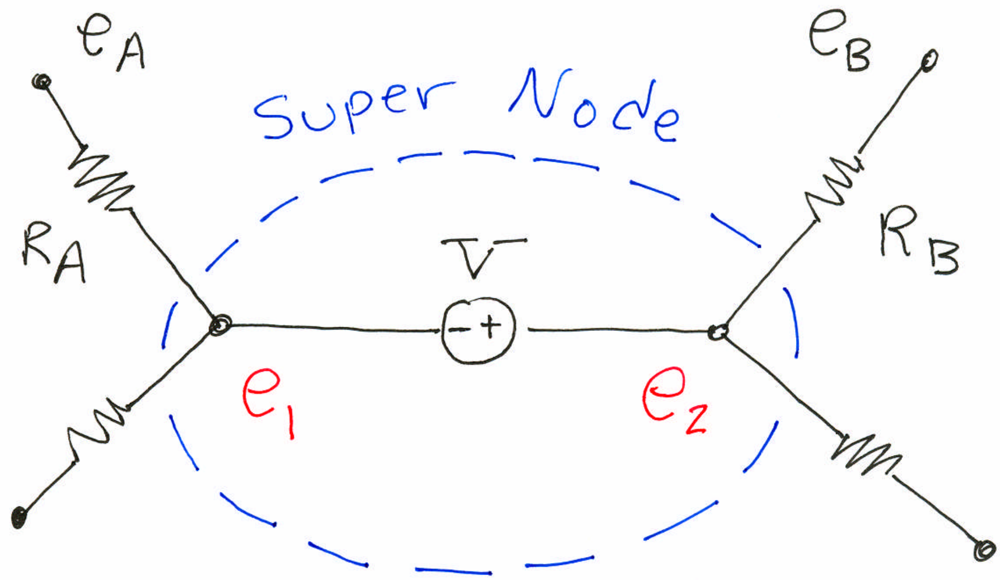
6.002 - Lecture 3

Circuit Analysis Simplifications

- Superposition
- Thevenin & Norton Equivalence

Node Analysis & Floating Voltage Sources

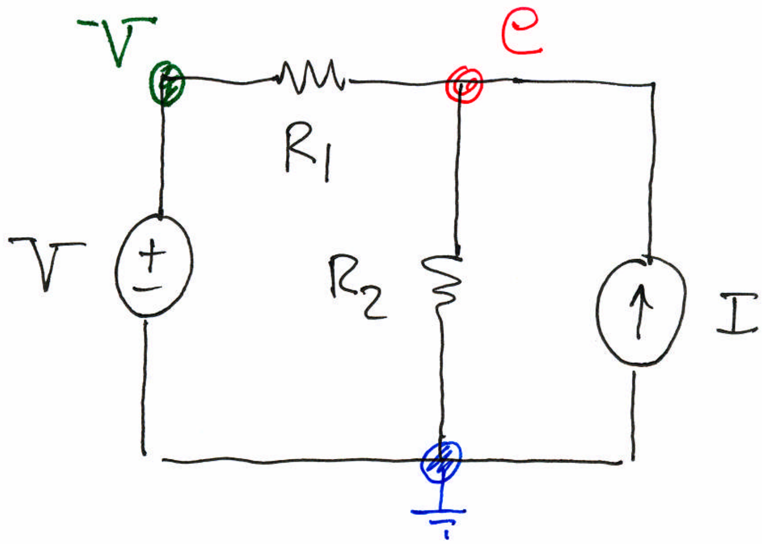


$$e_2 = e_1 + V$$

Write only one KCL for each super node.

$$\frac{e_1 - e_A}{R_A} + \frac{\overbrace{e_1 + V}^{e_2} - e_B}{R_B} + \cdots = 0$$

Simple Analysis Example



Linear in $\bar{e}$ and $\bar{i}$.

Conductance Matrix

Coefficient Matrix

$$\frac{e-V}{R_1}+\frac{e}{R_2}-I=0$$

$$e\left(\frac{1}{R_1}+\frac{1}{R_2}\right)=\frac{V}{R_1}+I \quad \cdots\to \quad G\bar{e}=H\bar{i}$$

Node Voltage Vector

Source Vector

$$e=\frac{R_2}{R_1+R_2}V+\frac{R_1R_2}{R_1+R_2}I$$

Linearity $\Rightarrow$ Homogeneity & Superposition

Homogeneity: If $\bar{x} \longrightarrow \boxed{\text{Linear System}} \longrightarrow \bar{y}$

Then $\alpha\bar{x} \longrightarrow \boxed{\text{Linear System}} \longrightarrow \alpha\bar{y}$

Superposition: If $\bar{x}_1 \longrightarrow \boxed{\text{Linear System}} \longrightarrow \bar{y}_1$

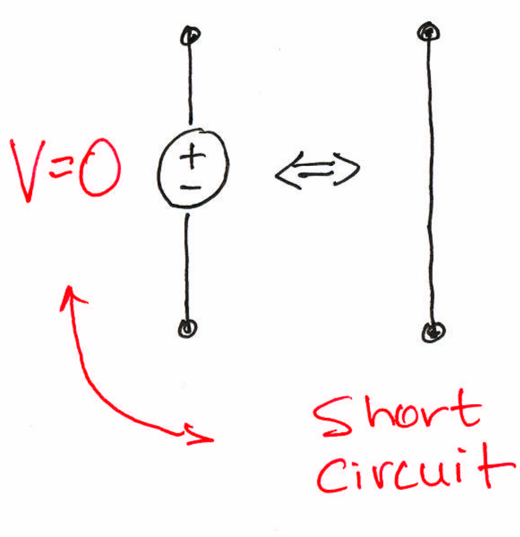
$\bar{x}_2 \longrightarrow \boxed{\text{Linear System}} \longrightarrow \bar{y}_2$

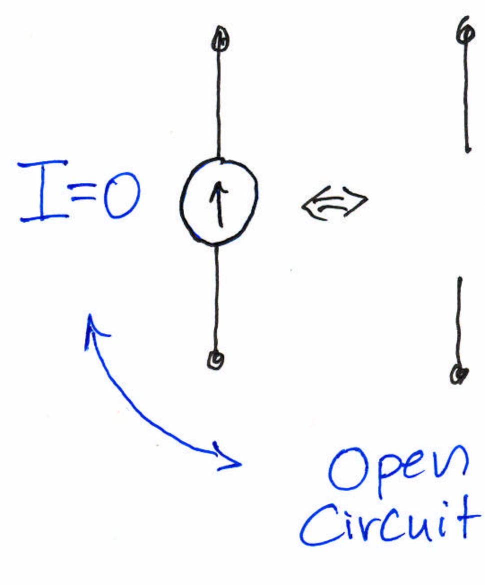
Then $\bar{x}_1 + \bar{x}_2 \longrightarrow \boxed{\text{Linear System}} \longrightarrow \bar{y}_1 + \bar{y}_2$

Divide & Conquer

Use superposition to analyze a linear circuit one source at a time. (All sources except one are set to zero each time.)

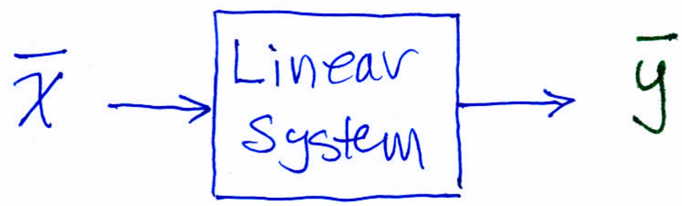
How to set a source to zero?

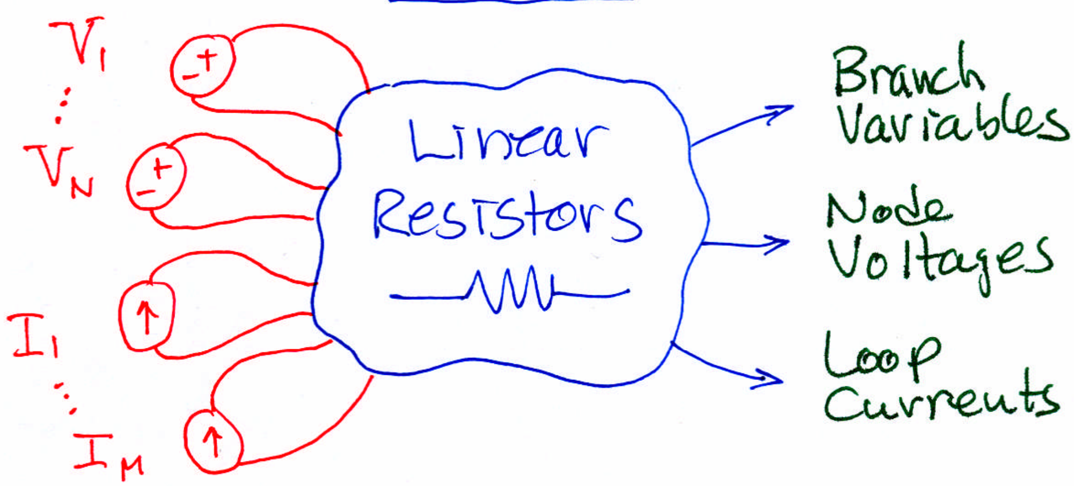




Linear Networks & Systems

Linear system $\Rightarrow$ all stimuli are external inputs. For example, $y = x_1 + x_2$ is linear from $\bar{x} \to y$, but $y = x_1 + x_2 + 1$ is not.

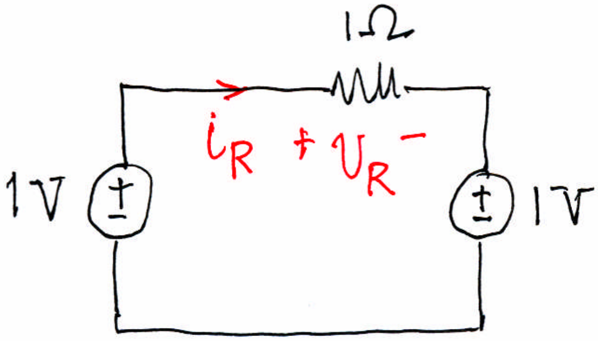




No source should be left within the network for the purposes of superposition. Superposition is carried out over all sources.

No Superposition For Power

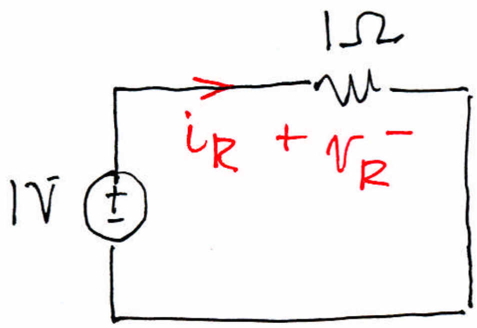
Power is a nonlinear network property, and so the superposition of power is not possible.



$v_R = 0\ \text{V}$

$i_R = 0\ \text{A}$

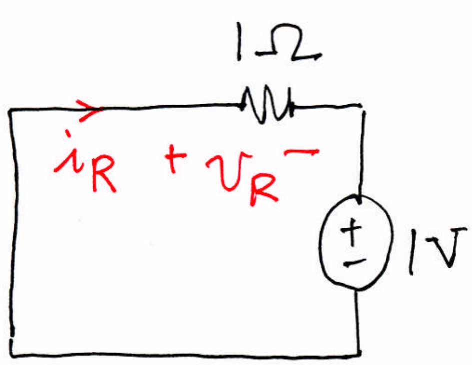
$v_R i_R = 0\ \text{W}$



$v_R = 1\ \text{V}$

$i_R = 1\ \text{A}$

$v_R i_R = 1\ \text{W}$

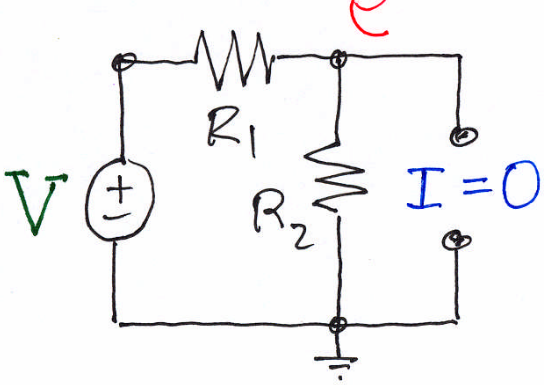


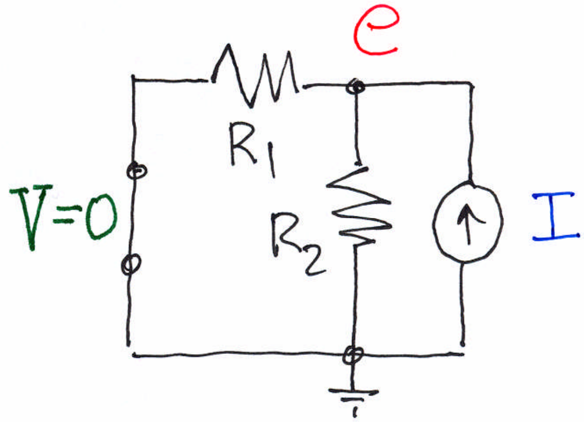
$v_R = -1\ \text{V}$

$i_R = -1\ \text{A}$

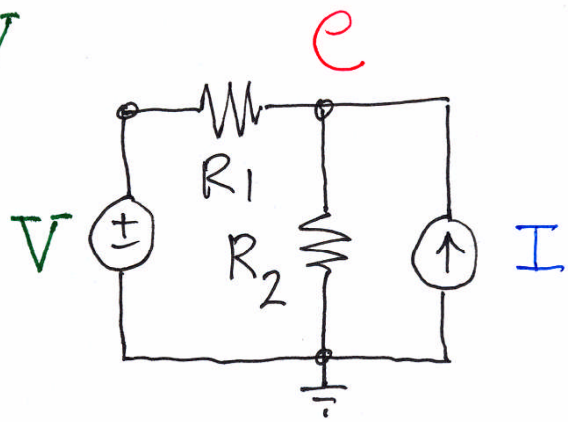
$v_R i_R = 1\ \text{W}$

Superposition Examples



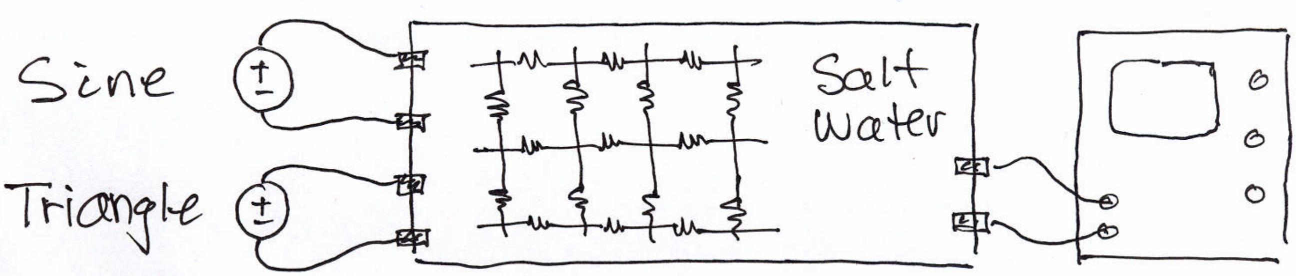


$$e = \frac{R_2}{R_1+R_2} V$$

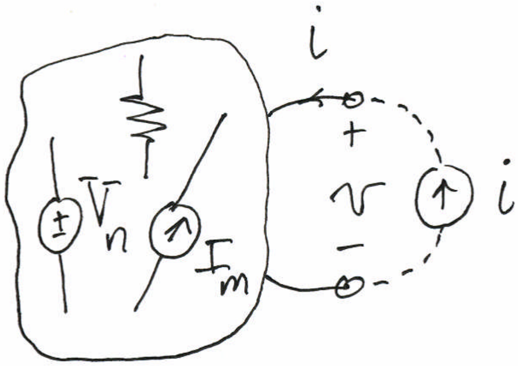


$$e = \frac{R_1 R_2}{R_1+R_2} I$$

$$e = \frac{R_2}{R_1+R_2} V + \frac{R_1 R_2}{R_1+R_2} I$$

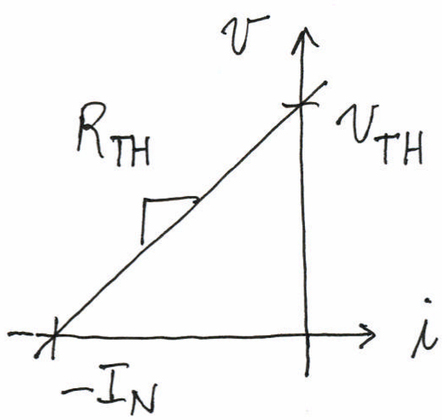


Thevenin & Norton Equivalence



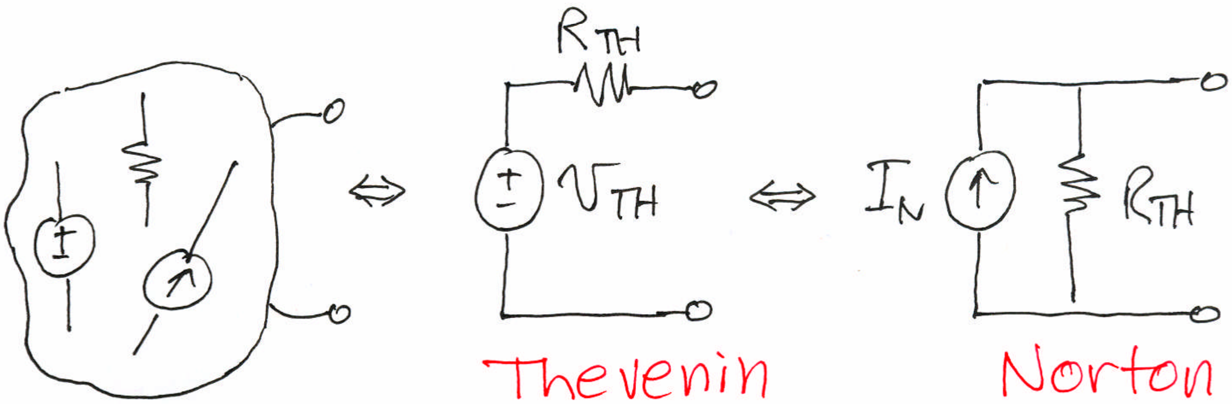
Linear Network

$$v = \underbrace{\sum_n \alpha_n \bar{V}_n + \sum_m \beta_m I_m + R_{TH}\, i}_{\text{Superposition}}$$



Open Circuit $(i=0)$ $\Rightarrow$ $v_{TH} = \sum_n \alpha_n \bar{V}_n + \sum_m \beta_m I_m$

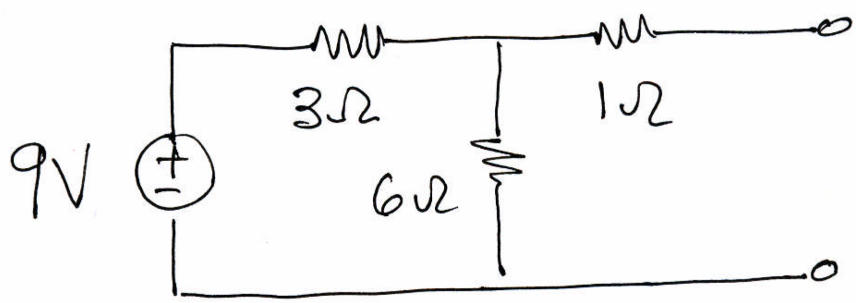
Short Circuit $(v=0)$ $\Rightarrow$ $I_N = \dfrac{v_{TH}}{R_{TH}}$



Thevenin

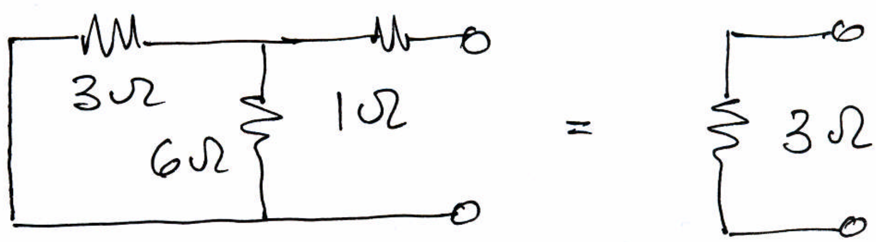
Norton

THEVENIN / NORTON EXAMPLE

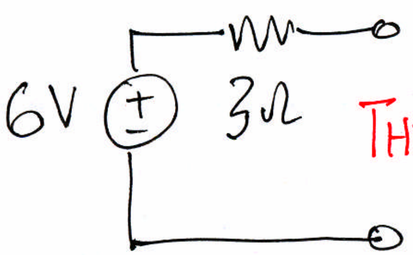


Open-circuit voltage = 6 V via voltage divider ⇒ $v_{TH} = 6\ V$

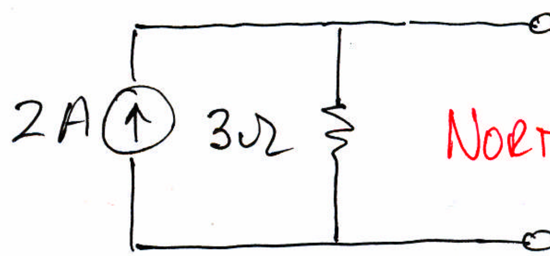
R_{TH} ⇒



$I_N = v_{TH} / R_{TH} = 2\ A$



THEVENIN



NORTON

3 - Lecture Summary

- Two-terminal devices: V, I, R
- KCL and KVL
- Brute-force analysis
- Node analysis
- Parallel and series reductions
- Voltage and current dividers
- Superposition
- Thevenin and Norton equivalence