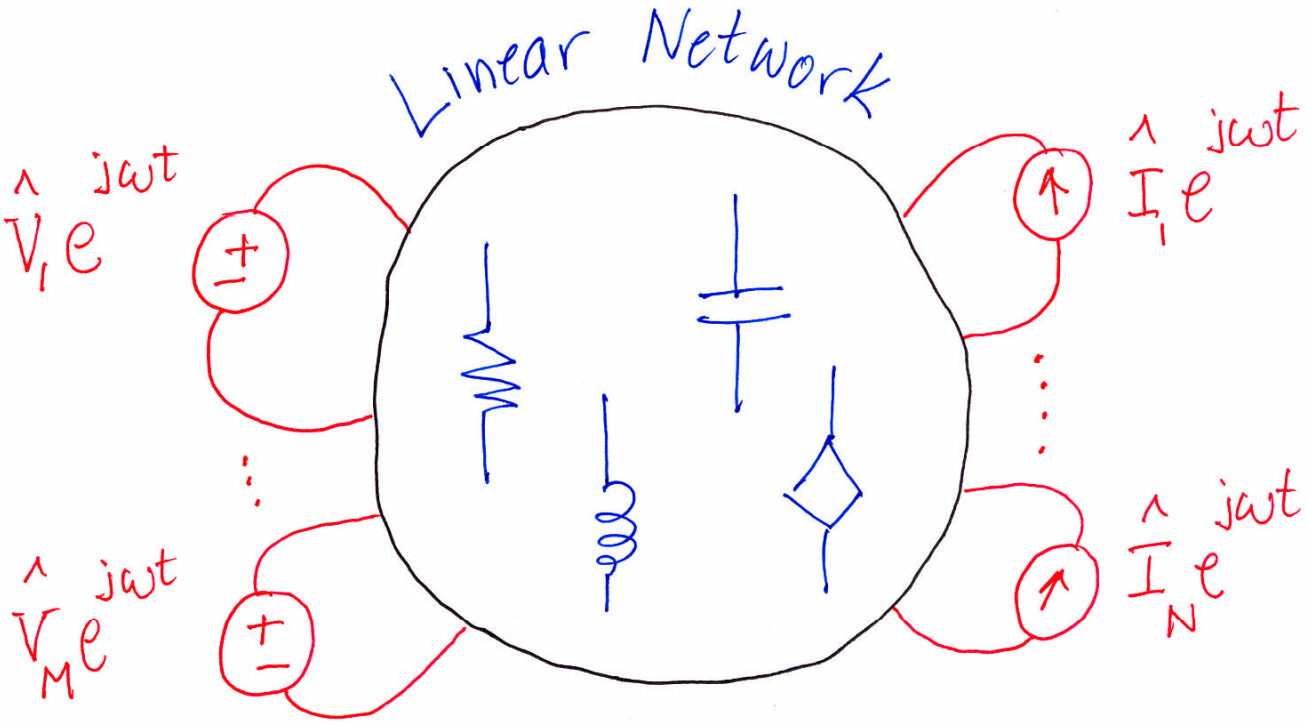
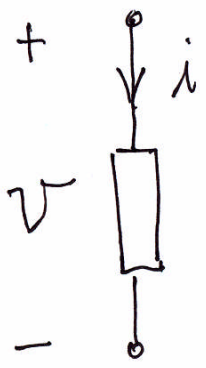
6.002 - Lecture 17

Impedance Methods

- SSS Review
- Impedance & Admittance
- Foundations & Results Of Resistor-Source Network Analysis
- Impedance Methods Of Analysis For Networks In SSS

SSS Review



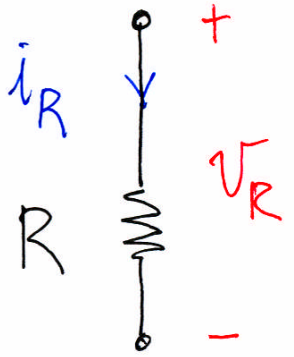


All branch variables will take the form of

$v = \hat{v} e^{j\omega t}$ or $i = \hat{i} e^{j\omega t}$

Re { } notation omitted for brevity.

Resistor

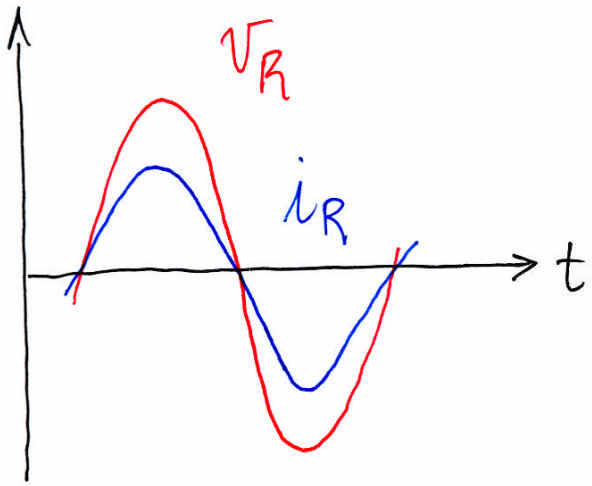


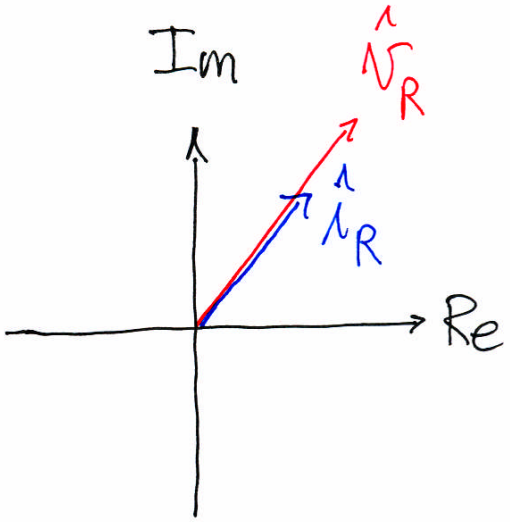
$$v_R = R\, i_R$$

$$\hat{v}_R e^{j\omega t} = R\, \hat{i}_R e^{j\omega t}$$

$$\hat{v}_R = R\, \hat{i}_R$$

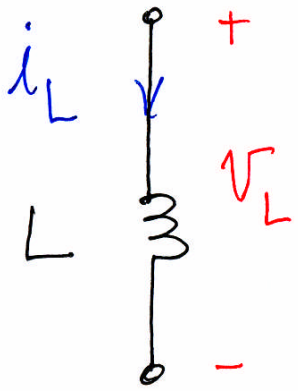
Branch Variables





$$v_R(t) = |\hat{v}_R| \cos(\omega t + \angle \hat{v}_R) \qquad i_R(t) = |\hat{i}_R| \cos(\omega t + \angle \hat{i}_R)$$

Inductor



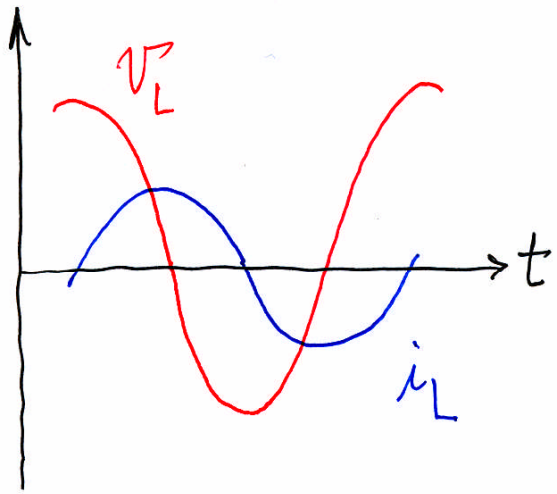
$$v_L = L \frac{di_L}{dt}$$

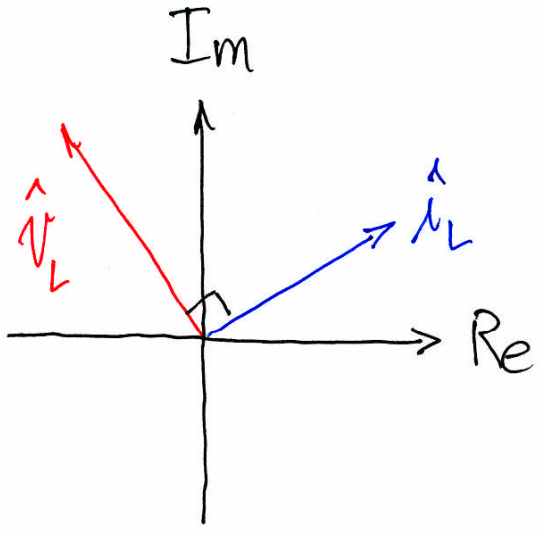
$$\hat{v}_L e^{j\omega t} = L \frac{d}{dt}\left(\hat{i}_L e^{j\omega t}\right)$$

$$= j\omega L \, \hat{i}_L e^{j\omega t}$$

$$\hat{v}_L = j\omega L \, \hat{i}_L$$

Branch Variables

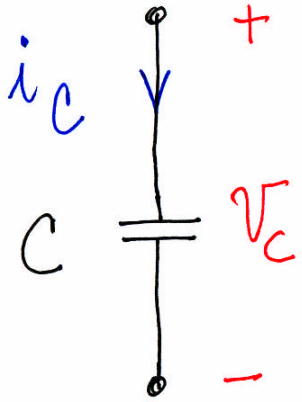




$$v_L(t) = |\hat{v}_L| \cos(\omega t + \angle \hat{v}_L)$$

$$i_L(t) = |\hat{i}_L| \cos(\omega t + \angle \hat{i}_L)$$

Capacitor



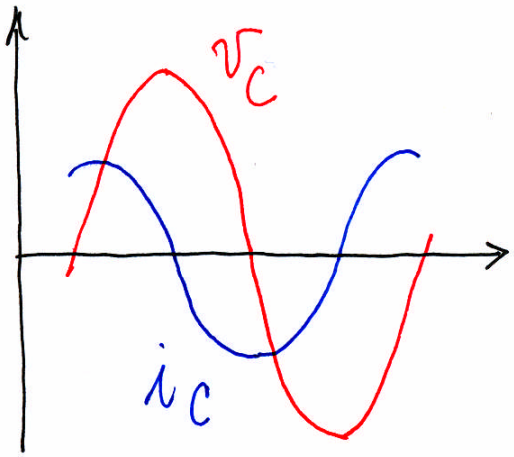
$$i_C = C \frac{dv_C}{dt}$$

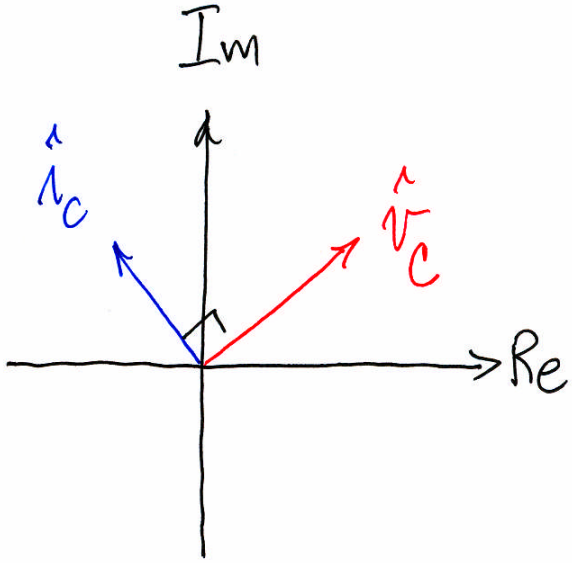
$$\hat{i}_C e^{j\omega t} = C \frac{d}{dt}\left(\hat{v}_C e^{j\omega t}\right)$$

$$= j\omega C \hat{v}_C e^{j\omega t}$$

$$\hat{i}_C = j\omega C \hat{v}_C$$

Branch Variables

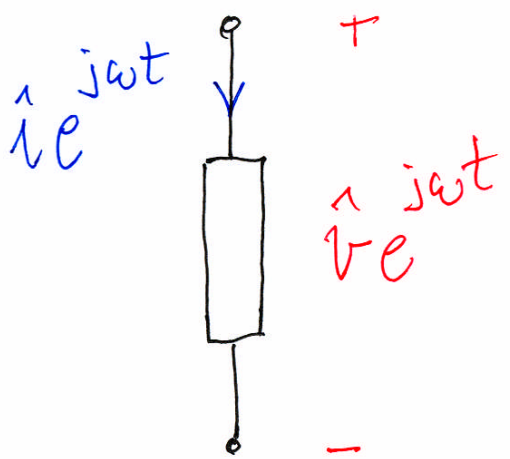




$$v_C(t) = |\hat{v}_C| \cos(\omega t + \angle \hat{v}_C)$$

$$i_C(t) = |\hat{i}_C| \cos(\omega t + \angle \hat{i}_C)$$

Impedance & Admittance



$$\text{Impedance } Z \equiv \frac{\hat{v}}{\hat{\imath}}$$

$$\text{Admittance } Y \equiv \frac{\hat{\imath}}{\hat{v}}$$

$$Z = \frac{\hat{v}}{\hat{\imath}} = R + jX$$

R: Resistance; X: Reactance

$$Y = \frac{\hat{\imath}}{\hat{v}} = G + jB$$

G: Conductance; B: Susceptance

Impedance & Admittance

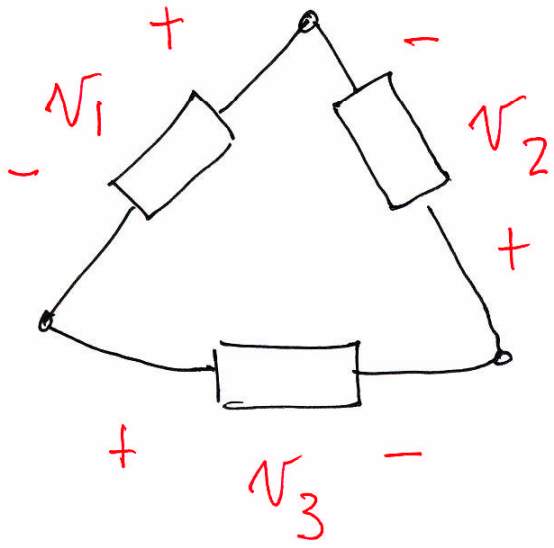
Resistor: $\hat{v} = R\hat{i}$

Inductor: $\hat{v} = j\omega L\,\hat{i}$

Capacitor: $\hat{i} = j\omega C\,\hat{v}$

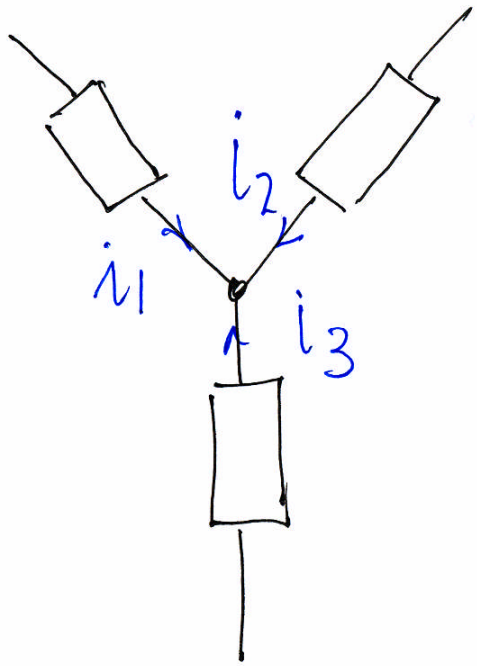
	Impedance	Admittance
Resistor	R	$G = \frac{1}{R}$
Inductor	$j\omega L$	$1/j\omega L$
Capacitor	$1/j\omega C$	$j\omega C$

KVL & KCL



$$\hat{v}_1 e^{j\omega t} + \hat{v}_2 e^{j\omega t} + \hat{v}_3 e^{j\omega t} = 0$$

$$\hat{v}_1 + \hat{v}_2 + \hat{v}_3 = 0$$



$$\hat{i}_1 e^{j\omega t} + \hat{i}_2 e^{j\omega t} + \hat{i}_3 e^{j\omega t} = 0$$

$$\hat{i}_1 + \hat{i}_2 + \hat{i}_3 = 0$$

Static Resistor-Source Networks

Foundations: Linear Device Laws

KVL & KCL

Results: Direct & Node Analyses

Linearity & Superposition

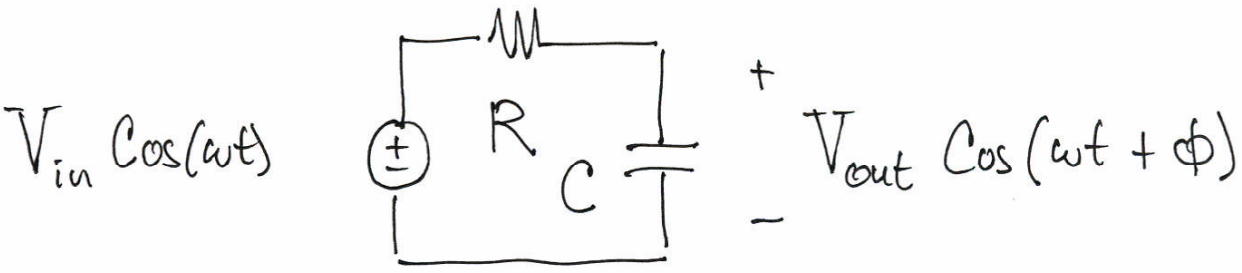
Thevenin & Norton

Parallel & Series

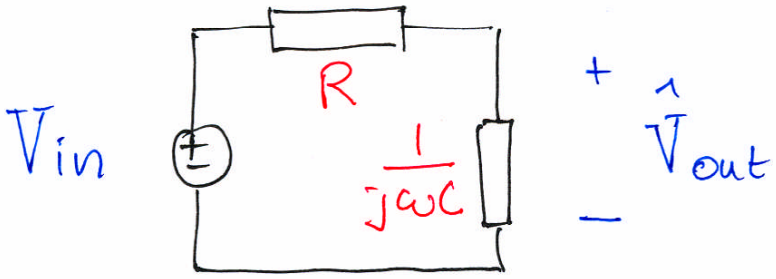
Dividers

Direct extension to SSS via impedances.

RC Example I



Complex Amplitudes
Impedances

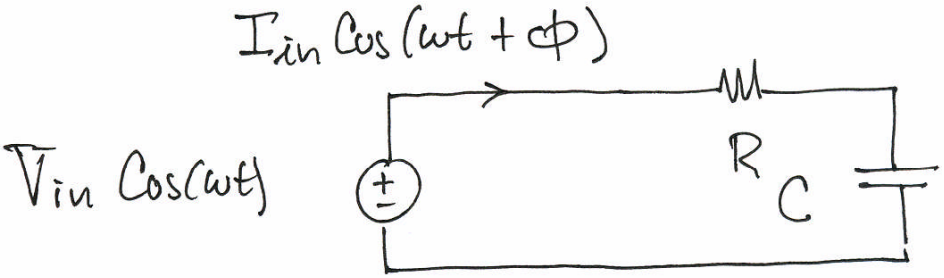


$$\hat{V}_{out} = \frac{1/j\omega C}{R + 1/j\omega C} V_{in} = \frac{1}{1 + j\omega RC} V_{in}$$

$$V_{out} = |\hat{V}_{out}| = \frac{V_{in}}{\sqrt{1+\omega^2 R^2 C^2}}$$

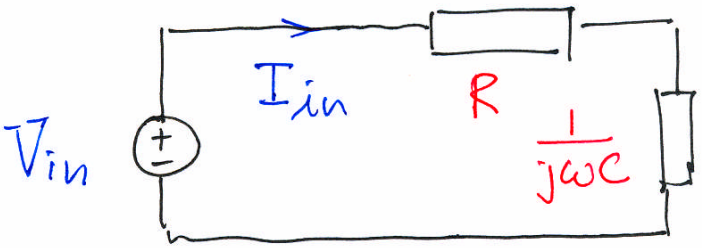
$$\phi = \angle \hat{V}_{out} = -\text{Tan}^{-1}(\omega RC)$$

RC Example II



Complex Amplitudes

Impedances

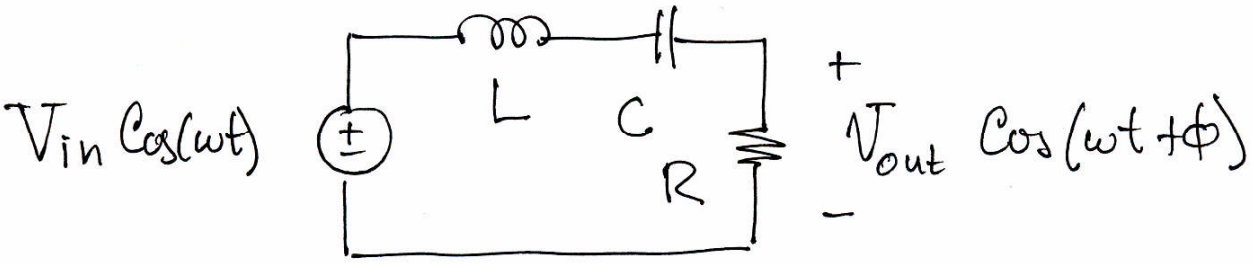


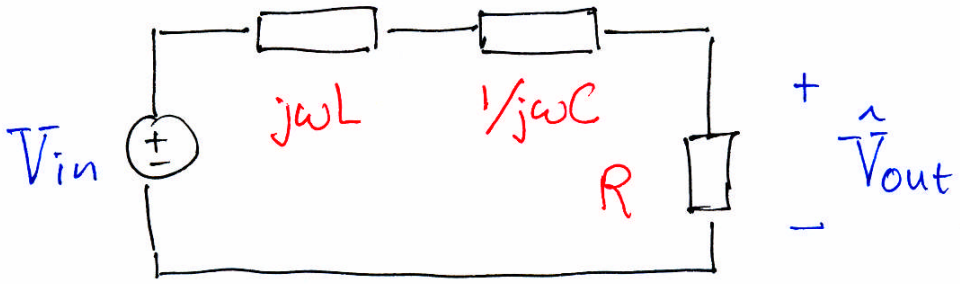
$$\hat{I}_{in} = \frac{1}{R + 1/j\omega C} V_{in} = \frac{j\omega C}{1 + j\omega RC} V_{in}$$

$$I_{in} = |\hat{I}_{in}| = \frac{\omega C}{\sqrt{1 + \omega^2 R^2 C^2}} V_{in}$$

$$\phi = \angle \hat{I}_{in} = \mathrm{Tan}^{-1}(1/\omega RC)$$

RLC Example I



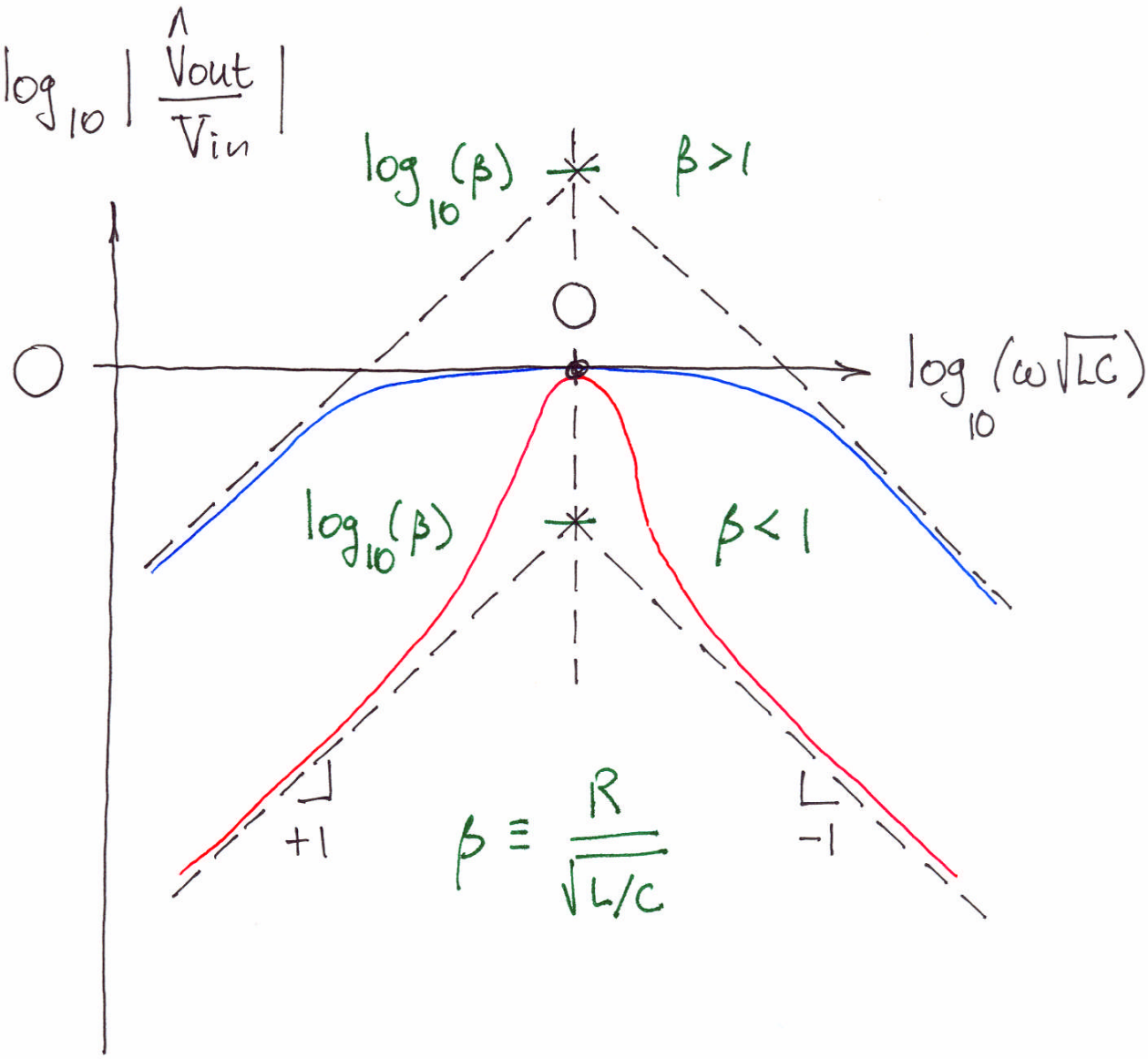


$$\hat{\bar{V}}_{out} = \frac{R}{R + j\omega L + 1/j\omega C}\bar{V}_{in} = \frac{j\omega RC}{1-\omega^2 LC + j\omega RC}V_{in}$$

$$V_{out} = \left|\hat{\bar{V}}_{out}\right| = \frac{\omega RC}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}V_{in}$$

$$\phi = \angle\,\hat{\bar{V}}_{out} = \mathrm{Tan}^{-1}\left(\frac{1-\omega^2 LC}{\omega RC}\right)$$

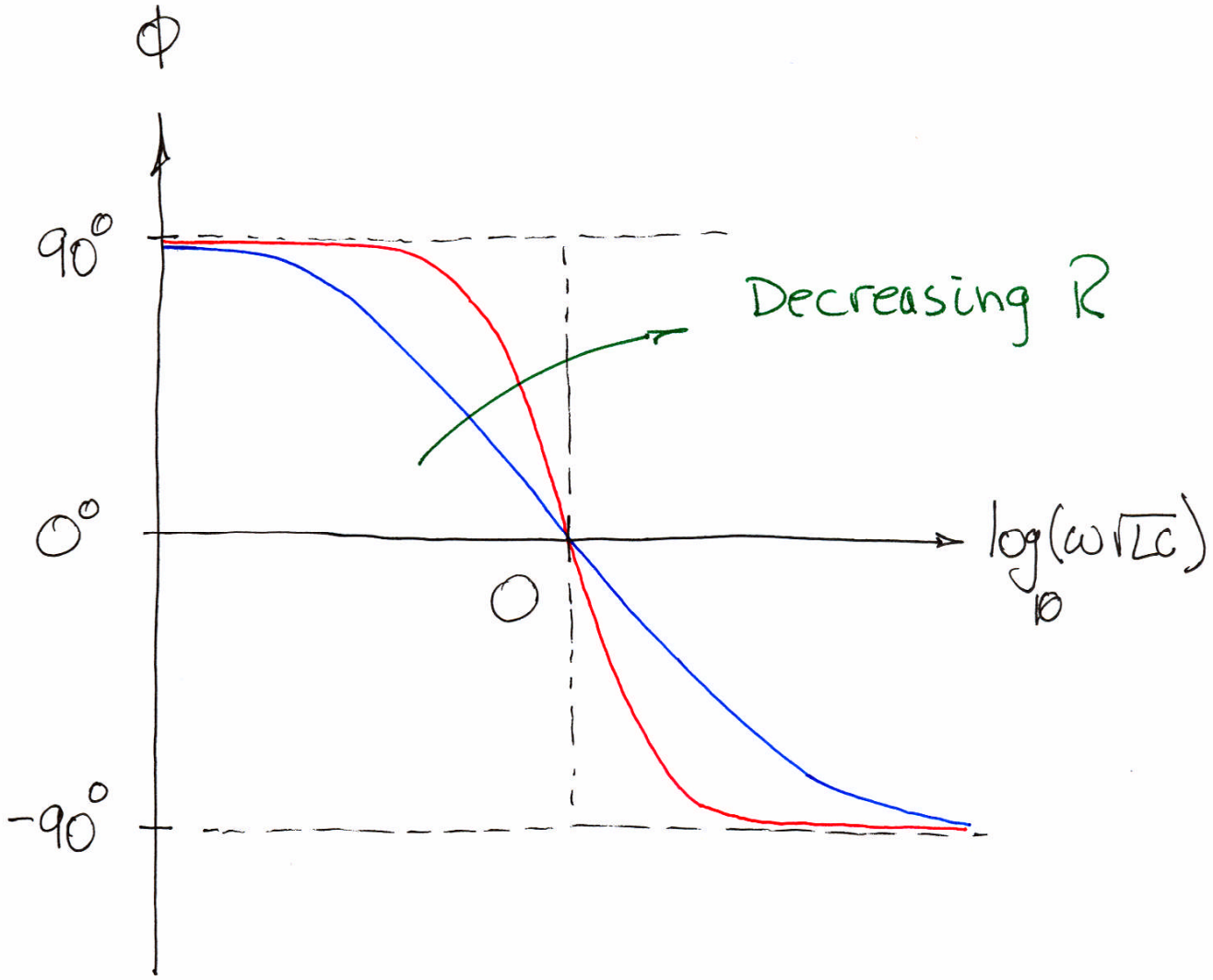
RLC Example II



Low Frequency

$$\frac{|\hat{V}_{out}|}{V_{in}} \sim \omega RC$$

High Frequency

$$\frac{|\hat{V}_{out}|}{V_{in}} \sim \frac{R}{\omega L}$$

RLC Example III