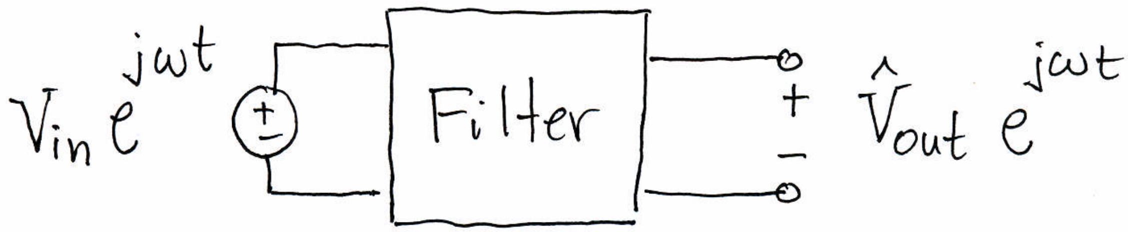
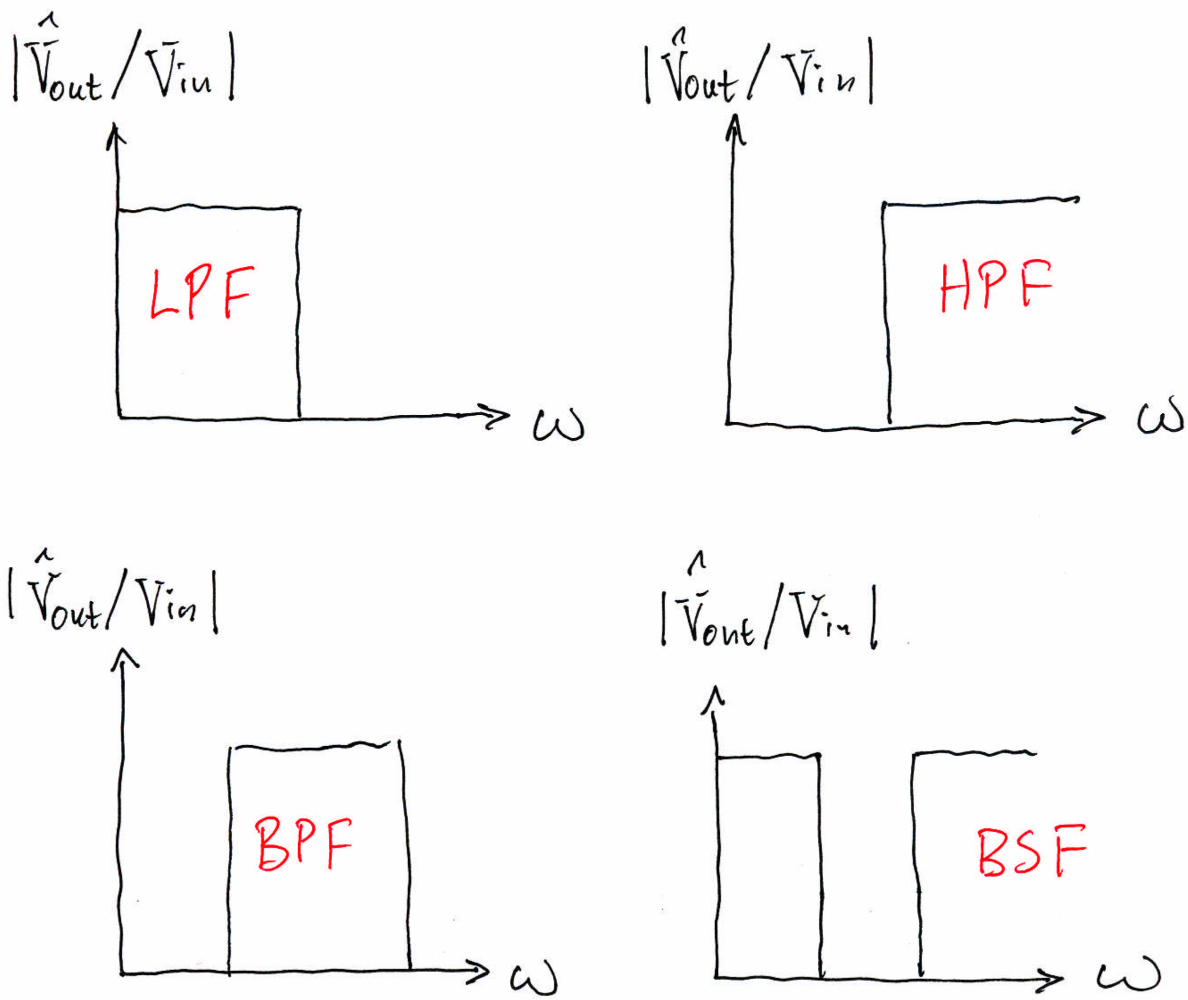
6.002 - Lecture 18

Filtering

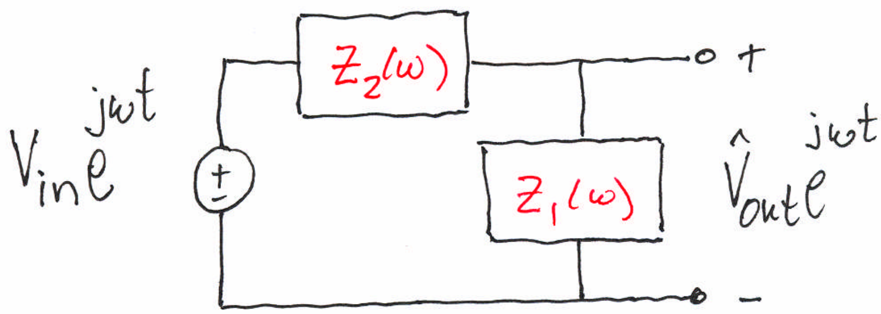
- Simple Objectives:
  (LPF, HPF, BPF, BSF)
- Simple Synthesis
- LC Resonators
- Radio Examples

Simple Filtering Objectives

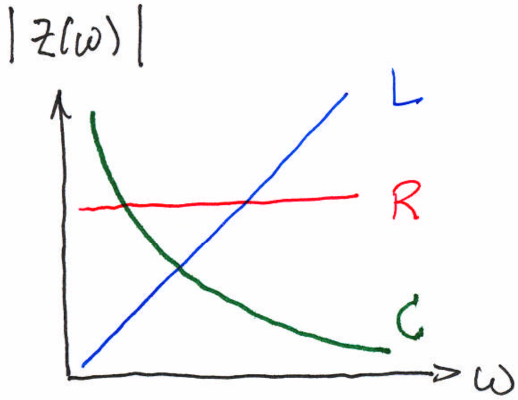


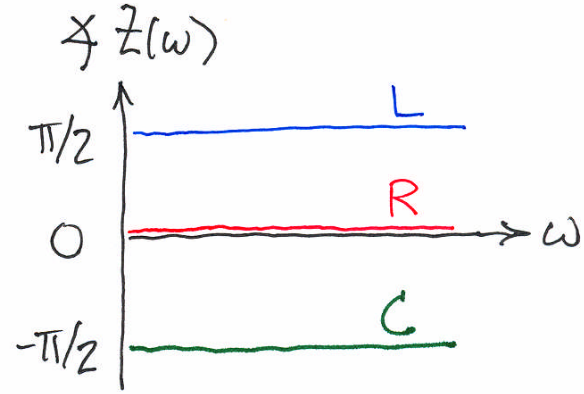


Simple Filtering Synthesis

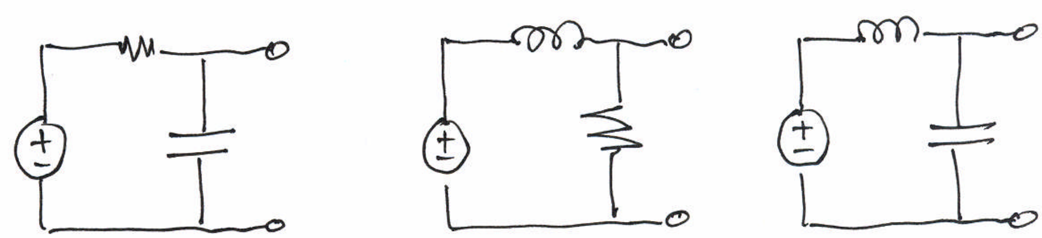


$$\frac{\hat{V}_{out}}{V_{in}} = \frac{Z_1(\omega)}{Z_2(\omega) + Z_1(\omega)}$$

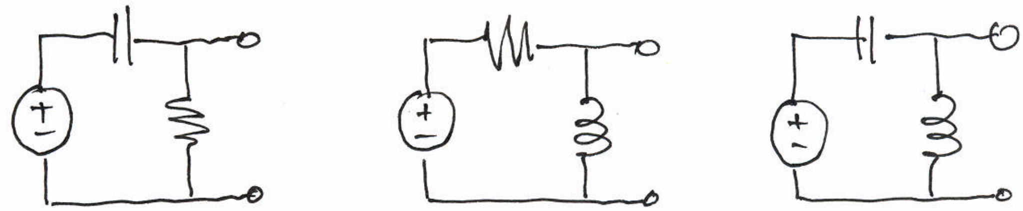




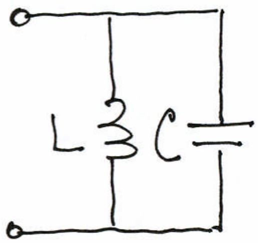
LPF:



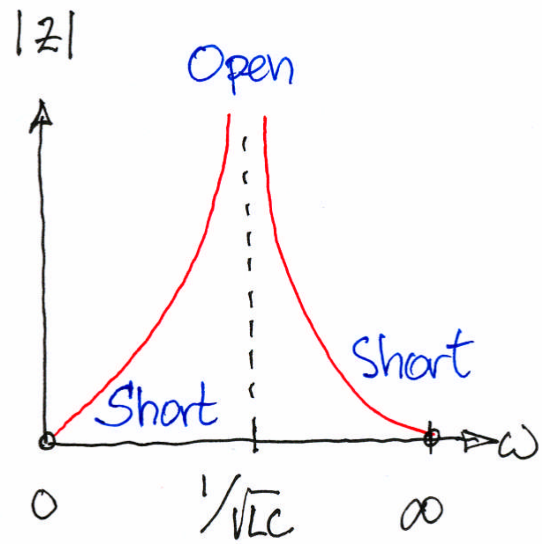
HPF:

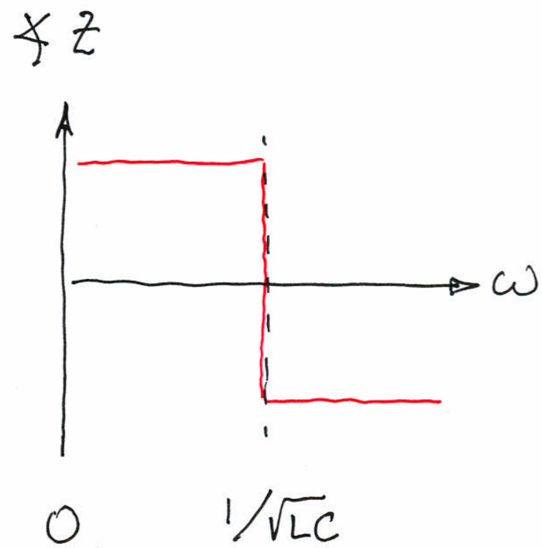


Parallel LC Resonator

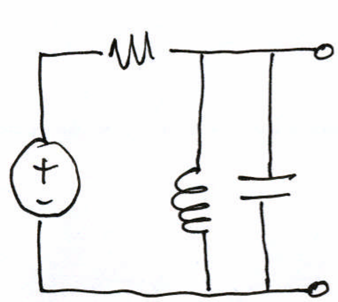


$$Z = \frac{j\omega L \cdot \frac{1}{j\omega C}}{j\omega L + \frac{1}{j\omega C}} = \frac{j\omega L}{1 - \omega^2 LC}$$

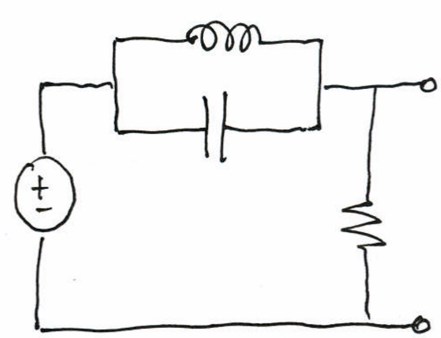




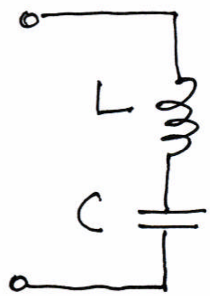
BPF:



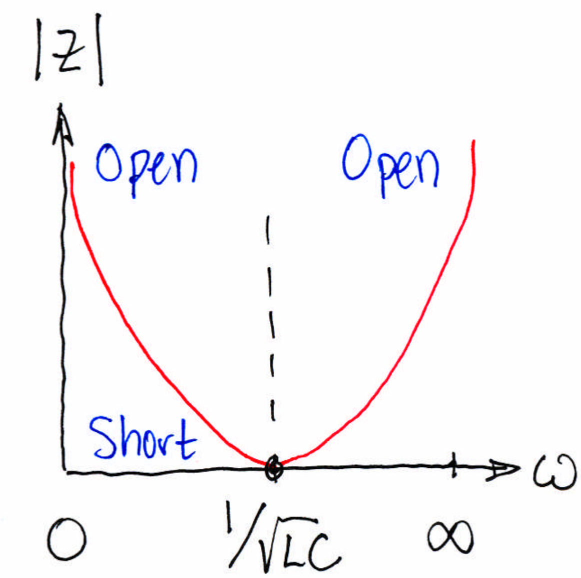
BSF:

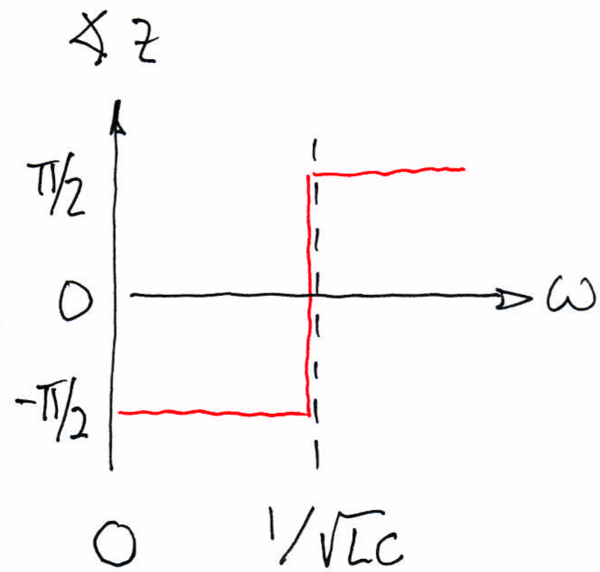


Series LC Resonator

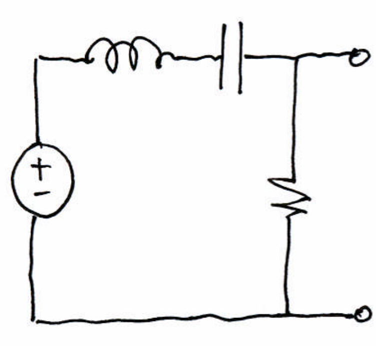


$$Z = j\omega L + \frac{1}{j\omega C} = \frac{1-\omega^2 LC}{j\omega C}$$

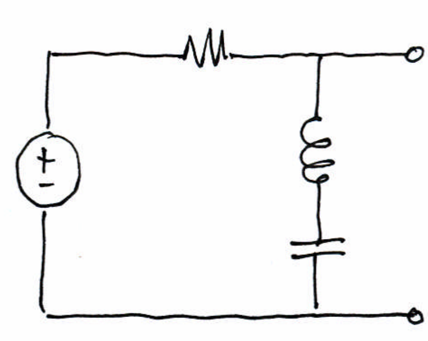




BPF:

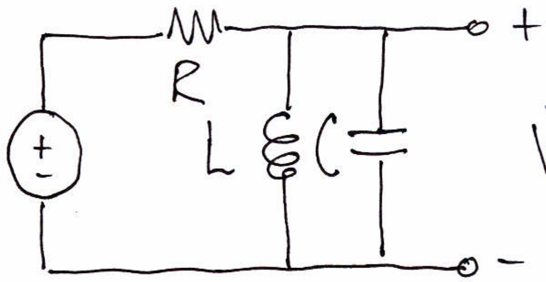


BSF:



BPF Example

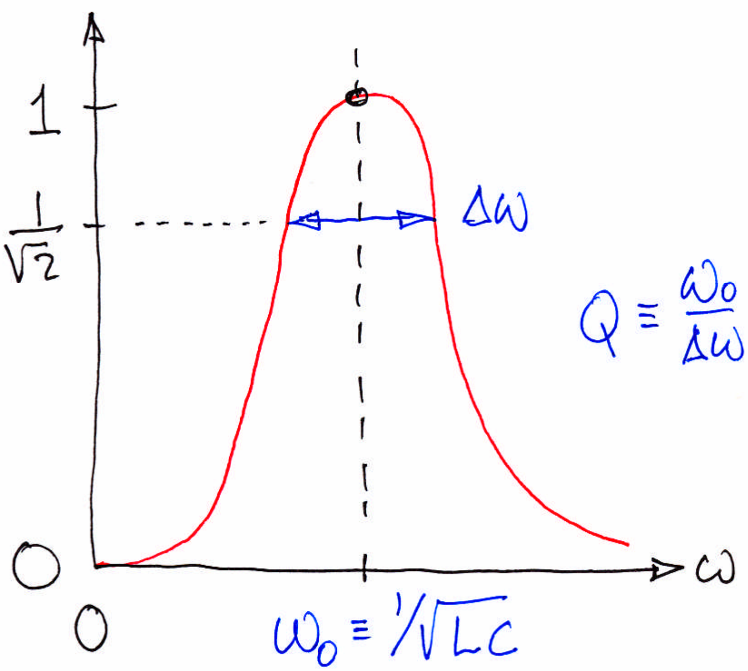
$V_{in}\, e^{j\omega t}$



$\hat{V}_{out}\, e^{j\omega t}$

$$\frac{\hat{V}_{out}}{V_{in}} = \frac{1/R}{1/R + 1/j\omega L + j\omega C} = \frac{j\omega L/R}{1-\omega^2 LC + j\omega L/R}$$

$$\left|\frac{\hat{V}_{out}}{V_{in}}\right| = \frac{(\omega L/R)}{\sqrt{(1-\omega^2 LC)^2 + (\omega L/R)^2}}$$



$$Q \equiv \frac{\omega_0}{\Delta\omega} = \frac{R}{\sqrt{L/C}}$$

Numbers:

$L = 0.1\ H$

$C = 0.25\ \mu F$

$R = 10\ k\Omega$

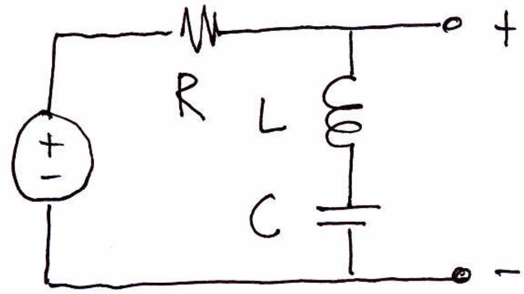
$\frac{1}{2\pi\sqrt{LC}} = 1\ kHz$

$\sqrt{L/C} = 632\ \Omega$

$Q = 16$

BSF Example

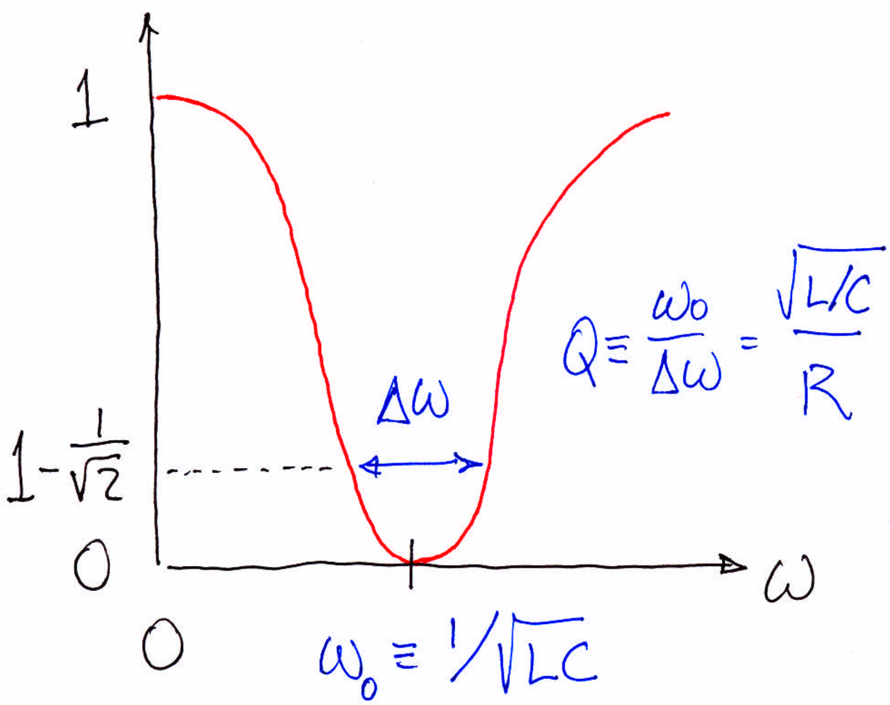
$V_{in} e^{j\omega t}$



$\hat{V}_{out} e^{j\omega t}$

$$\frac{\hat{V}_{out}}{V_{in}} = \frac{j\omega L + 1/j\omega C}{R + j\omega L + 1/j\omega C} = \frac{1-\omega^2 LC}{1-\omega^2 LC + j\omega RC}$$

$$\left|\frac{\hat{V}_{out}}{V_{in}}\right| = \frac{1-\omega^2 LC}{\sqrt{(1-\omega^2 LC)^2 + (\omega RC)^2}}$$



Numbers:

$L = 0.1\ H$

$C = 0.25\ \mu F$

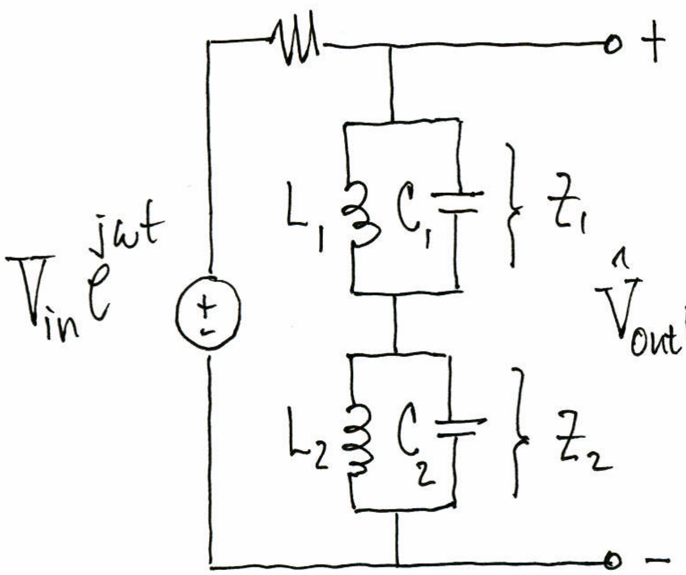
$R = 100\ \Omega$

$\frac{1}{2\pi\sqrt{LC}} = 1\,kHz$

$\sqrt{L/C} = 632\ \Omega$

$Q = 6.3$

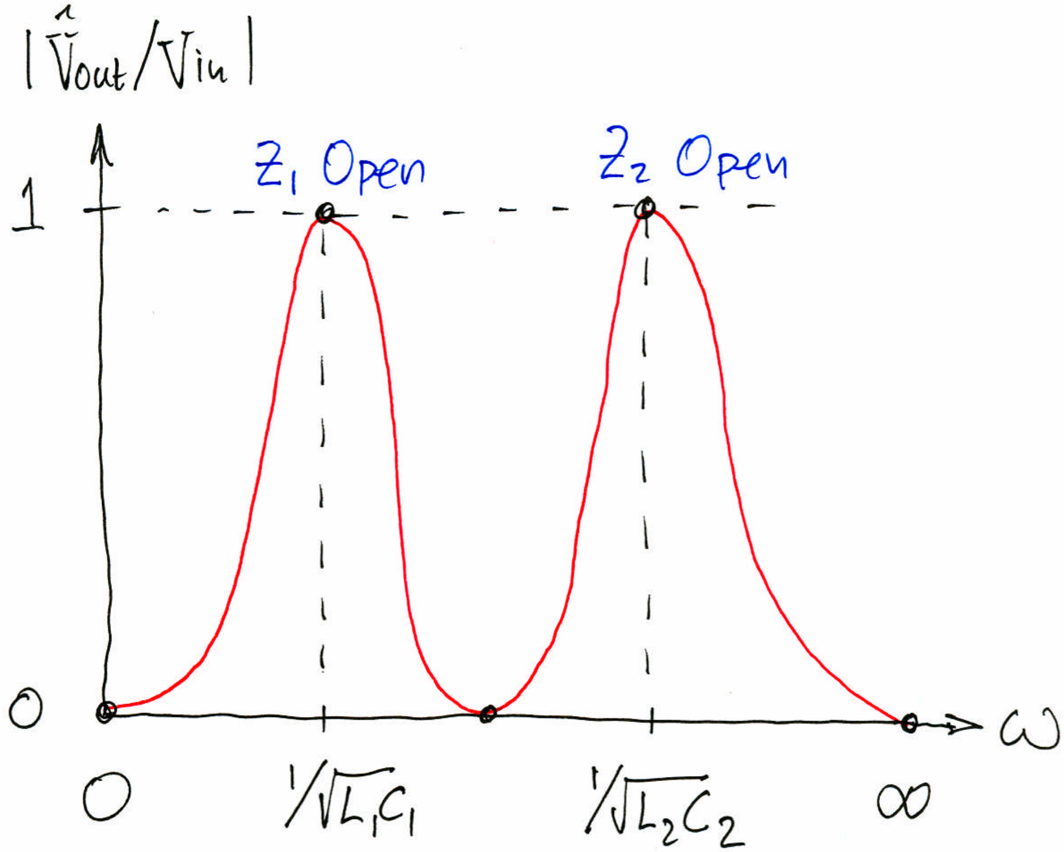
A More Complex Filter



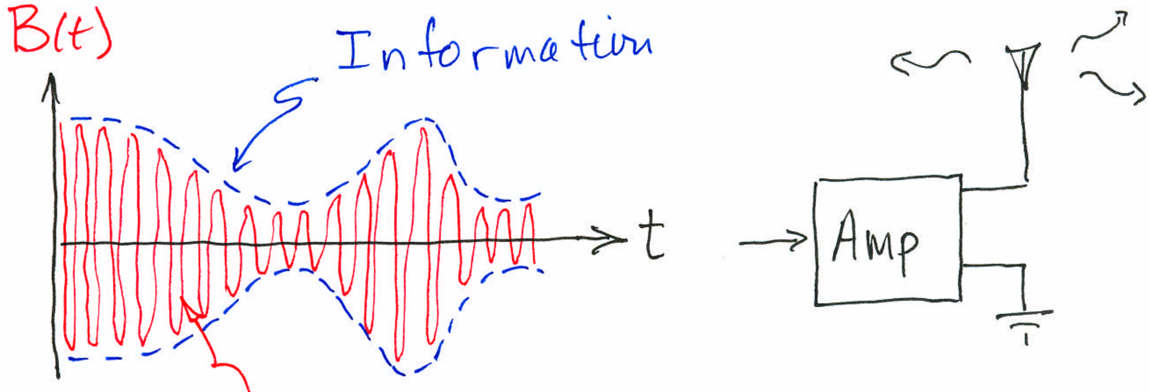
Z_1 = Open at $\omega = \frac{1}{\sqrt{L_1C_1}}$

Z_2 = Open at $\omega = \frac{1}{\sqrt{L_2C_2}}$

$Z_1 + Z_2$ = Short at $\omega = 0, \infty$ and between the two opens



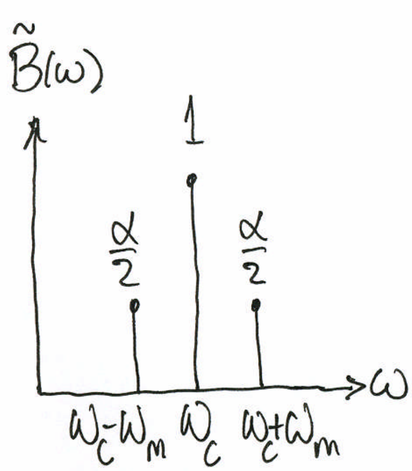
AM Radio Transmission

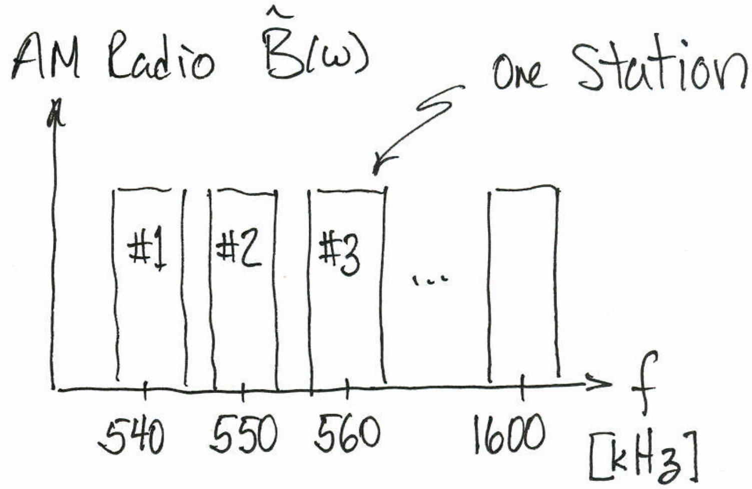


$$B(t) = \overbrace{\left(1 + \underbrace{S(t)}_{\text{Information}}\right) \underbrace{\cos(\omega_c t)}_{\text{Carrier}}}^{\text{Modulated Carrier}} \qquad |S(t)| < 1$$

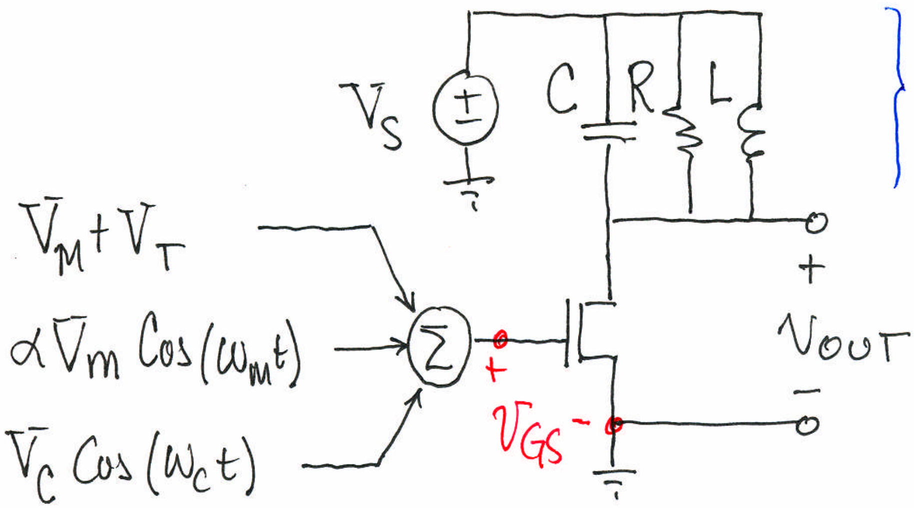
Suppose $S(t) = \alpha \cos(\omega_m t) \qquad |\alpha| < 1$

Then $B(t) = \cos(\omega_c t) + \frac{\alpha}{2}\Big(\cos((\omega_c + \omega_m)t) + \cos((\omega_c - \omega_m)t)\Big)$





AM Radio Transmitter



Largest impedance at $\omega = 1/\sqrt{LC}$

$$C\frac{d}{dt}\left(v_{OUT}-\bar{V}_S\right) + \frac{1}{R}\left(v_{OUT}-\bar{V}_S\right) + \frac{1}{L}\int_{-\infty}^{t}\left(v_{OUT}-\bar{V}_S\right)dt + \frac{K}{2}\left(v_{GS}-\bar{V}_T\right)^2 = 0$$

$$LC\frac{d^2 v_{OUT}}{dt^2} + \frac{L}{R}\frac{d v_{OUT}}{dt} + v_{OUT} = \bar{V}_S - KL\left(v_{GS}-\bar{V}_T\right)\frac{d v_{GS}}{dt}$$

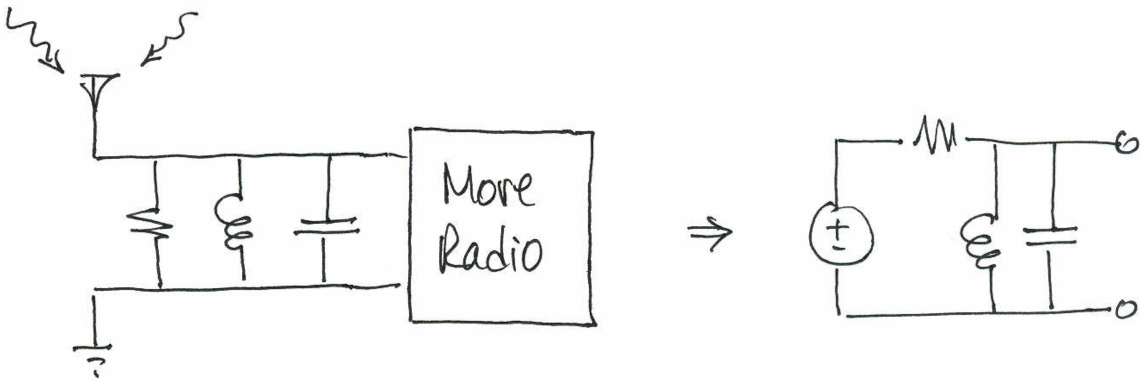
$$-\left(v_{GS}-\bar{V}_T\right)\frac{d v_{GS}}{dt} = \omega_c \bar{V}_m \bar{V}_c \left(1+\alpha\cos(\omega_m t)\right)\sin(\omega_c t)$$ Desired

$$+\ \alpha\omega_m \bar{V}_m \bar{V}_c \sin(\omega_m t)\cos(\omega_c t)$$ Small For $\omega_m \ll \omega_c$

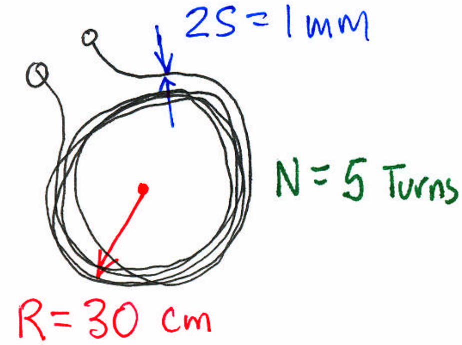
$$+\ \omega_c \bar{V}_c^2 \sin(\omega_c t)\cos(\omega_c t)$$ Filtered Out

$$+\ \alpha\omega_m \bar{V}_m^2 \left(1+\alpha\cos(\omega_m t)\right)\sin(\omega_m t)$$ Filtered Out

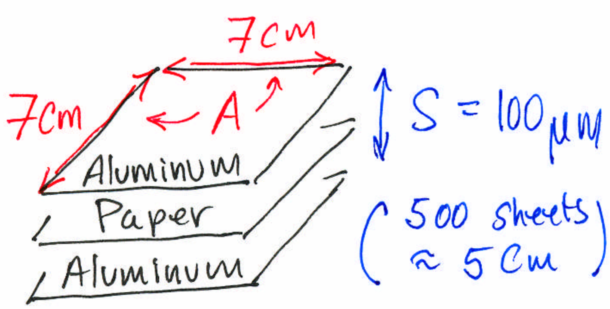
AM Radio Receiver



RLC Filter $\Rightarrow$ BPF around $\omega = 1/\sqrt{LC}$



$$L = \mu_0 N^2 R \left(\ln\left(\frac{8R}{S} - 2 \right) \right)$$
$$= 61\ \mu H \quad (\text{Actually } 52\,\mu H)$$



$$C = \frac{\epsilon_0 A}{S} = 434\ pF$$
$$+\ 150\ pF$$
$$\overline{584\ pF}$$

$\omega = 1/\sqrt{LC}$ .... 910 kHz as measured

Parasitic