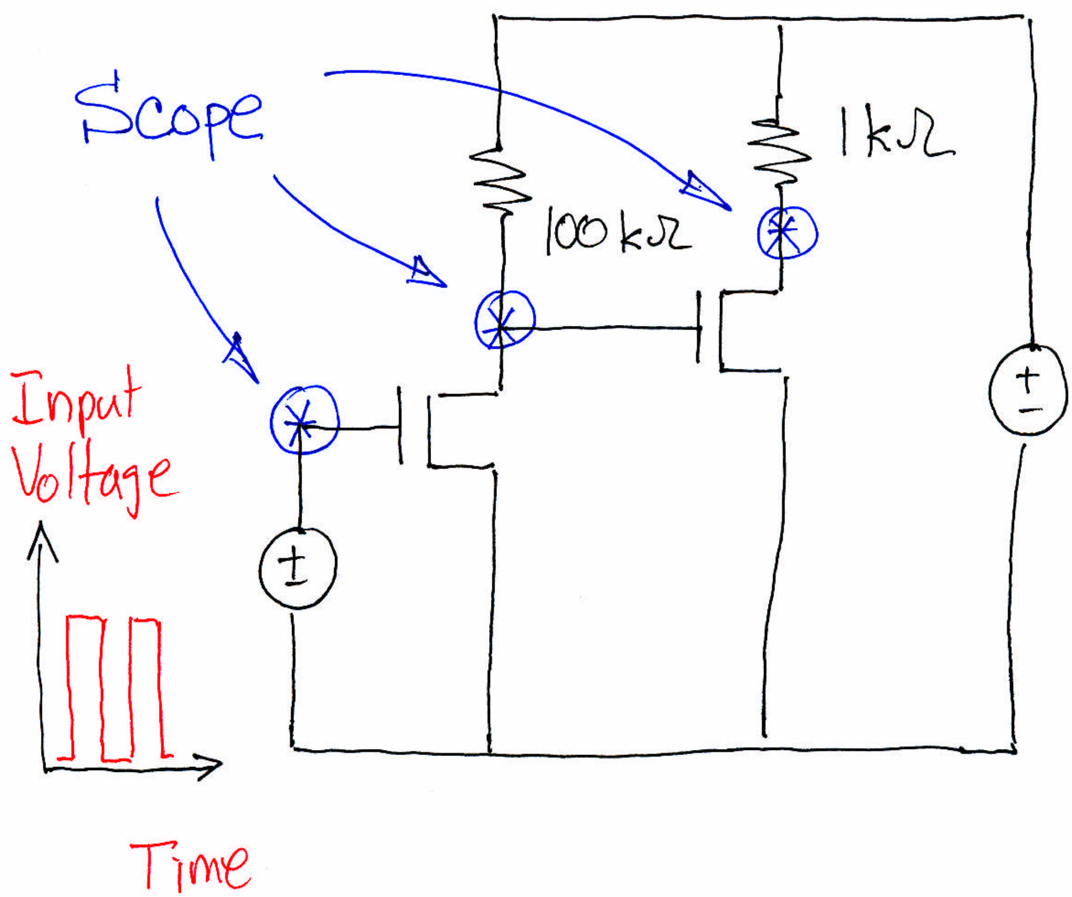
6.002 - Lecture 10

Time Does Matter

- Logic Gate Delays
- MOSFET Physics
- Capacitors, Inductors, Transformers
- First-Order ODE

Logic Gate Delays



Observe voltages as frequency increases.

Time Delays

Time Delays

$\Updownarrow$

Memory

$\Updownarrow$

Energy storage

E & M

Circuits

Mechanics

Fluidics

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Memoryless* Devices

Sources

Resistors

Ideal MOSFETs

Ideal Transformers

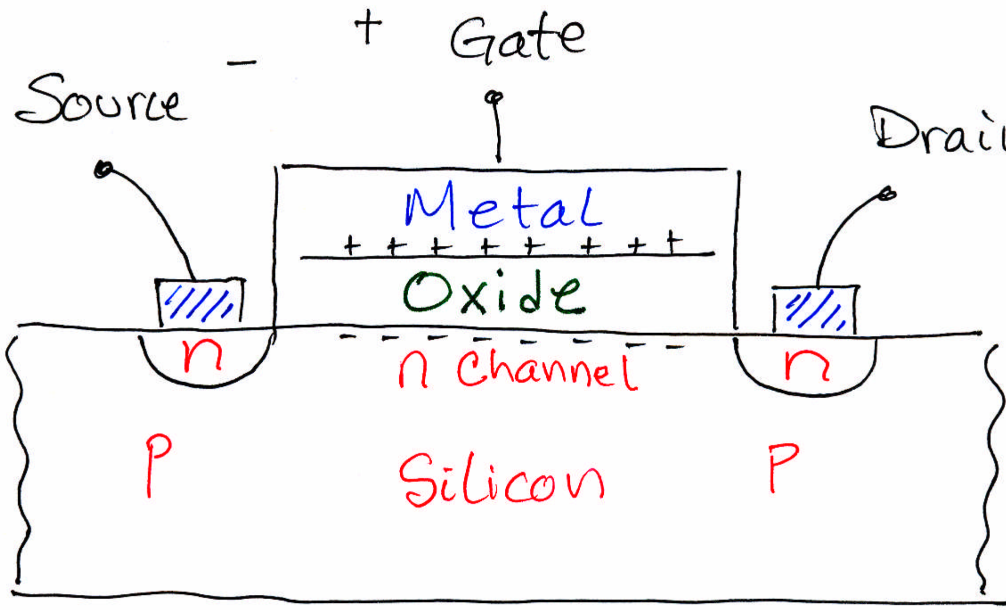
Memory Devices

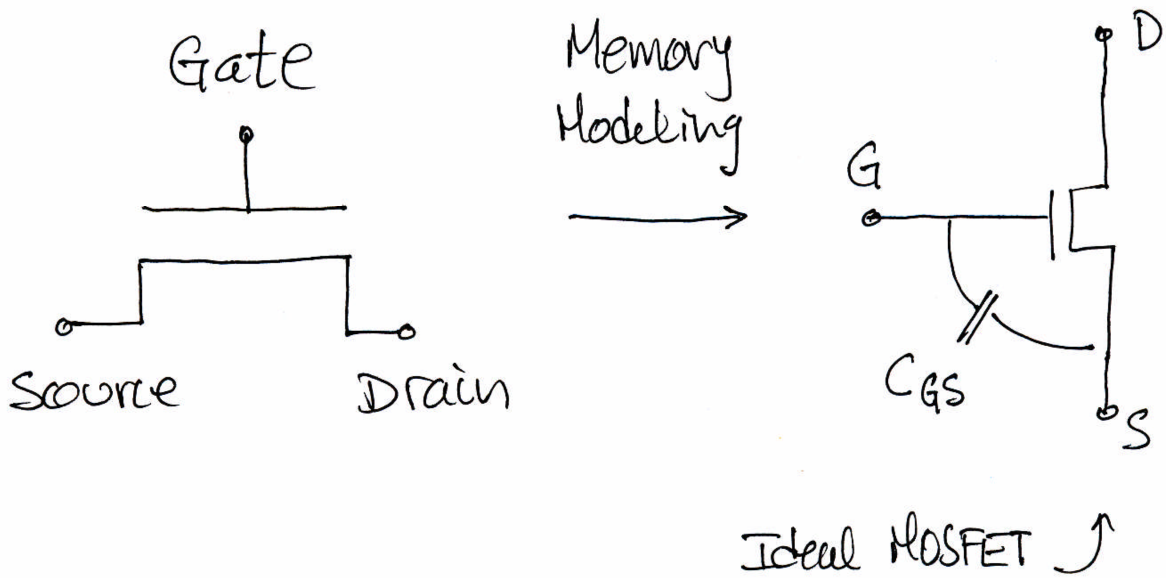
Capacitors

Inductors

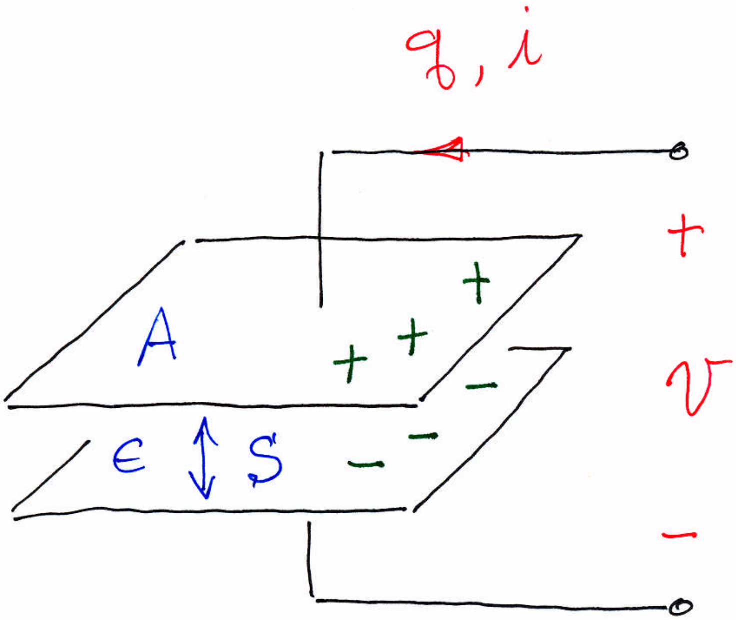
* Not considering thermal memory!

MOSFET Physics





Ideal Capacitor



$$\overset{\text{Farads}}{q = C\,v}$$

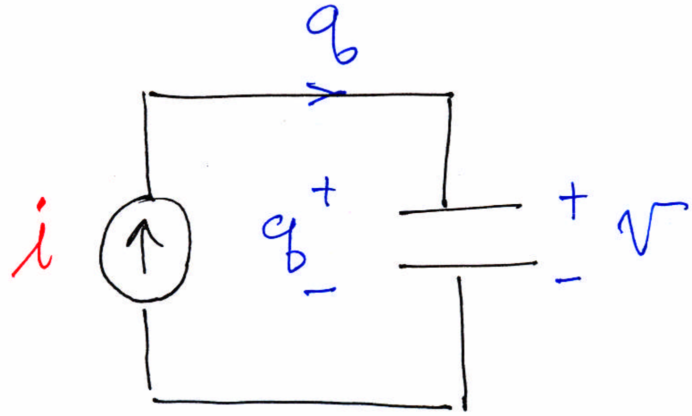
Coulombs   Vots

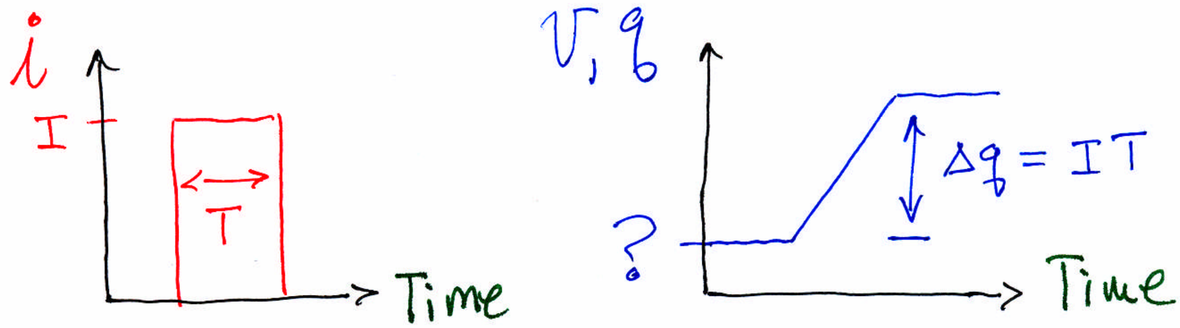
$$C = \frac{\epsilon A}{S}$$

$$\underbrace{q = \int_{-\infty}^{t} i\,dt \qquad i = \frac{dq}{dt}}_{\text{Memory}} = \frac{d}{dt}(Cv) = \underbrace{C\frac{dv}{dt}}_{6.002}$$

$$P = vi = C\,\underbrace{v\frac{dv}{dt}}_{\text{Two signs Possible} \Rightarrow \text{Reversible}} = \frac{d}{dt}\Big(\underbrace{\tfrac{1}{2}Cv^2}_{\text{Stored Energy}}\Big)$$

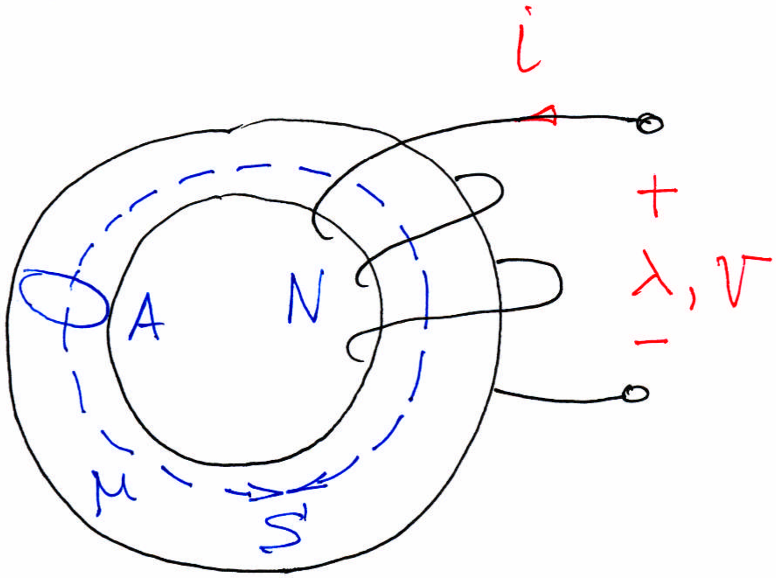
Capacitor Memory & Delay





- Charge and voltage depend on entire current history.
- Charge and voltage can not change abruptly without an infinite current.
- Delay is sometimes useful and sometimes annoying.

Ideal Inductor



Henries

$$\lambda = L\, i$$

webers　　Amps

$$L = \frac{\mu N^2 A}{S}$$

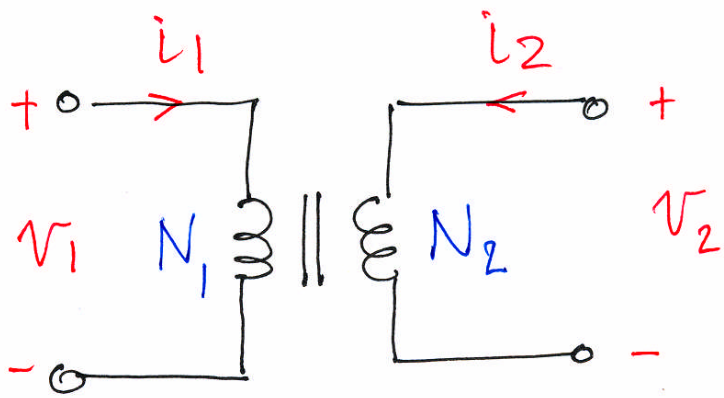
$$\underbrace{\lambda = \int_{-\infty}^{t} v\, dt \qquad v = \frac{d\lambda}{dt}}_{\text{Memory}} = \frac{d}{dt}(Li) = L\frac{di}{dt}$$

6.002

$$P = vi = \underbrace{Li\,\frac{di}{dt}}_{\substack{\text{Two Signs Possible} \\ \Rightarrow \text{Reversible}}} = \frac{d}{dt}\Big(\underbrace{\tfrac{1}{2}Li^2}_{\text{Stored Energy}}\Big)$$

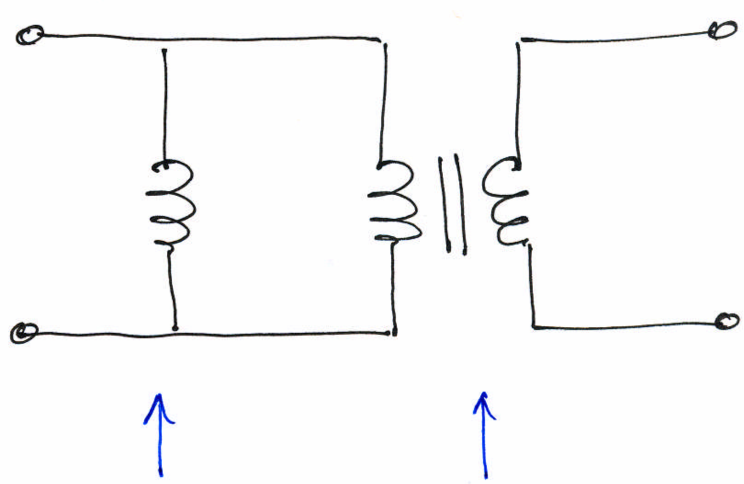
Transformer

Ideal

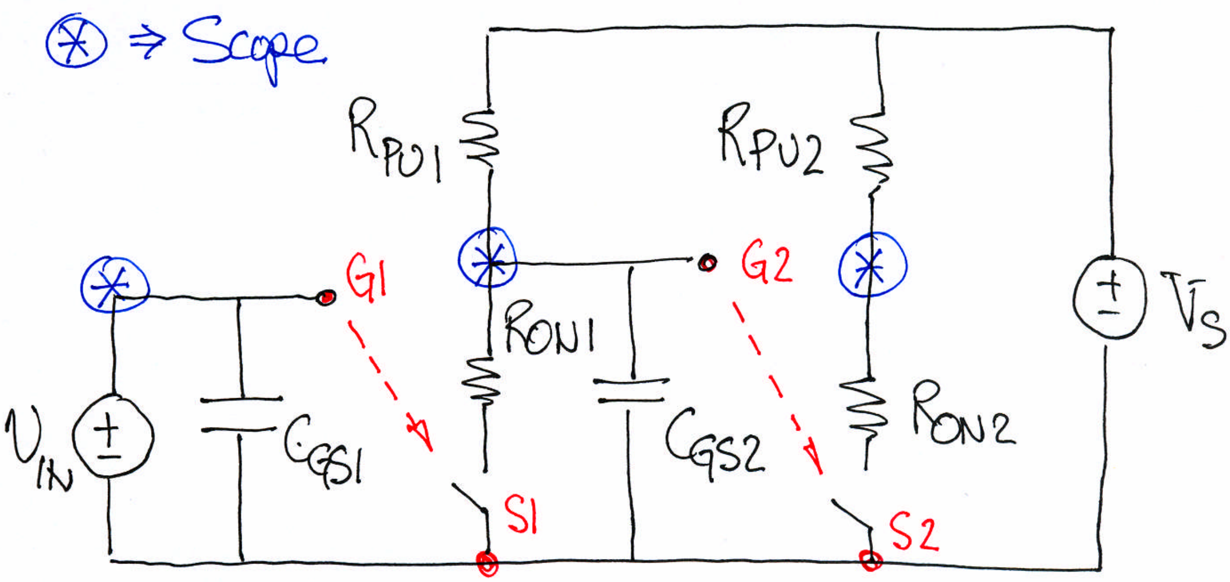


$$\frac{v_1}{v_2} = \frac{N_1}{N_2} \qquad \frac{i_1}{i_2} = -\frac{N_2}{N_1} \qquad \frac{P_1}{P_2} = -1$$

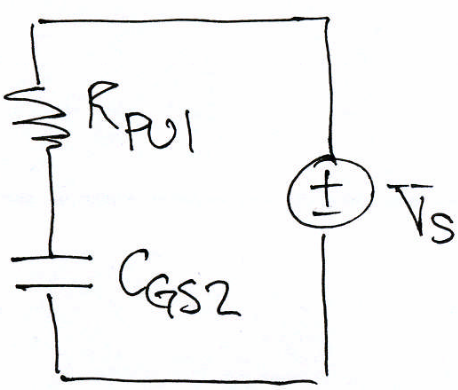
Non-Ideal
(Simplified)



Logic Delay Explained

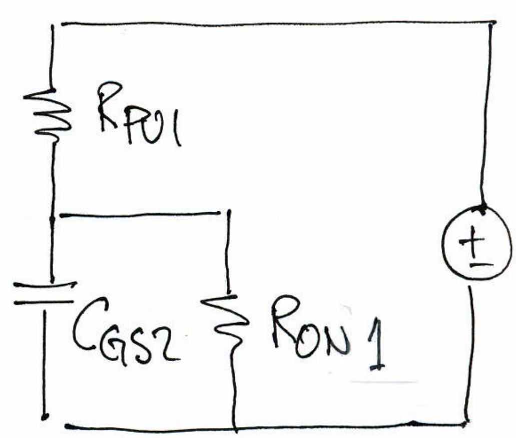


C_{GS2} Charge Up



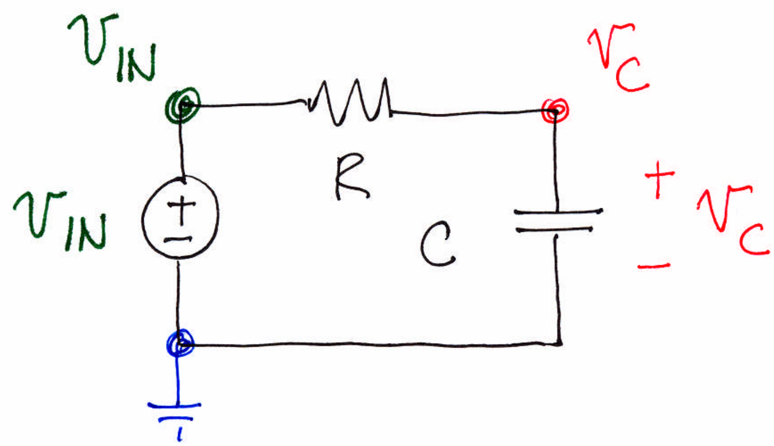
MOSFET 1 OPEN

C_{GS2} Charge Down



MOSFET 1 CLOSED

Transient Analysis I



Thevenin

Node Method $\Rightarrow$ $$\frac{v_C - v_{IN}}{R} + C\,\frac{dv_C}{dt} = 0$$

$$\underbrace{RC}_{\text{Time}}\,\frac{dv_C}{dt} + v_C = v_{IN}$$

Well-Posed Problem $\Rightarrow$

- $v_C(0)$ Given
- $v_{IN}(t)$ Given For $t \geq 0$

Transient Analysis II

Example: $v_C(0)$ Given & $v_{IN}(t) \equiv V$ For $t \geq 0$

Solution: $v_C(t) = v_P(t) + v_H(t)$

Particular Homogeneous

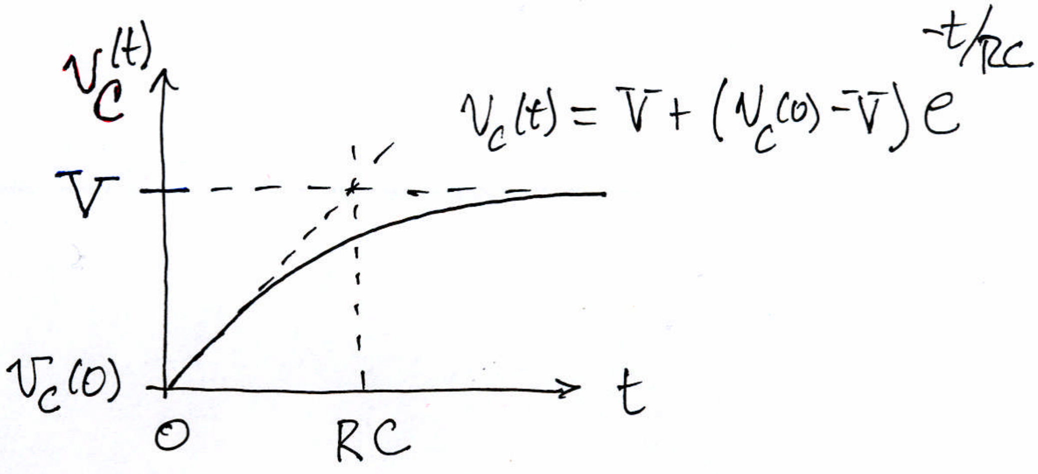
Particular: $v_P(t) = V$ ... By Inspection

Homogeneous: $RC \dfrac{d v_H(t)}{dt} + v_H(t) = 0$

$$v_H(0) = v_C(0) - v_P(0) = v_C(0) - V$$

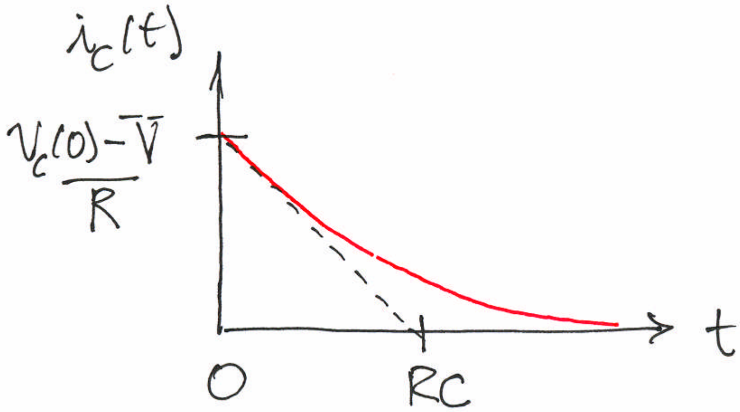
$$v_H(t) = (v_C(0) - V)\, e^{-t/RC}$$

Solution:



Transient Analysis III

$$i_C(t) = C\frac{dv_C(t)}{dt} = \frac{v_C(0) - \bar{V}}{R} e^{-t/RC}$$



Short Times: $t \ll RC$

Capacitor charge and voltage are approximately constant.

Capacitor $\approx$ short if $q(0) = 0$.

Example: $i_C(0) \approx \frac{\bar{V} - V_C(t)}{R}$.

Long Times: $t \gg RC$

Capacitor current $\to 0$ and capacitor behaves as an open.

Example: $v_C \to \bar{V}$ and $i_c \to 0$.