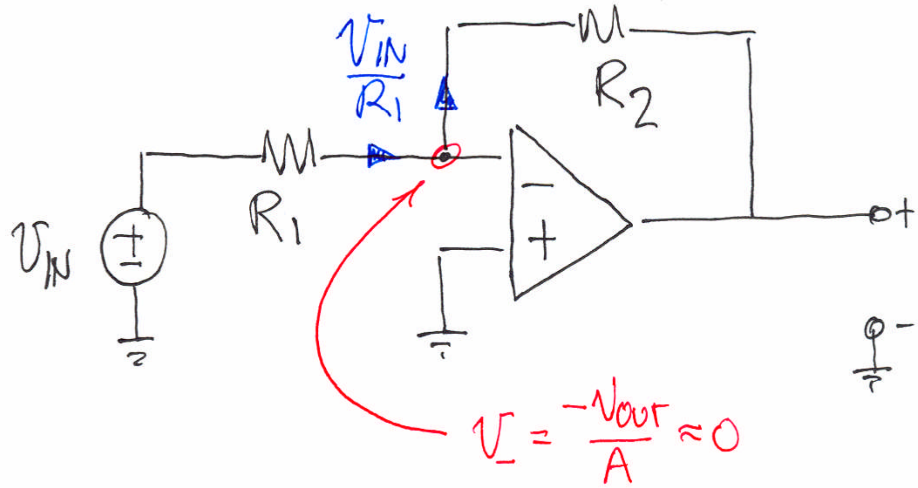
6.002 - Lecture 21

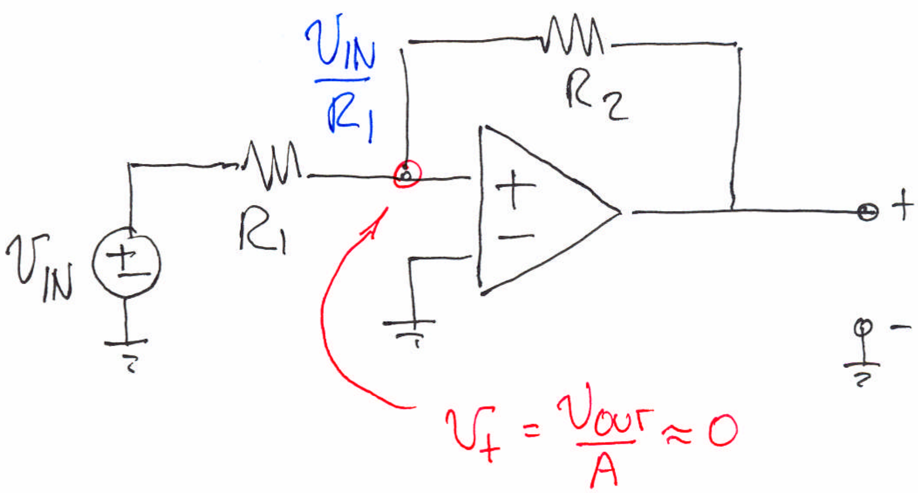
Stability

- Equilibria
- Dynamics & Stability
- Comparators
- Positive Feedback
- Schmitt Trigger
- Relaxation Oscillator

Equilibria



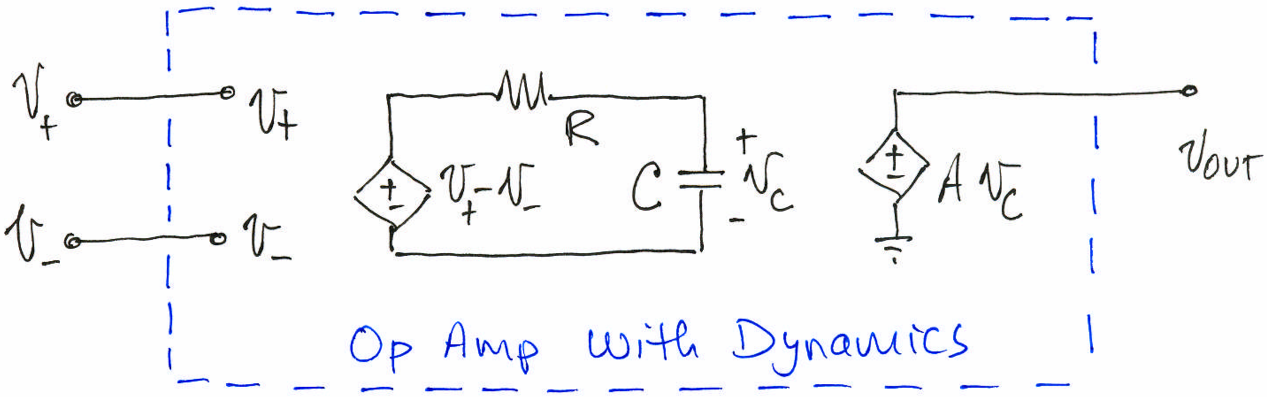
$$V_{OUT} = -\frac{R_1}{R_2} V_{IN}$$



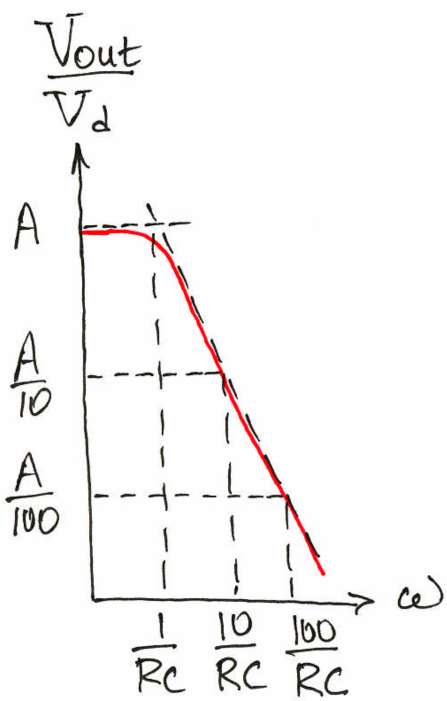
$$V_{OUT} = -\frac{R_1}{R_2} V_{IN}$$

Do these two circuits really behave in the same way?

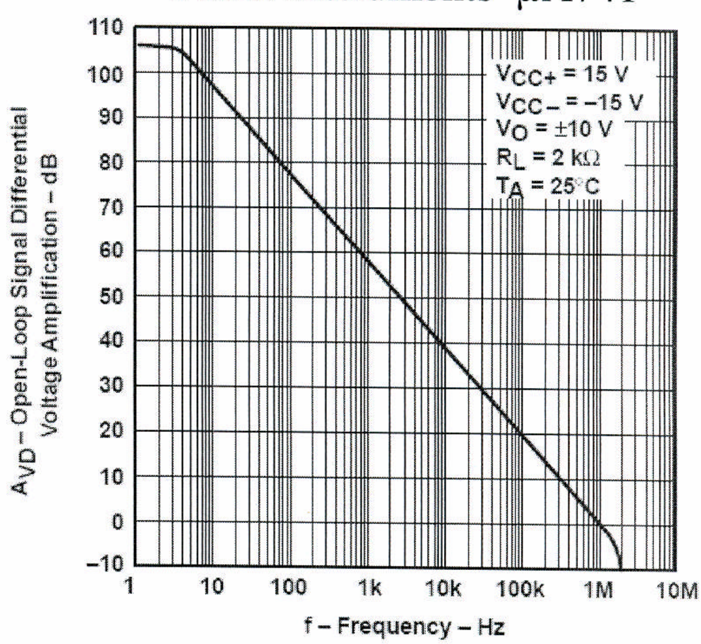
Op-Amp Dynamics



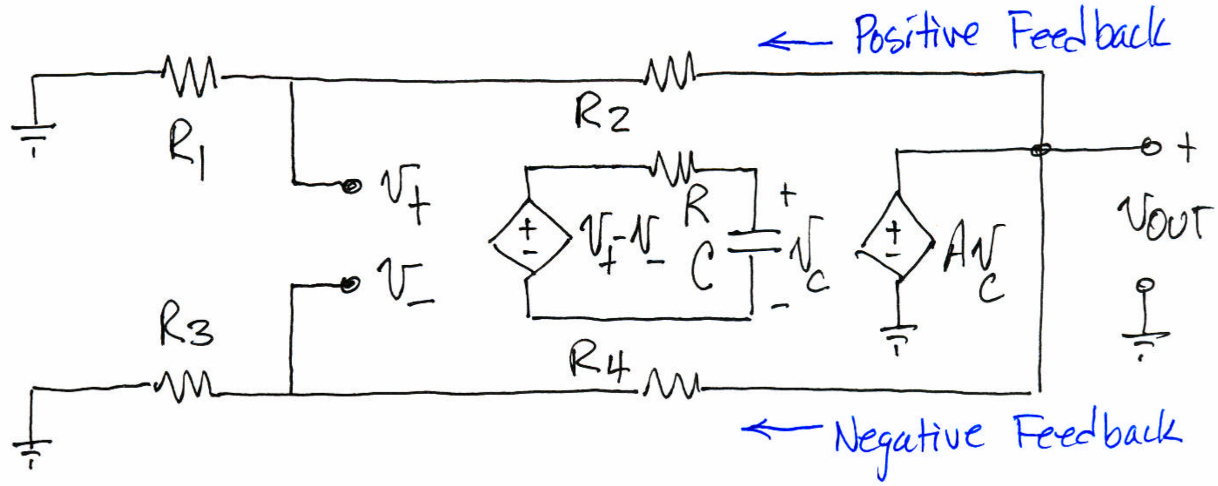
$$v_+ - v_- \equiv v_D \rightarrow V_d e^{j\omega t} \quad \Rightarrow \quad v_{OUT} \rightarrow \frac{A\, V_d}{1 + j\omega RC} e^{j\omega t}$$



Texas Instruments μA741



Amplifier Dynamics I



$$v_{OUT} = A \, v_C$$

$$RC \frac{dv_C}{dt} + v_C = v_+ - v_-$$

$$v^+ = \frac{R_1}{R_1+R_2} v_{OUT} \equiv \overset{+}{\gamma} \, v_{OUT}$$ (Positive Feedback)

$$v^- = \frac{R_3}{R_3+R_4} v_{OUT} \equiv \overset{-}{\gamma} \, v_{OUT}$$ (Negative Feedback)

Small as $A \to \infty$

$$\frac{RC}{A} \frac{dv_{OUT}}{dt} + \left[\frac{1}{A} + \overset{-}{\gamma} - \overset{+}{\gamma} \right] v_{OUT} = 0$$

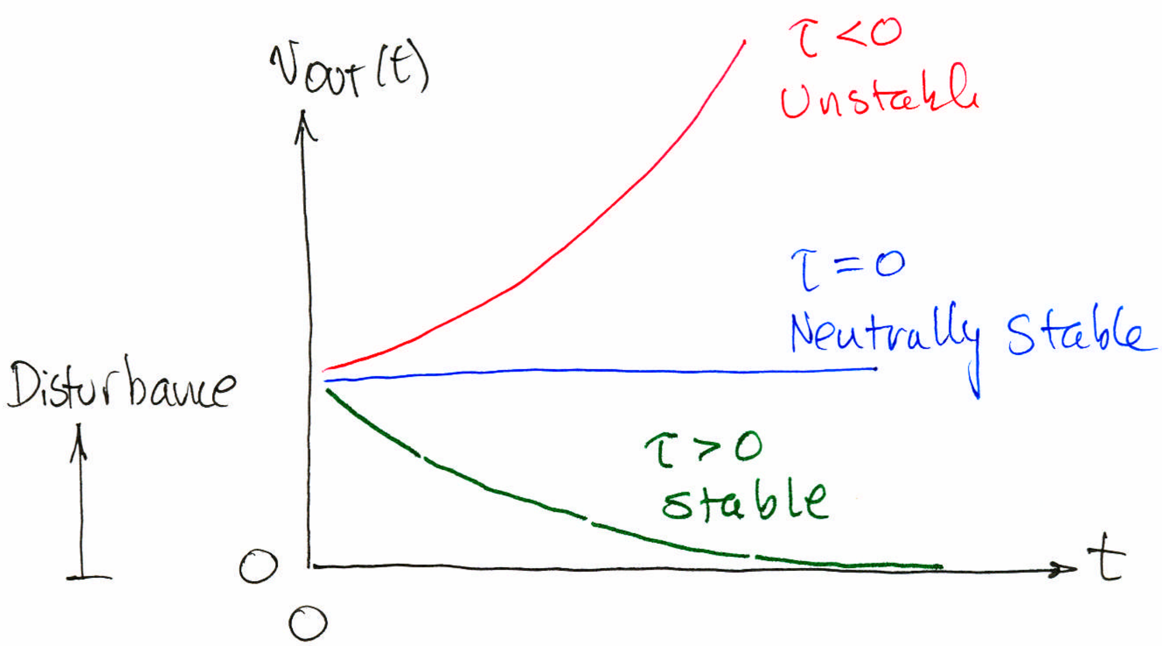
$$\frac{dv_{OUT}}{dt} + \frac{A(\overset{-}{\gamma} - \overset{+}{\gamma})}{RC} v_{OUT} = 0$$

Amplifier Dynamics II

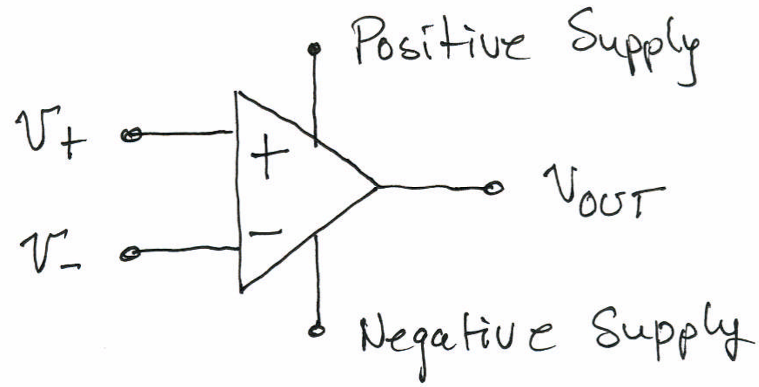
$$\frac{dV_{out}}{dt} + \frac{1}{\tau} V_{out} = 0 \qquad \tau \equiv \frac{RC}{A(\bar{\gamma} - \overset{+}{\gamma})}$$

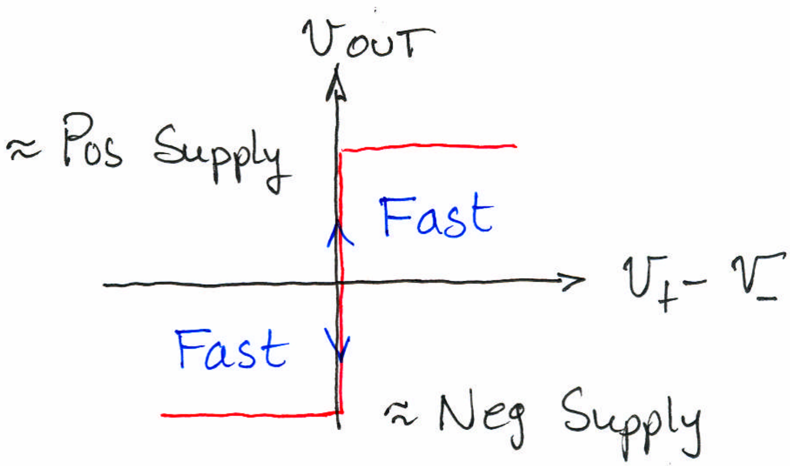
$$V_{out}(t) = V_{out}(0)\, e^{-t/\tau}$$

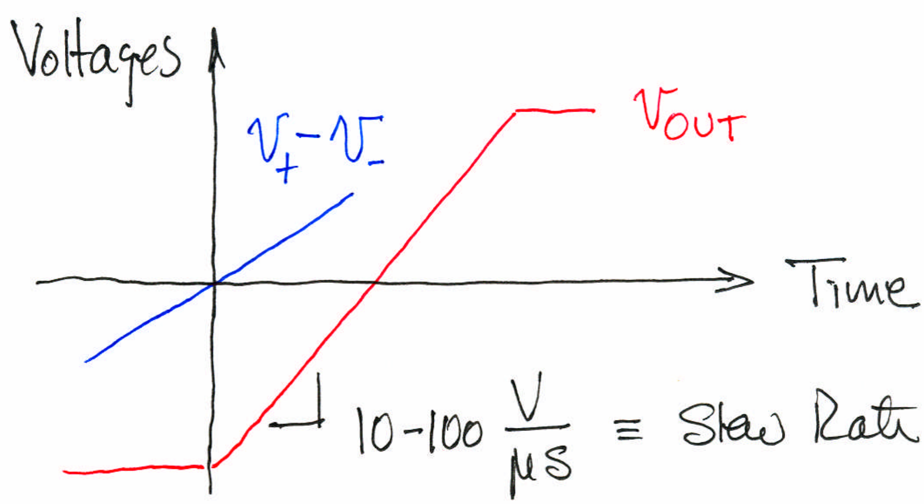
$$\tau \begin{cases} > 0 & \bar{\gamma} > \overset{+}{\gamma} \quad \text{Negative Feedback} \\ = 0 & \bar{\gamma} = \overset{+}{\gamma} \quad \text{Neutral Feedback} \\ < 0 & \bar{\gamma} < \overset{+}{\gamma} \quad \text{Positive Feedback} \end{cases}$$



Comparators





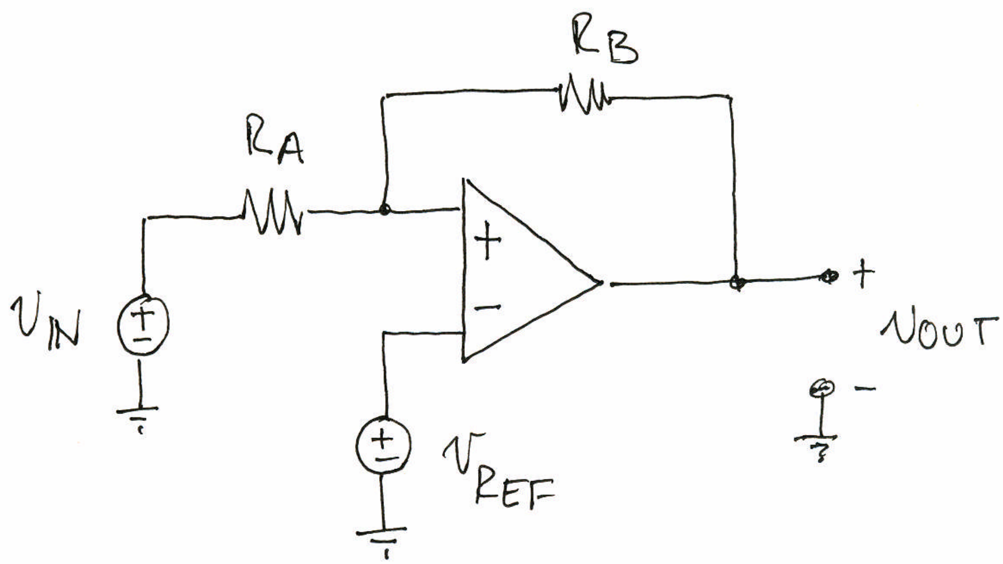


Op Amp Versus Comparator

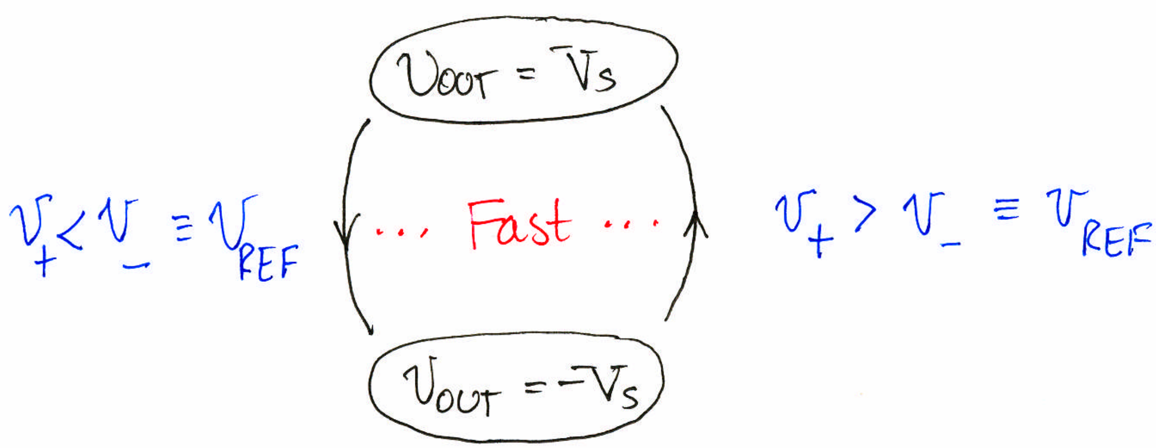
Op-Amp ⇒ Designed to be largely linear, and for use in negative feedback applications. Implemented with a few high-gain stages having modest bandwidth.

Comparators ⇒ Designed to be very fast, and for use in open-loop applications or with positive feedback. Often implemented with many very fast stages having modest gain. May employ positive feedback for high-speed.

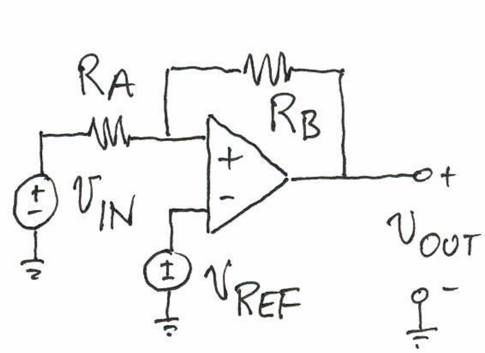
Schmitt Trigger I



Could interchange roles of v_{IN} and v_{REF}.



Schmitt Trigger II



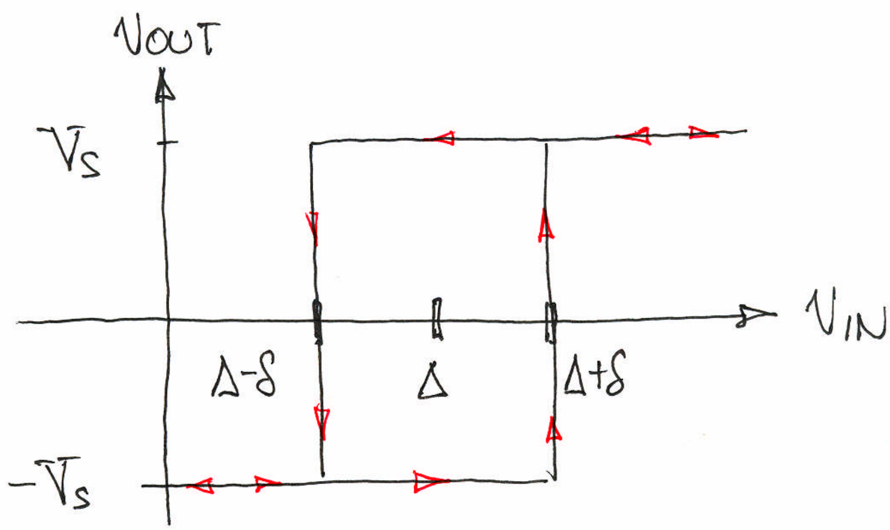
$-\bar{V}_S \rightarrow +\bar{V}_S$ Transition:

$$v_{OUT}\frac{R_A}{R_A+R_B} + v_{IN}\frac{R_B}{R_A+R_B} > v_{REF} \Rightarrow v_{IN} > \frac{R_A+R_B}{R_A}v_{REF} + \frac{R_A}{R_B}\bar{V}_S$$

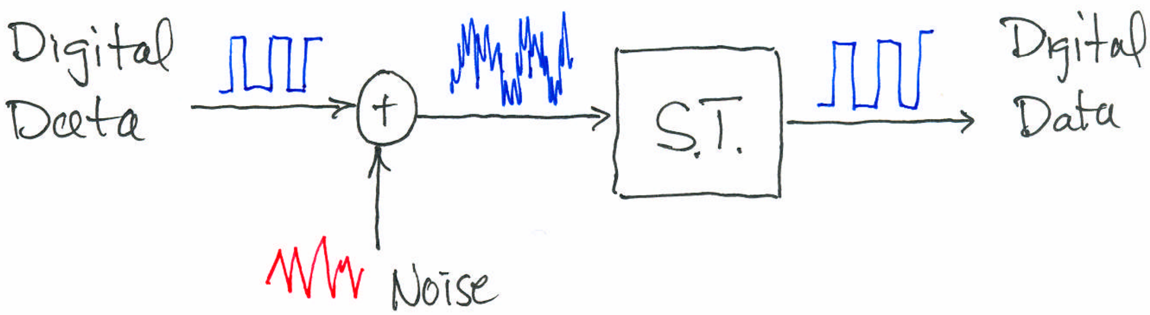
$$\Delta \pm \delta$$

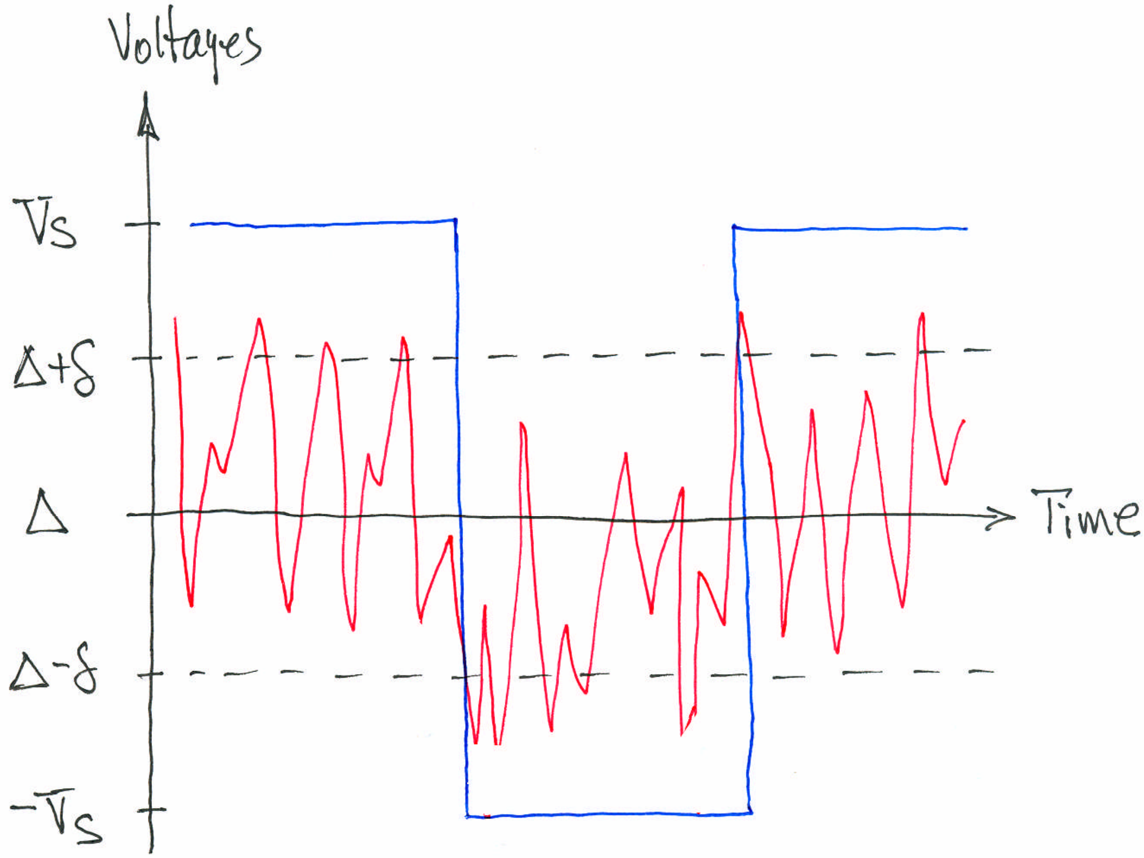
$+\bar{V}_S \rightarrow -\bar{V}_S$ Transition:

$$v_{OUT}\frac{R_A}{R_A+R_B} + v_{IN}\frac{R_B}{R_A+R_B} < v_{REF} \Rightarrow v_{IN} < \frac{R_A+R_B}{R_A}v_{REF} - \frac{R_A}{R_B}\bar{V}_S$$



Schmitt Trigger Application





Oscillator