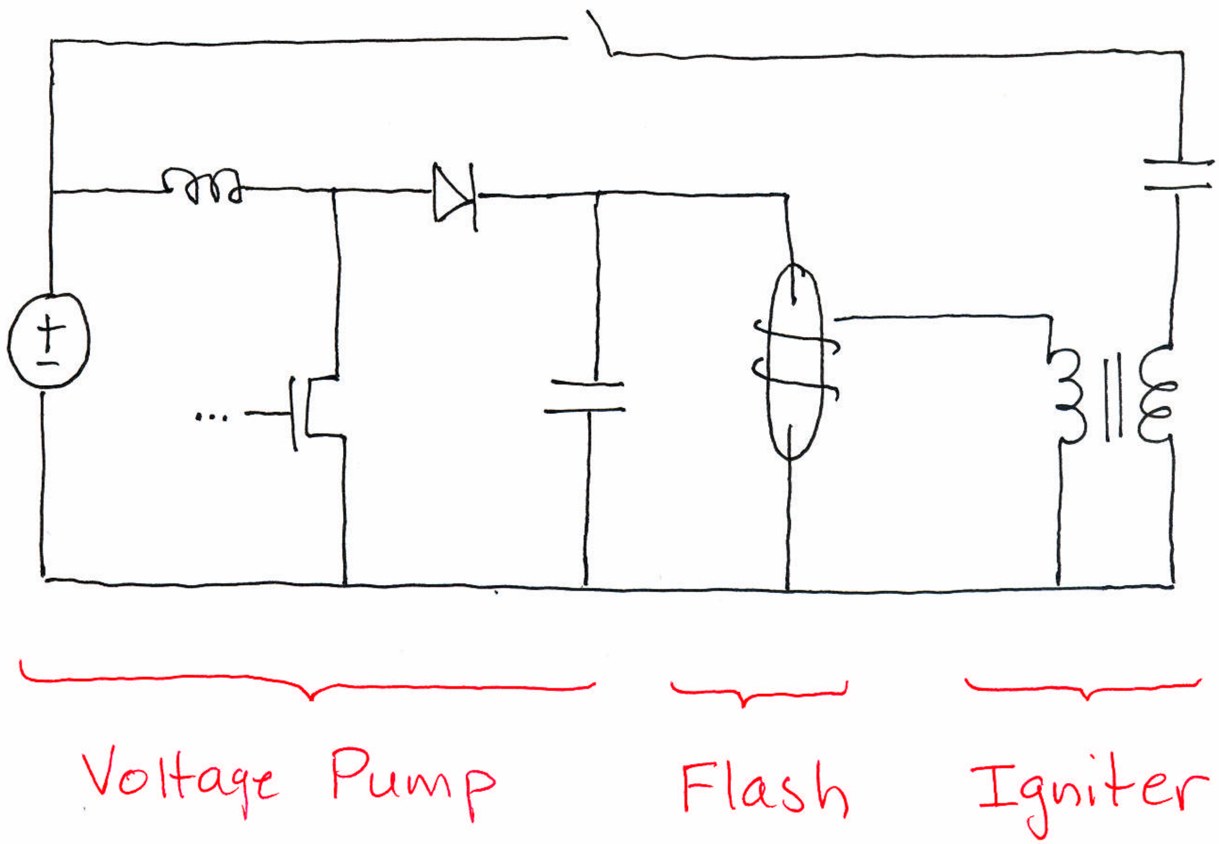
6.002 - Lecture 24

Diodes & LC Networks

- Review Of LC Networks
- Diodes As Uncontrolled Switches
- Energy Processing & Conservation
- Preview Of Power Electronics
- Voltage Pump Example

Camera Flash Circuit

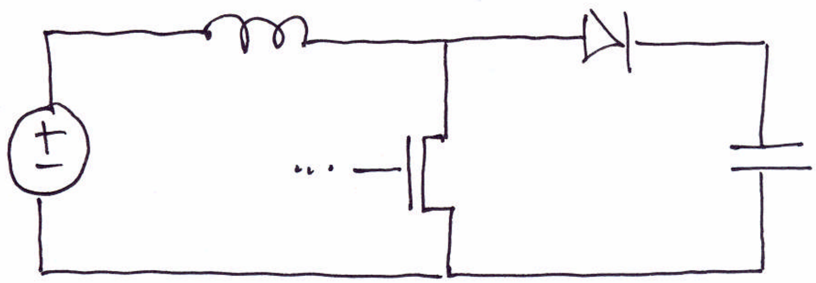


Typical Operation: 1.5 V @ Battery

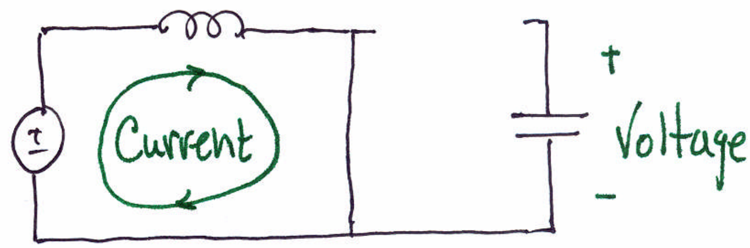
300 V @ Capacitor

10 kV @ Igniter

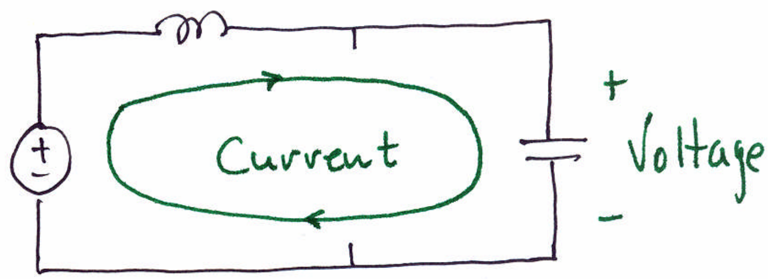
Voltage Pump (Boost Converter)



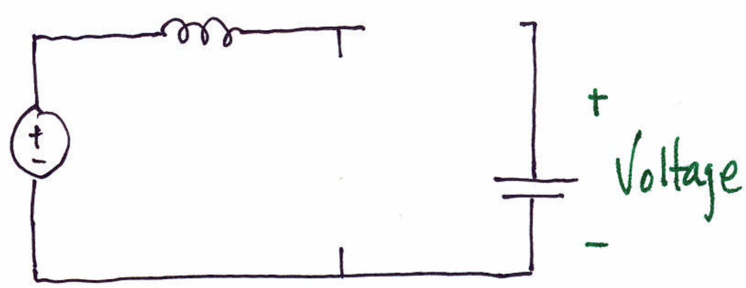
State I
MOSFET On
Diode Off



State II
MOSFET Off
Diode On



State III
MOSFET Off
Diode Off

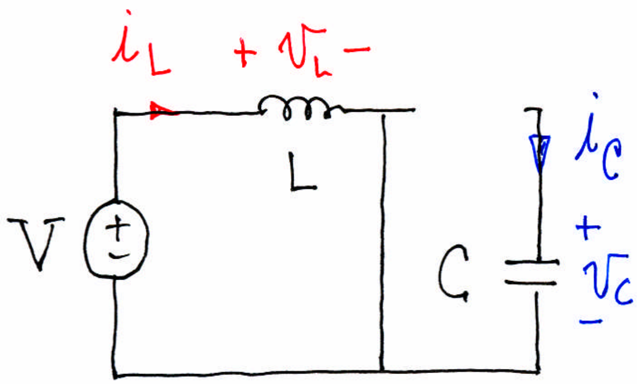


Observations

- MOSFET ⇒ Ideal controlled switch
- Diode ⇒ Ideal uncontrolled switch
- Inductor ⇒ Energy storage
- Capacitor ⇒ Energy storage
- No use for lossy resistors

Power electronics preview!

State I



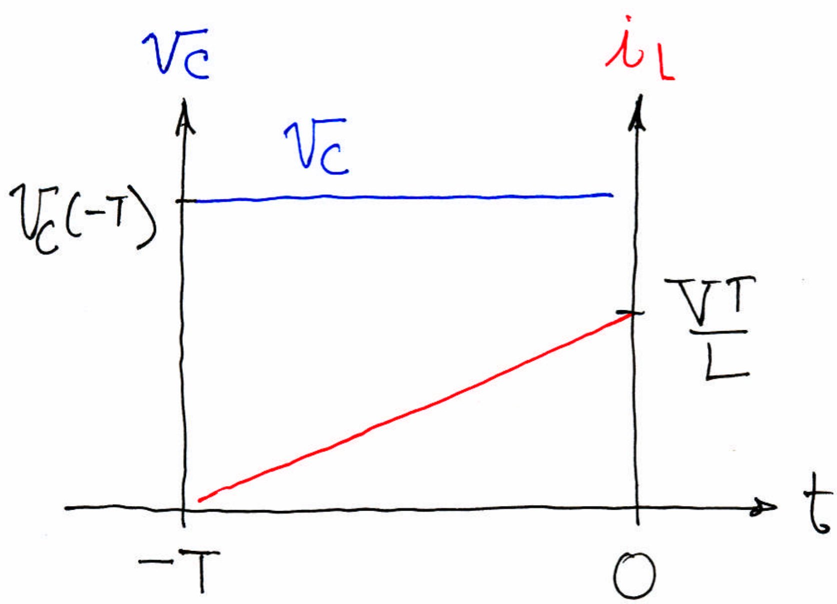
$-T \le t \le 0$

$i_L(-T) = 0$

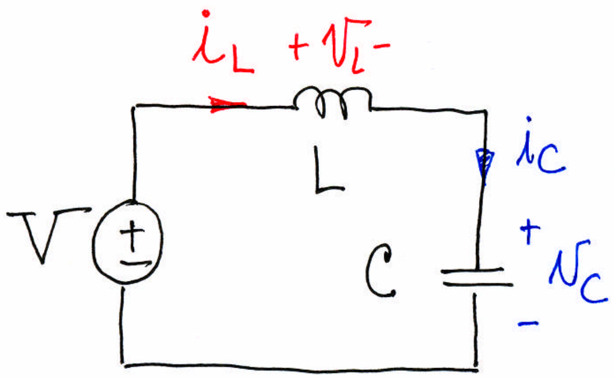
$v_C(-T) = \text{Given}$

$$V = v_L = L\frac{di_L}{dt} \quad \Rightarrow \quad i_L(t) = \frac{V}{L}(t+T)$$

$$0 = i_C = C\frac{dv_C}{dt} \quad \Rightarrow \quad v_C(t) = v_C(-T)$$



State II



$0 \le t \le$ Diode Turn Off

$i_L(0) = $ Given

$v_C(0) = $ Given

$$V = LC\frac{d^2 v_C}{dt^2} + v_C \quad \& \quad i_C = C\frac{dv_C}{dt}$$

$$v_C = V + A\sin\left(\frac{t}{\sqrt{LC}}\right) + B\cos\left(\frac{t}{\sqrt{LC}}\right)$$

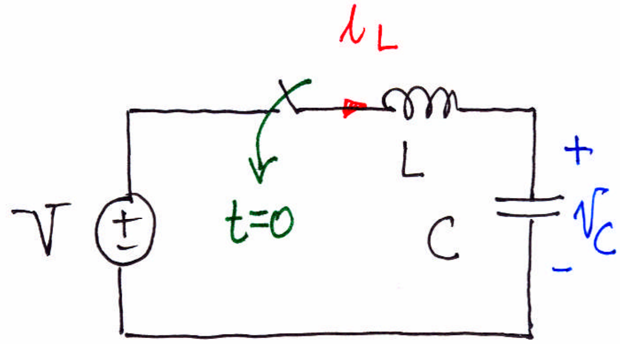
$$i_L = C\frac{dv_C}{dt} = \sqrt{\frac{C}{L}}A\cos\left(\frac{t}{\sqrt{LC}}\right) - \sqrt{\frac{C}{L}}B\sin\left(\frac{t}{\sqrt{LC}}\right)$$

$$v_C = V + \sqrt{\frac{L}{C}}\,i_L(0)\sin\left(\frac{t}{\sqrt{LC}}\right) + \left(v_C(0) - V\right)\cos\left(\frac{t}{\sqrt{LC}}\right)$$

$$i_L = i_L(0)\cos\left(\frac{t}{\sqrt{LC}}\right) - \sqrt{\frac{C}{L}}\left(v_C(0) - V\right)\sin\left(\frac{t}{\sqrt{LC}}\right)$$

Demo

Ideal:

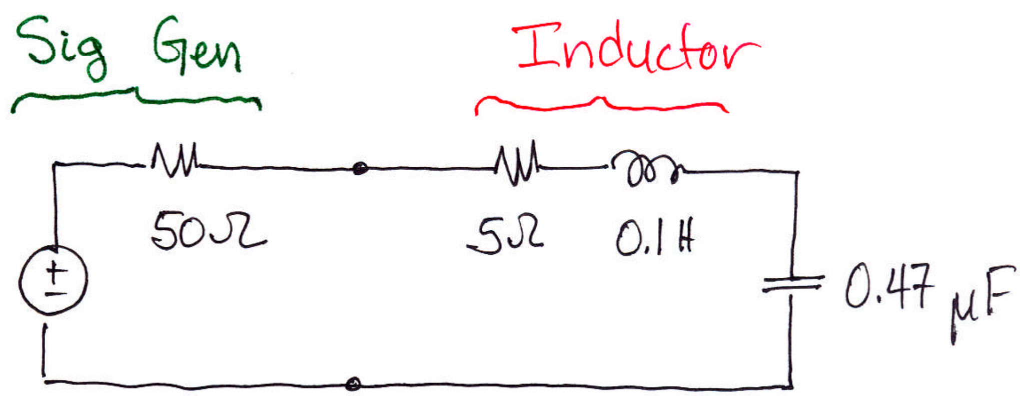


$i_L(0) = 0$

$v_C(0) = 0$

$$v_C = V\left(1 - \cos\left(\frac{t}{\sqrt{LC}}\right)\right) \qquad i_L = \sqrt{\frac{C}{L}}\, V \sin\left(\frac{t}{\sqrt{LC}}\right)$$

Actual:



$$2\pi\sqrt{LC} = 1.3 \text{ ms} \qquad \frac{\pi}{2}\sqrt{LC} = 0.34 \text{ ms}$$

$$Q = \frac{\sqrt{L/C}}{R} = \frac{461\,\Omega}{55\,\Omega} = 8.4 \Rightarrow \text{Underdamped}$$

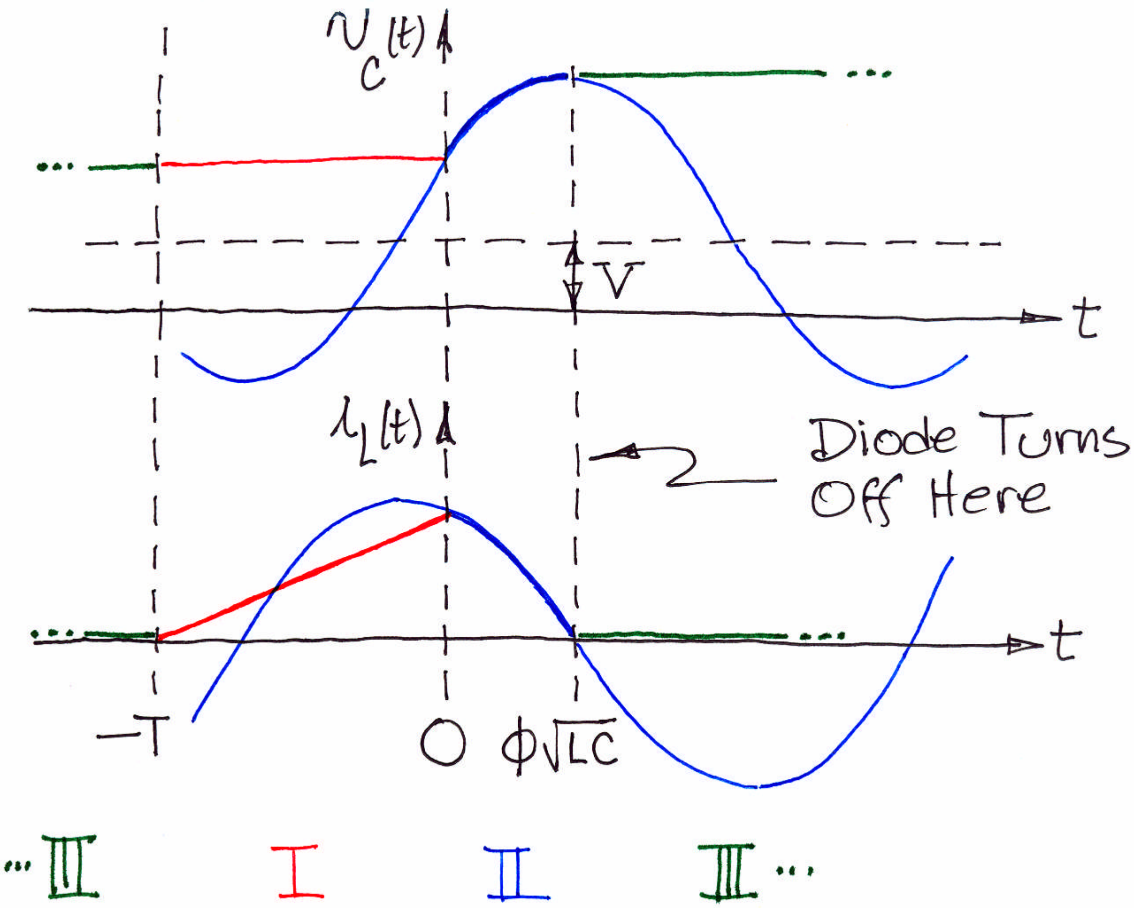
$$e^{-\frac{Rt}{2L}} \Rightarrow 50\% \text{ Decay Over } 2.5 \text{ ms}$$

All States Together

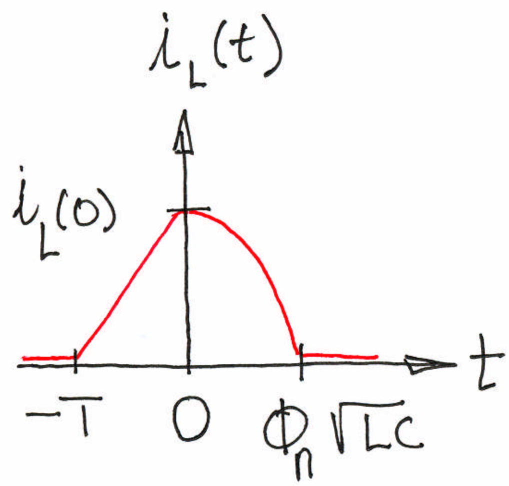
State II Review:

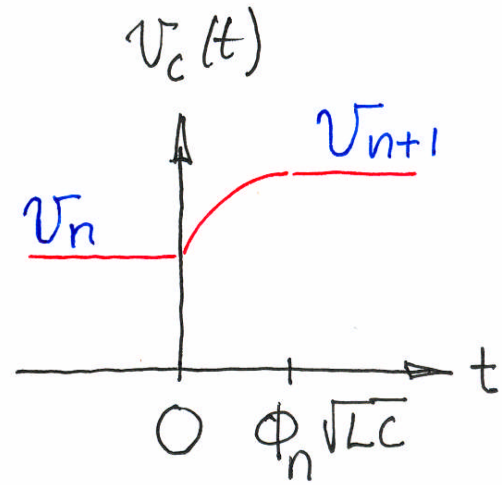
$$v_C(t) = V + \sqrt{(v_C(0)-V)^2 + \frac{L}{C} i_L^2(0)}\ \operatorname{Cos}\left(\frac{t}{\sqrt{LC}} - \underbrace{\operatorname{Tan}^{-1}\left(\sqrt{\frac{L}{C}}\frac{i_L(0)}{v_C(0)-V}\right)}_{\phi\ =\ \text{Positive}}\right)$$

$$i_L(t) = -\sqrt{\frac{C}{L}(v_C(0)-V)^2 + i_L^2(0)}\ \operatorname{Sin}\left(\frac{t}{\sqrt{LC}} - \operatorname{Tan}^{-1}\left(\sqrt{\frac{L}{C}}\frac{i_L(0)}{v_C(0)-V}\right)\right)$$



Cycle Analysis





$$v_{n+1} = V + \sqrt{(v_n - V)^2 + \frac{L}{C} i_L^2(0)}$$

$$= V + \sqrt{(v_n - V)^2 + \frac{V^2 T^2}{LC}}$$

$$(v_{n+1} - V)^2 = (v_n - V)^2 + \frac{V^2 T^2}{LC} \qquad (v_0 - V)^2 \equiv 0$$

$$v_n = V + V\sqrt{\frac{nT^2}{LC}}$$

Cycle Analysis Via Energy Conservation

Conservation: $\Delta W_C = -\Delta W_B + W_L$

Capacitor: $\Delta W_C = \frac{1}{2} C v_{n+1}^2 - \frac{1}{2} C v_n^2$

Inductor: $W_L = \frac{1}{2} L i_L^2(0)$

Battery: $-\Delta W_B = V \Delta Q$

Charge: $\Delta Q = C v_{n+1} - C v_n$

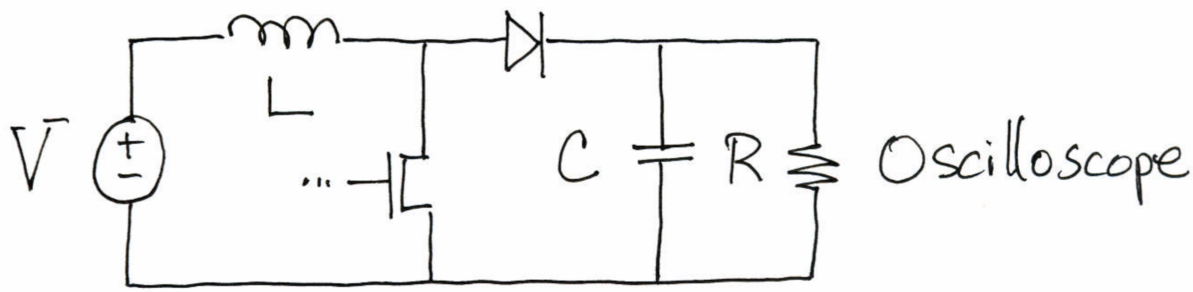
$$\frac{C}{2} v_{n+1}^2 - \frac{C}{2} v_n^2 = C V v_{n+1} - C V v_n + \frac{L}{2} i_L^2(0)$$

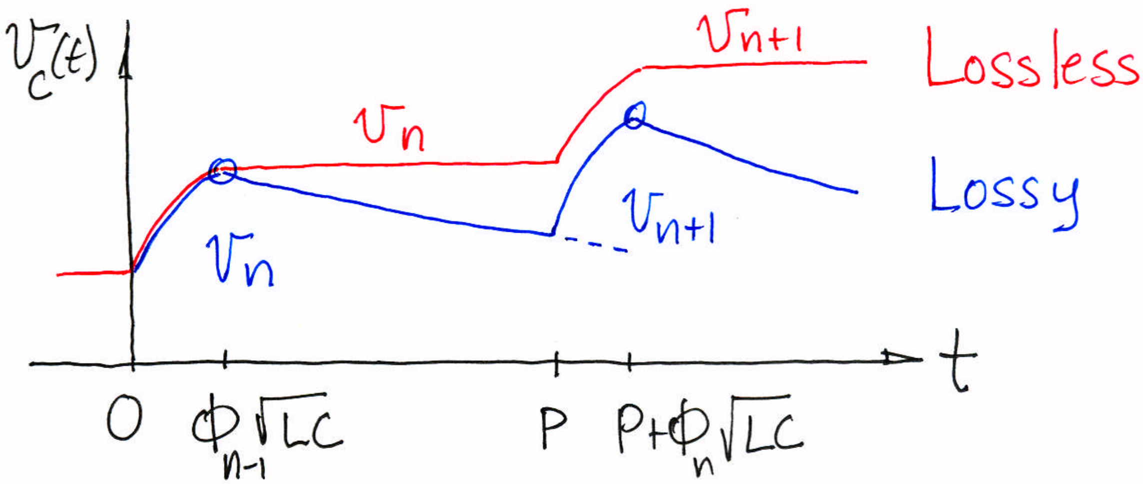
$$v_{n+1}^2 - 2 V v_{n+1} + V^2 = v_n^2 - 2 V v_n + V^2 + \frac{L}{C} i_L^2(0)$$

$$(v_{n+1} - V)^2 = (v_n - V)^2 + \frac{L}{C} i_L^2(0)$$

... As Before!

Demo





Assume $P \gg \phi_n\sqrt{LC} \Rightarrow v_n \to v_n e^{-P/RC}$
during energy conservation argument.

$$(v_{n+1} - V)^2 = (v_n e^{-P/RC} - V)^2 + \frac{L}{C} i_L^2(0)$$

$$v_n = V + \sqrt{(v_{n-1} \underbrace{e^{-P/RC}} - V) + \frac{V^2T^2}{LC}}$$

Except $n = 1$.

Demo Numbers

$L = 0.1\ H$

$C = 0.47\ \mu F$

$2\pi\sqrt{LC} = 1.36\ ms$

$\sqrt{L/C} = 460\ \Omega$

$V = 5.1\ V$

$R = 10\ M\Omega$

$T \approx 0.75\ \mu s$

$P = 50\ \mu s$

n	Predicted v_n [V]	Measured v_n [V]
0	5.1	5.1
1	23	24
2	30	30
3	35	35
4	40	39
5	44	42
6	47	44 (Clipped)