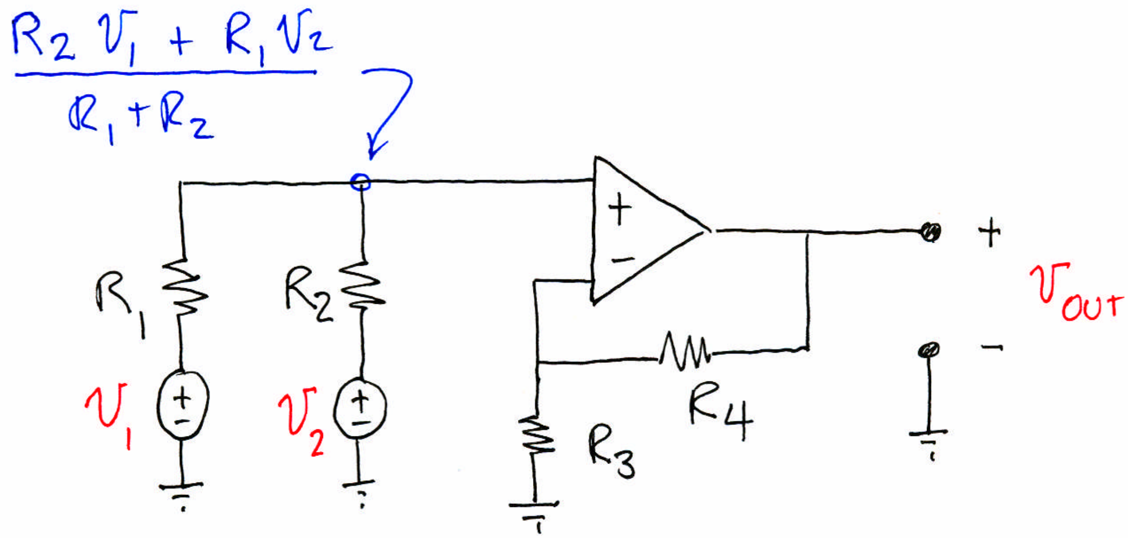
6.002 - Lecture 20

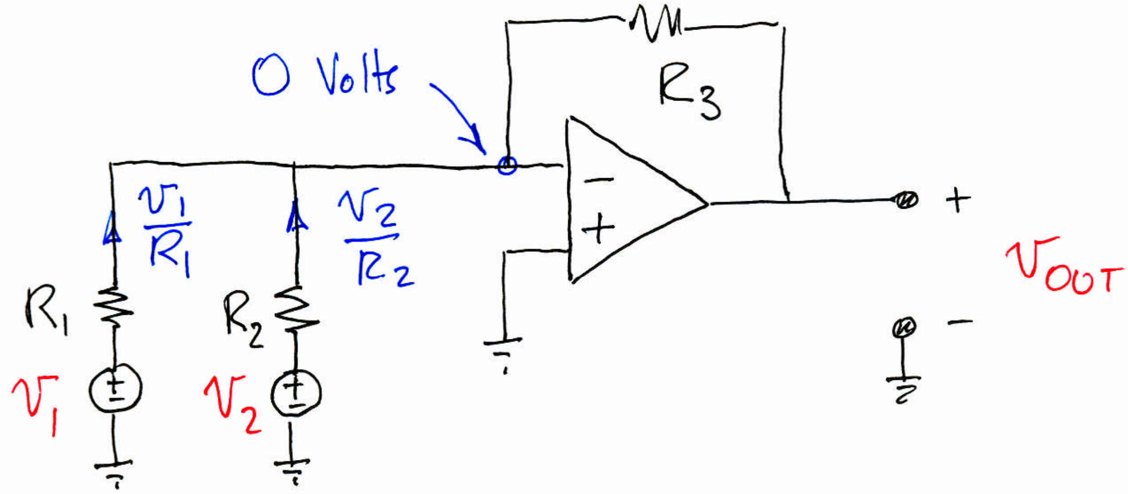
Analog Signal Processing

- Addition & Subtraction
- Integration & Differentiation
- Analog Computers
- Filter Synthesis

Addition

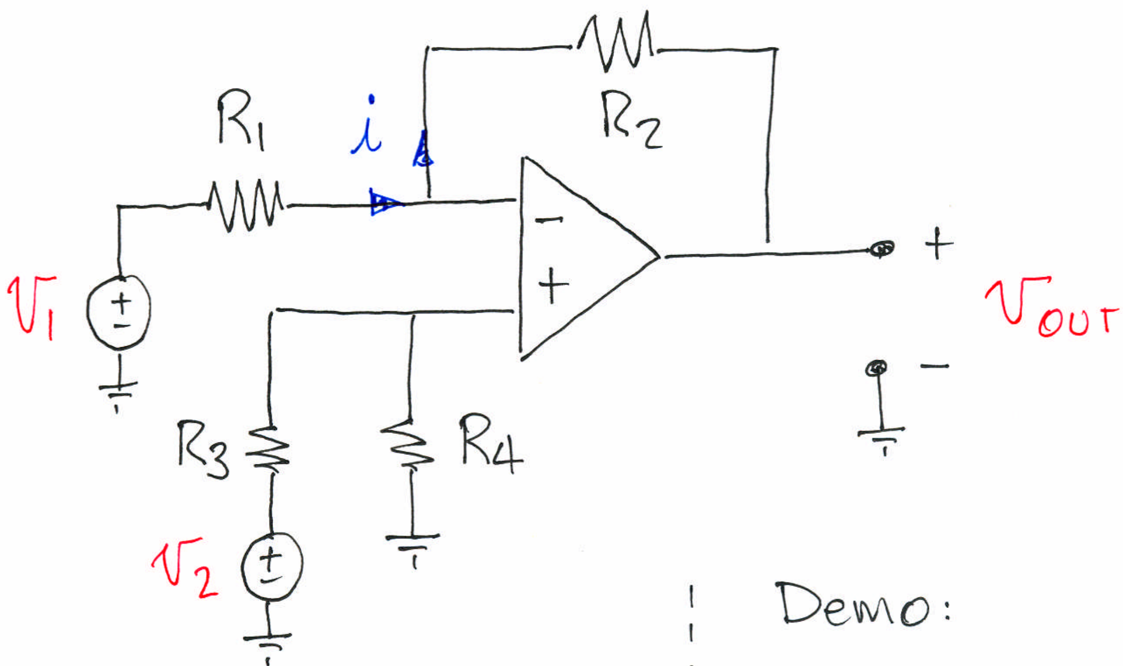


$$V_{OUT} = \frac{R_4 + R_3}{R_3} \left[\frac{R_2 V_1 + R_1 V_2}{R_1 + R_2} \right]$$



$$V_{OUT} = -R_3 \left[\frac{V_1}{R_1} + \frac{V_2}{R_2} \right]$$

Subtraction



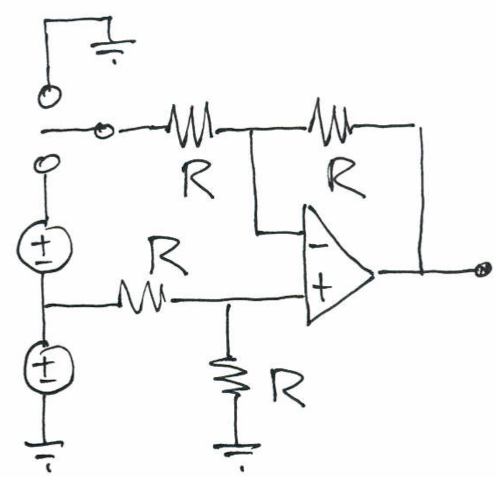
$$v_- \approx v_+ = \frac{R_4}{R_3 + R_4} v_2$$

$$i = \frac{v_1 - v_-}{R_1}$$

$$v_{out} = v_- - R_2 i$$

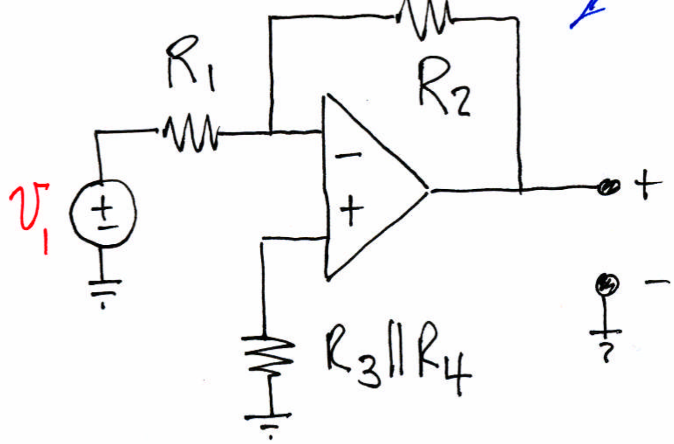
$$v_{out} = \frac{R_4}{R_3 + R_4} \cdot \frac{R_1 + R_2}{R_1} v_2 - \frac{R_2}{R_1} v_1$$

Demo:



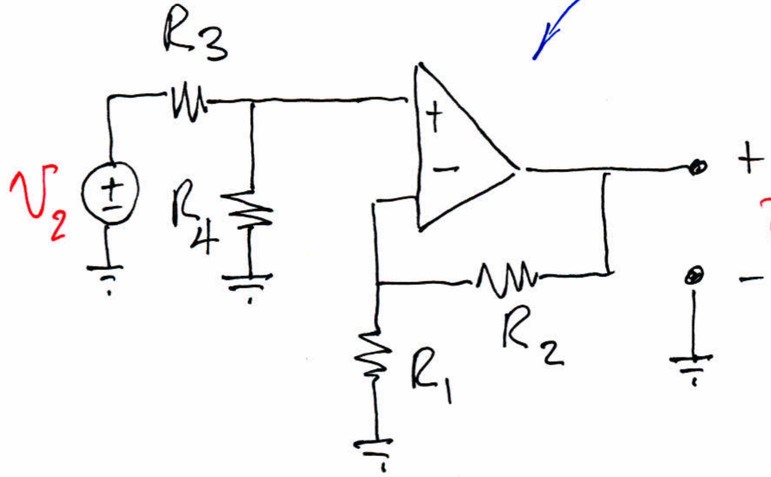
Superposition

Subtracter with $V_2 \equiv 0$



$$V_{OUT} = -\frac{R_2}{R_1} V_1$$

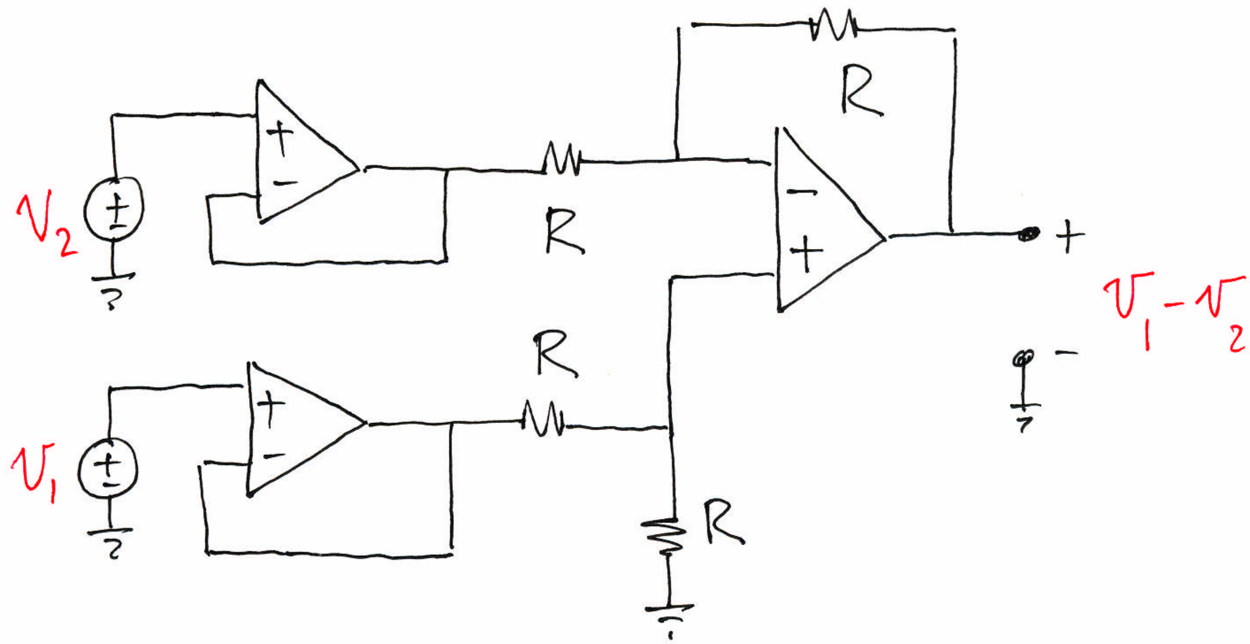
Subtracter with $V_1 = 0$

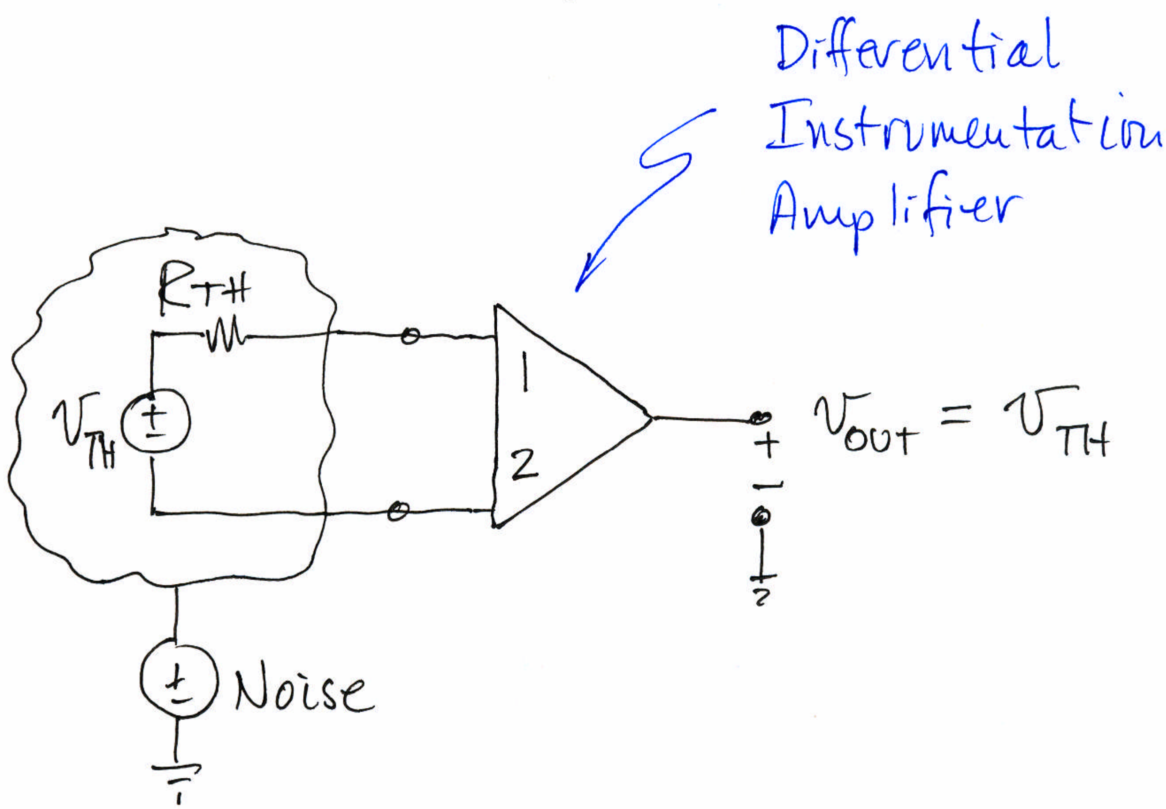


$$V_{OUT} = \frac{R_1 + R_2}{R_1} \cdot \frac{R_4}{R_3 + R_4} V_2$$

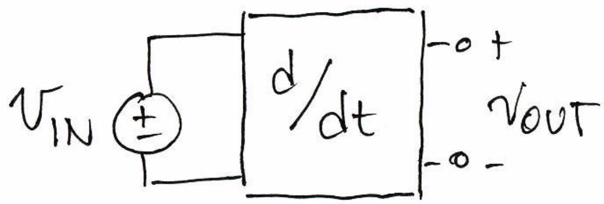
Subtracter: $V_{OUT} = \dfrac{R_1 + R_2}{R_1} \cdot \dfrac{R_4}{R_3 + R_4} V_2 - \dfrac{R_2}{R_1} V_1$

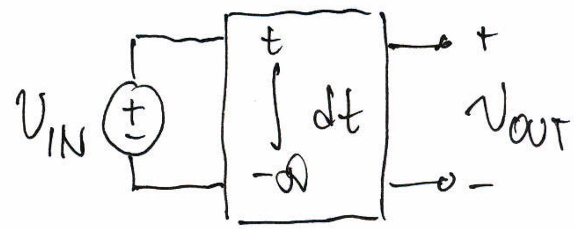
Differential Instrumentaon Amplifier

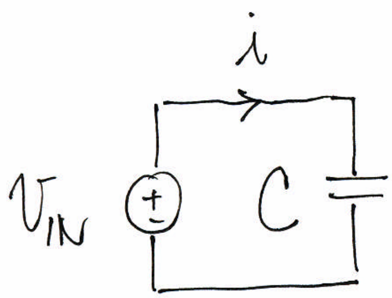


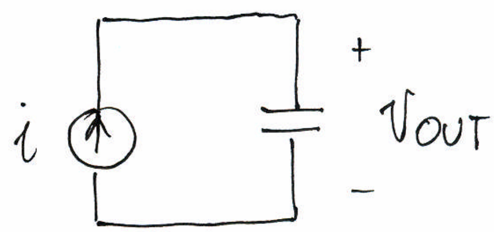


Differentiation & Integration I









$$i = C\frac{dv}{dt}$$

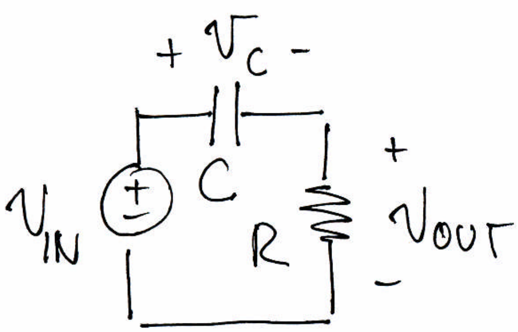
$$v_{OUT} = \frac{1}{C}\int_{-\infty}^{t} i\,dt$$

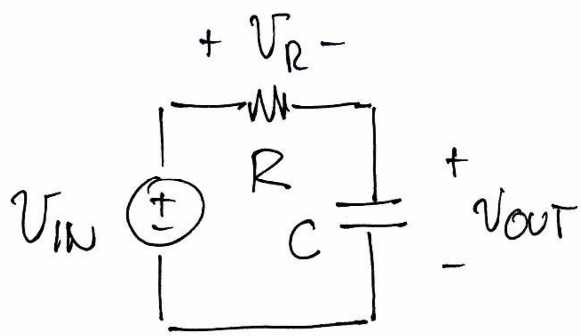
Needs conversion from i to v_{OUT}.

Needs conversion from v_{IN} to i.

(Inductors are generally not used because they are too big and too lossy.)

Differentiation & Integration II





Resistor converts current to voltage.

Resistor converts voltage to current.

Good Differentiation

$\Downarrow$

$V_C \gg V_{OUT}$

$\Downarrow$

$V_C \gg RC \frac{dV_C}{dt}$

$\Downarrow$

$\omega \ll \frac{1}{RC}$

Good Integration

$\Downarrow$

$V_R \gg V_{OUT}$

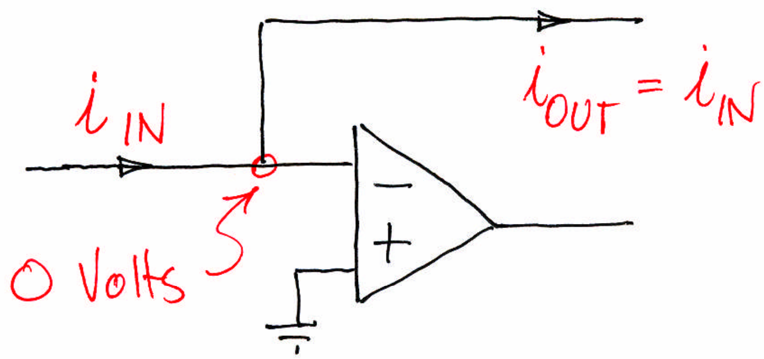
$\Downarrow$

$RC \frac{dV_{OUT}}{dt} \gg V_{OUT}$

$\Downarrow$

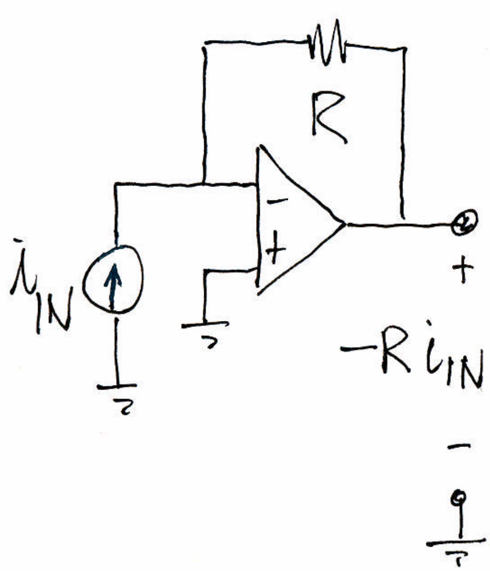
$\omega \gg \frac{1}{RC}$

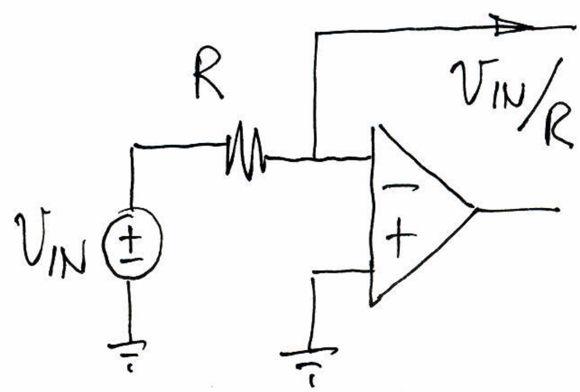
Better Converters



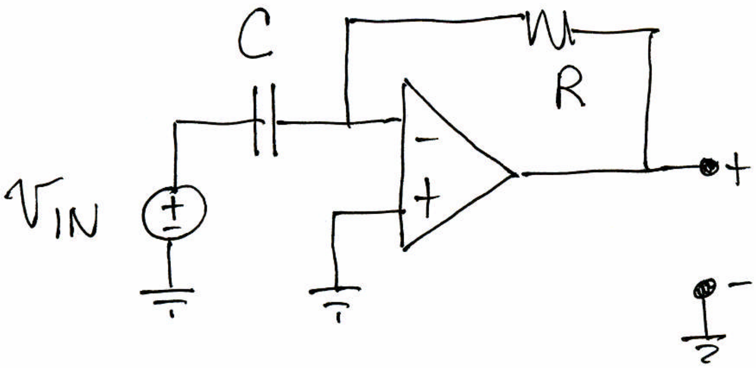
Current-To-Voltage Converter

Voltage-To-Current Converter





Differentiation III



$$V_{OUT} = -RC \frac{dV_{IN}}{dt}$$

Good Differentiation

$\Downarrow$

$$V_{IN} \gg V_- = \frac{-V_{OUT}}{A}$$

$\Downarrow$

$$V_{IN} \gg \frac{RC}{A} \frac{dV_{IN}}{dt}$$

$\Downarrow$

$$\omega \ll \frac{A}{RC}$$

Numbers:

$R = 1\,k\Omega$

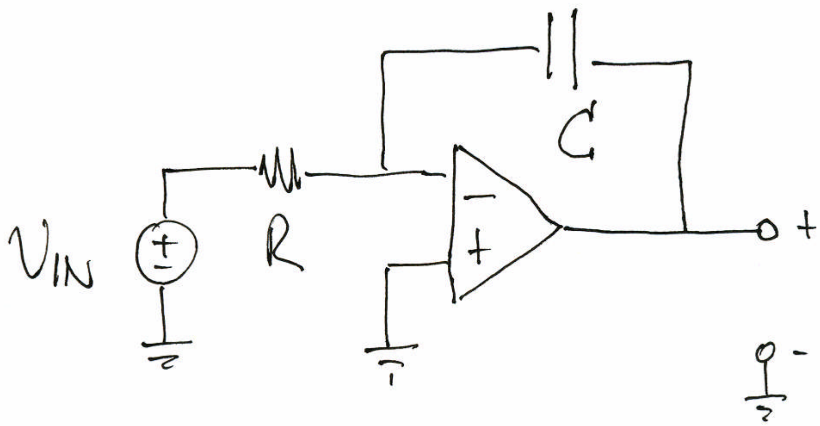
$C = 0.01\,\mu F$

$A = 10^6$

$\omega \ll 10^{11}\ \frac{rad}{s}$

Op Amp is probably too slow !!

Integration III



$$v_{OUT} = \frac{-1}{RC}\int_{-\infty}^{t} v_{IN}\,dt$$

Good Integration

$\Downarrow$

$$V_{IN} \gg V_- = -\frac{V_{OUT}}{A}$$

$\Downarrow$

$$V_{IN} \gg \frac{1}{ARC}\int_{-\infty}^{t} v_{IN}\,dt$$

$\Downarrow$

$$\omega \gg \frac{1}{ARC}$$

Numbers:

$R = 1\,k\Omega$

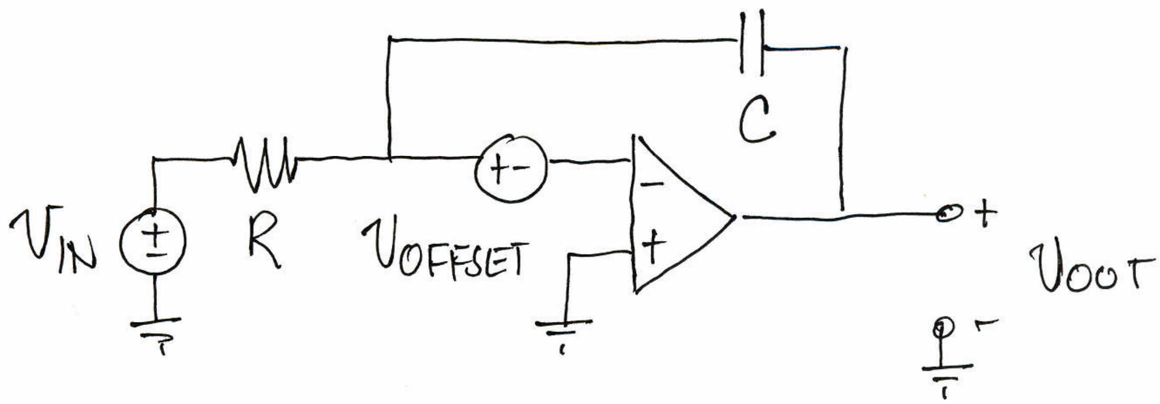
$C = 0.01\,\mu F$

$A = 10^6$

$\omega \gg 10^{-1}\ \frac{rad}{s}$

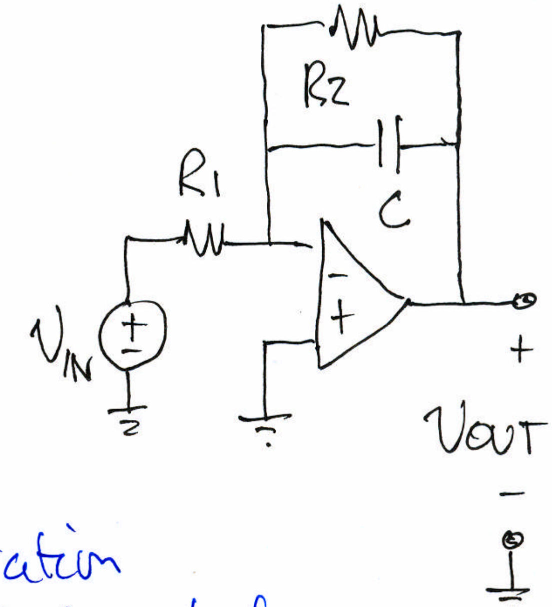
Beware of offsets!

Integration IV (Offsets)



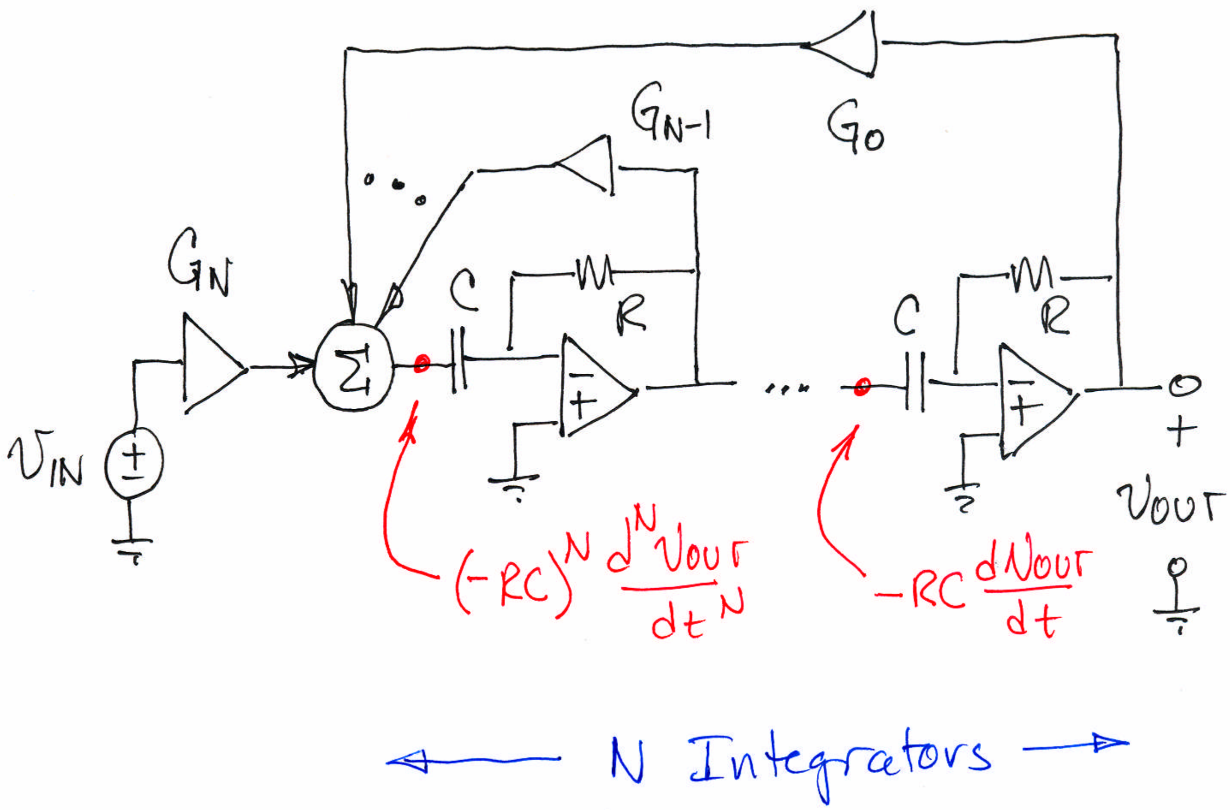
$$V_{OUT} = V_{OFFSET} + \frac{1}{RC}\int_{-\infty}^{t}(V_{OFFSET} - V_{IN})\,dt$$

V_{OFFSET} must be balanced, otherwise $V_{OUT} \to \pm\infty$ even when $V_{IN} \equiv 0$.



Use the circuit to the right, for example, for which good integration $\Rightarrow \omega \gg 1/R_2C$, but for which $V_{OUT} \approx \frac{R_2+R_1}{R_1} V_{OFFSET}$ when $V_{IN} \equiv 0$.

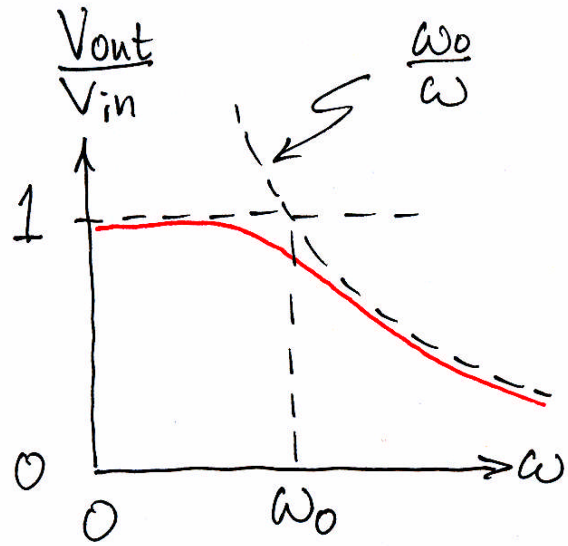
Analog Computers



$$(-RC)^N \frac{d^N V_{OUT}}{dt^N} = G_{N-1} (-RC)^{N-1} \frac{d^{N-1} V_{OUT}}{dt^{N-1}} + \cdots G_0 V_{OUT} + G_N V_{IN}$$

Note that V_{IN} can also be injected along the integrator string so as to inject its derivatives.

Filter Synthesis I



$$\Rightarrow \quad \frac{\tilde{V}_{out}}{V_{in}} = \frac{1}{1 + j\omega/\omega_0}$$

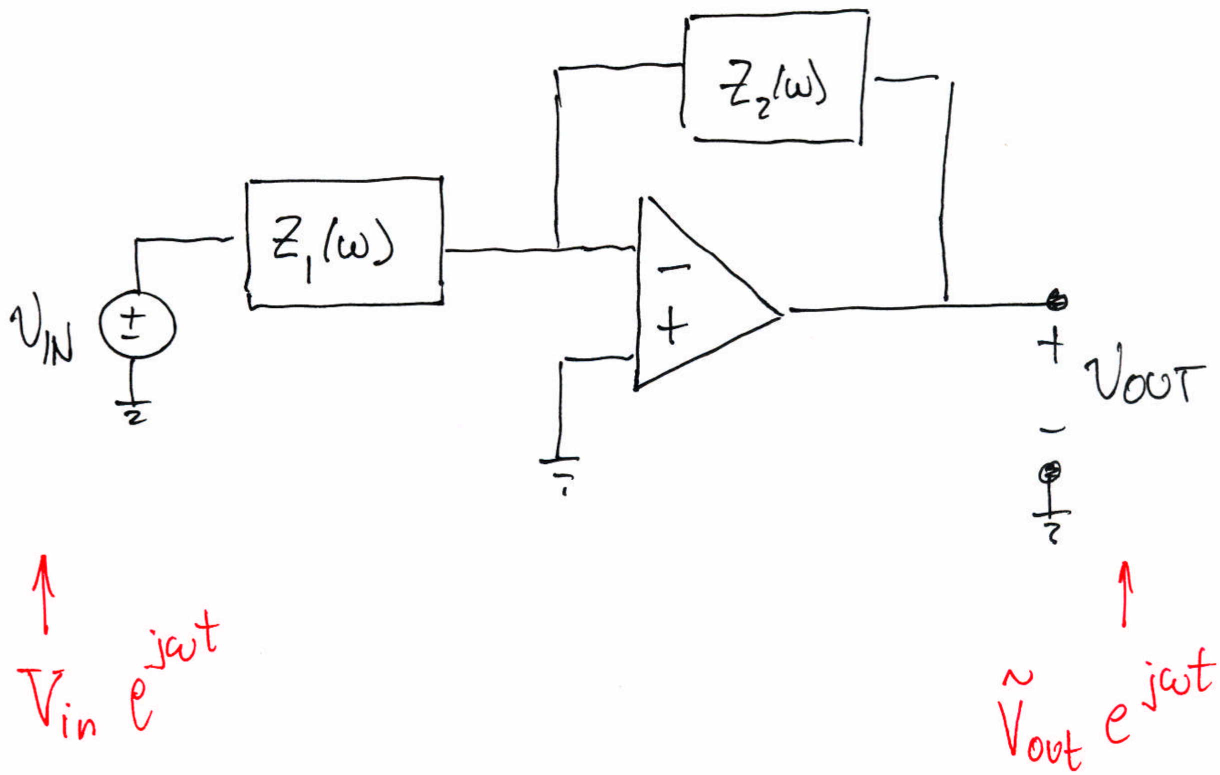
$$\tilde{V}_{out}\,(1 + j\omega/\omega_0) = V_{in}$$

$$j\omega \rightarrow \frac{d}{dt}$$

$$V_{OUT} + \frac{1}{\omega_0}\frac{dV_{OUT}}{dt} = V_{IN}$$

Construct an "analog computer" from the differential equation.

Filter Synthesis II



$$\tilde{V}_{out} = \frac{Z_2(\omega)}{Z_1(\omega)} V_{in}$$

Use parallel and series stages to obtain more complex filters.