6.002 - Lecture 9

Small-Signal Models

- Small-Signal Analysis Problem
- Small-Signal Model Solution
- Small-Signal MOSFET Model
- S-S MOSFET Amplifier Model

The Problem

- Small-signal analysis yields valuable insight. But ...

- Small-signal analysis via expansion or differentiation requires a complete closed-form nonlinear analysis.

- The expansion or differentiation can be very complex for a large circuit.

- Numerical analysis is possible but it yields little insight.

The Solution

- Rather than analyze first and then linearize the analysis, linearize first and then analyze.

- Remove the biases from the linearization to uncover the small-signal model.

- The linearization still requires an operating point, but it can come from a non-linear analysis, a numerical analysis, an approximate analysis, or an experiment.

Mathematical Example

$$x \longrightarrow \boxed{f(\cdot)} \xrightarrow{\;y = f(x)\;} \boxed{g(\cdot)} \longrightarrow z = g(f(x))$$

Bias: $Y = f(X) \quad , \quad Z = g(Y) = g(f(X))$

Analyze First:

$$\Delta z \approx \frac{d}{dx}\left(g(f(x))\right)\Big|_X \cdot \Delta x$$

$$\approx \frac{dg}{dy}\Big|_Y \cdot \frac{dy}{dx}\Big|_X \cdot \Delta x$$

$$z \approx Z + \frac{dg}{dy}\Big|_Y \cdot \frac{dy}{dx}\Big|_X \cdot \Delta x = Z + \Delta z$$

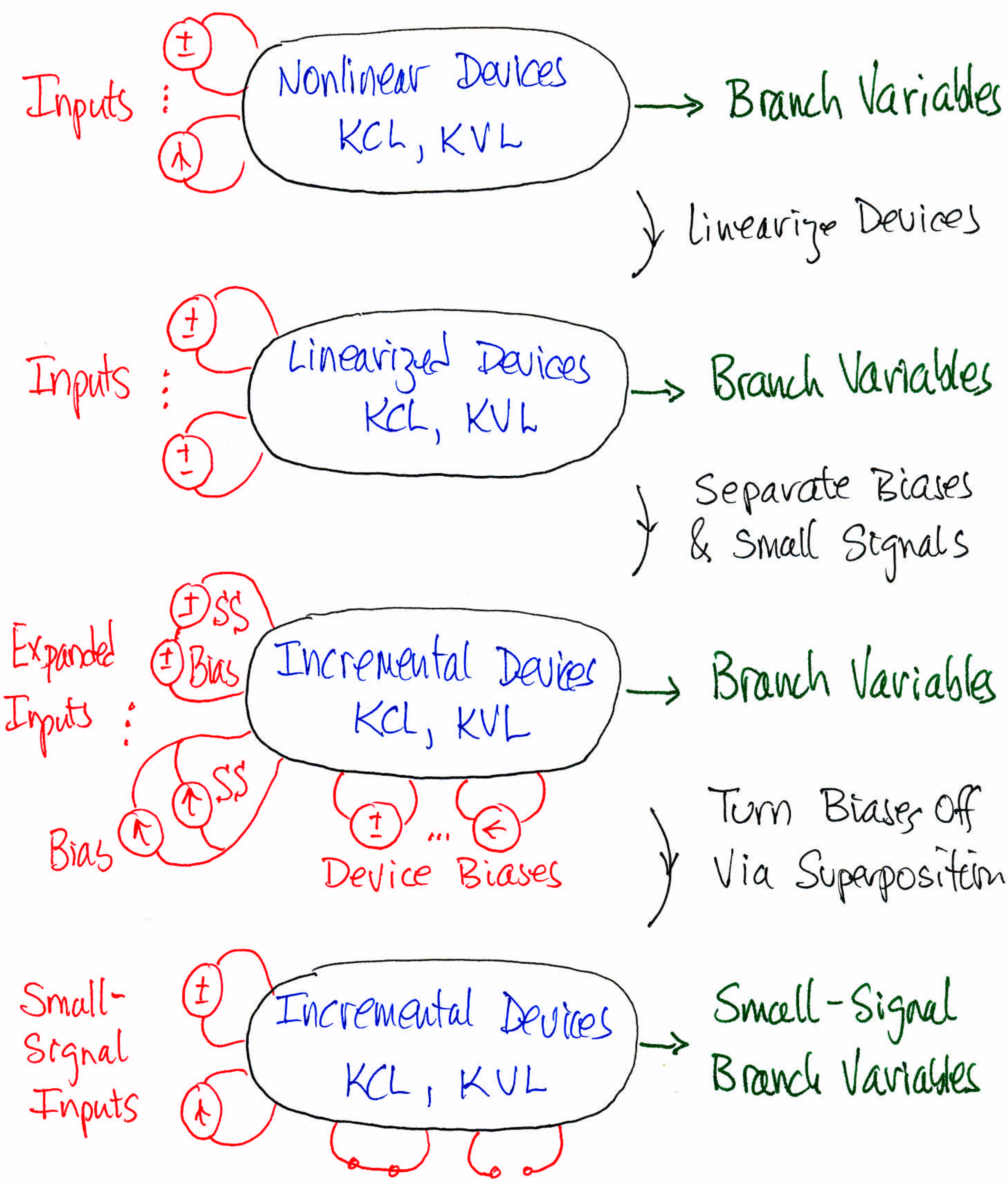
Linearize First:

$$\Delta y \approx \frac{df}{dx}\Big|_X \Delta x \quad , \quad \Delta z = \frac{dg}{dy}\Big|_Y \Delta y$$

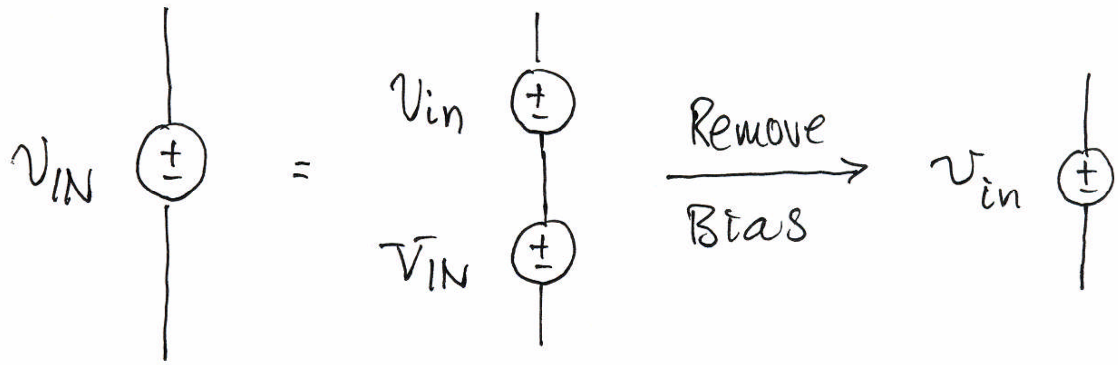
$$z \approx Z + \Delta z = Z + \frac{dg}{dy}\Big|_Y \cdot \frac{df}{dx}\Big|_X \Delta x$$

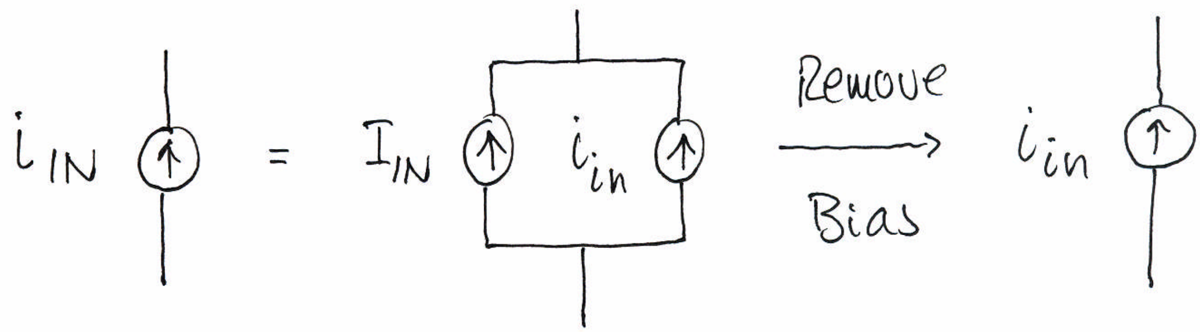
For circuit analysis, f and g represent KCL, KVL and the device laws.

Small-Signal-Model Method

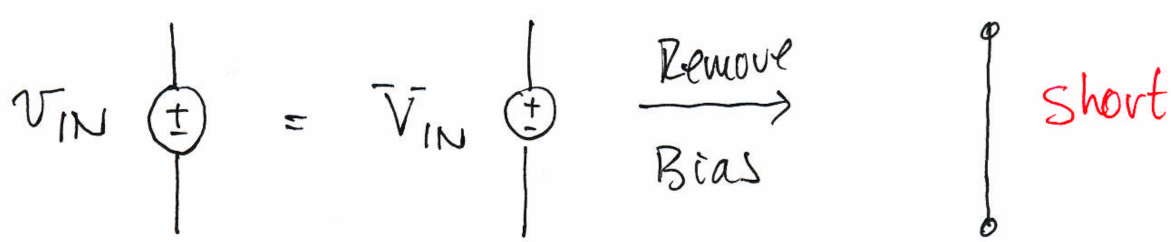


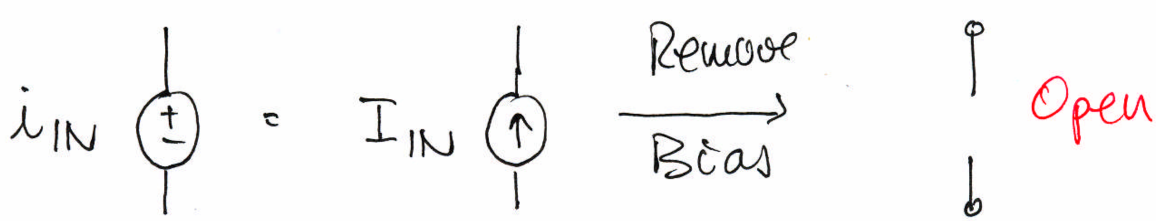
Treatment Of Independent Sources



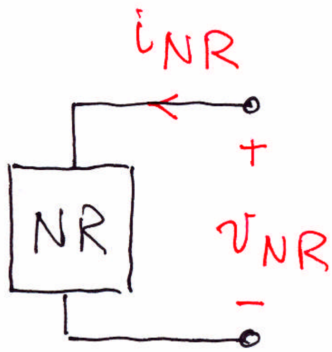


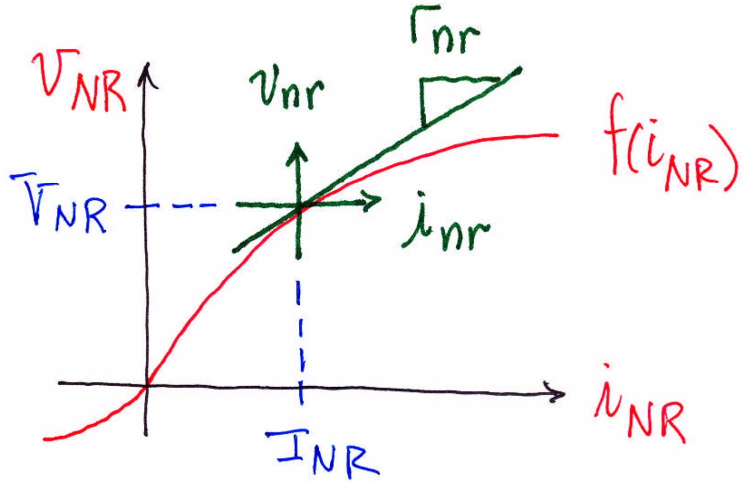
If the independent source has
no small-signal component, then ...





Linearizing A Nonlinear Resistor





Bias: $\quad V_{NR} = f(I_{NR})$

Off Bias: $\quad v_{NR} = f(i_{NR})$

$$= V_{NR} + v_{nr}$$

$$= f(I_{NR} + i_{nr})$$

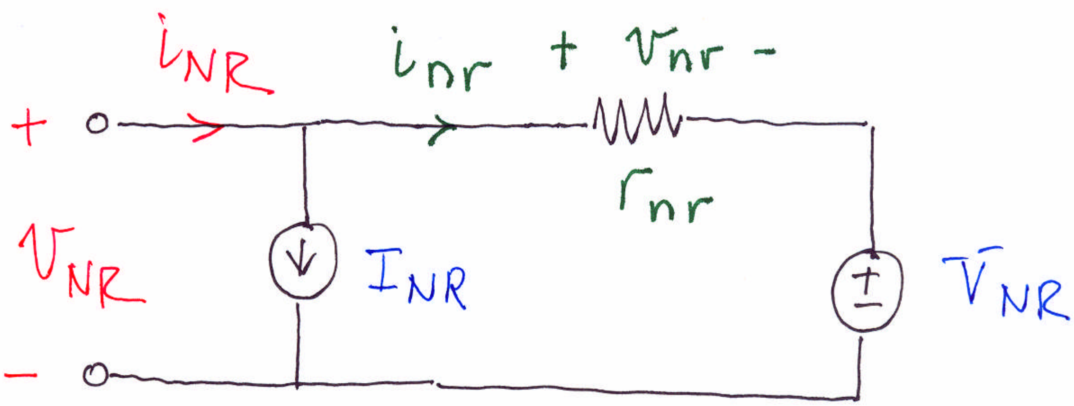
$$\approx f(I_{NR}) + \left.\frac{df}{di_{NR}}\right|_{I_{NR}} \cdot i_{nr}$$

$$= V_{NR} + r_{nr} \cdot i_{nr}$$

$$= V_{NR} + r_{nr}(i_{NR} - I_{NR})$$

Nonlinear-Resistor Small-Signal Model

$$v_{NR} = V_{NR} + r_{nr}\,(i_{NR} - I_{NR})$$

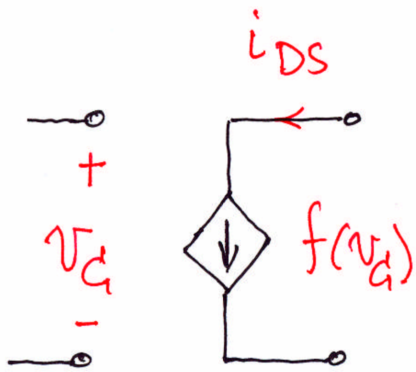


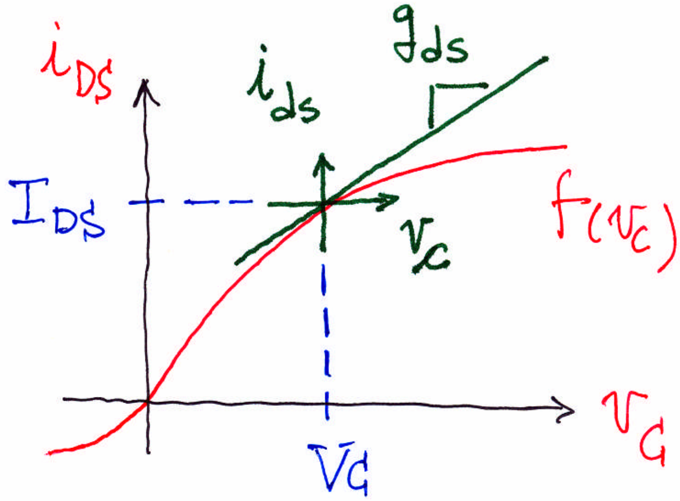
Set Biases To zero

Alternatively: $V_{NR} + v_{nr} = V_{NR} + r_{nr}\, i_{nr}$

Bias Cancels

Linearizing A Dependent Source





Bias: $I_{DS} = f(V_C)$

Off Bias: $i_{DS} = f(v_G)$

$$= I_{DS} + i_{ds}$$

$$= f(V_C + v_c)$$

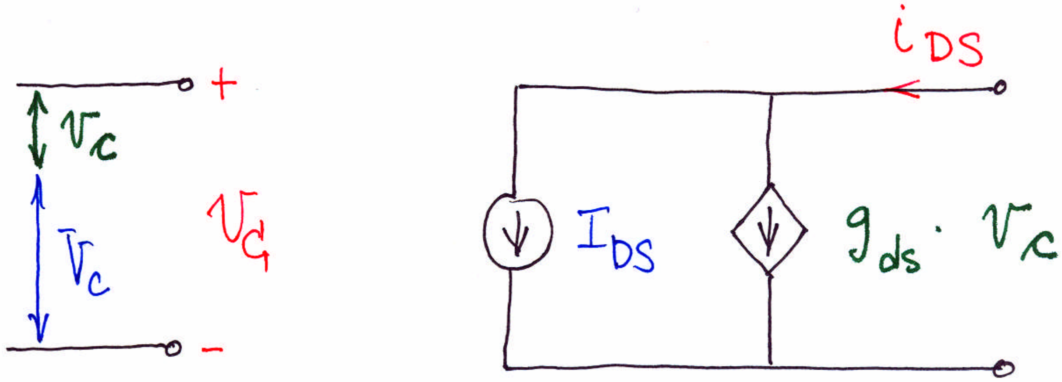
$$\approx f(V_C) + \frac{df}{dV_C}\Big|_{V_C} \cdot v_c$$

$$= I_{DS} + g_{ds} \cdot v_c$$

$$= I_{DS} + g_{ds}(v_C - V_G)$$

Dependent-Source Small-Signal Model

$$i_{DS} = I_{DS} + g_{ds}\,(v_G - V_G)$$

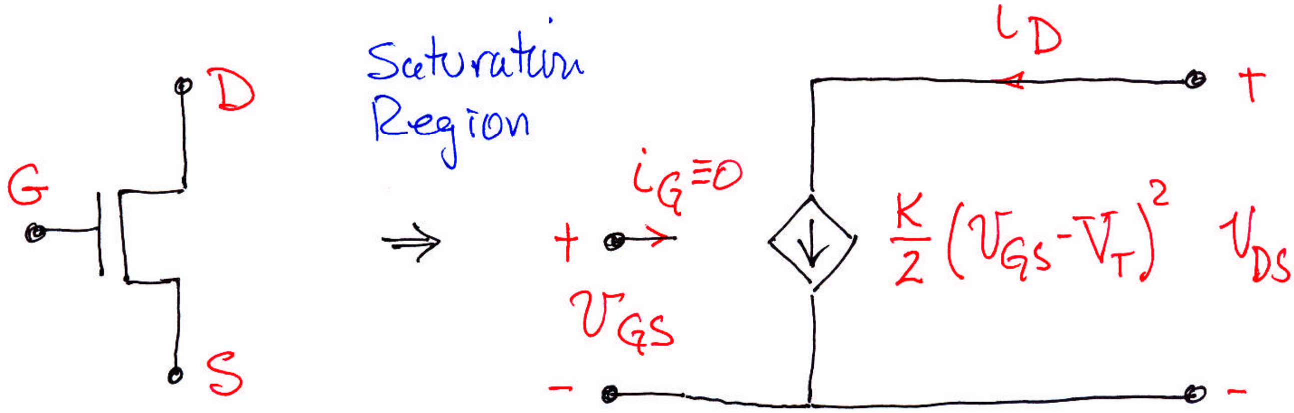


Set Biases To zero

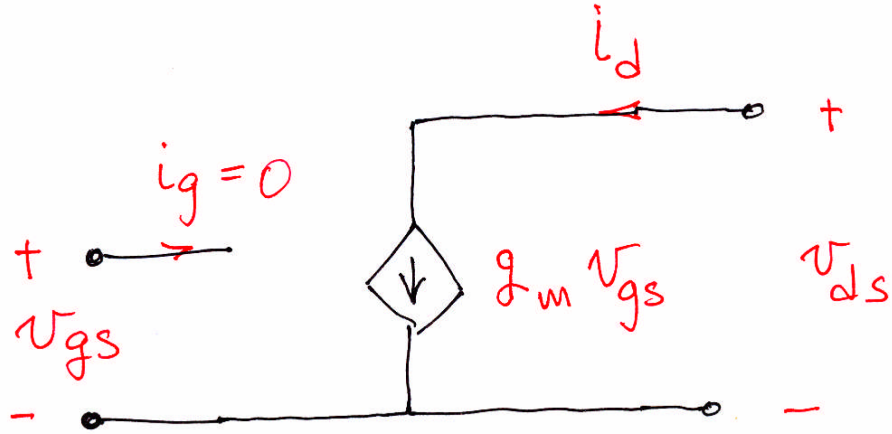
Alternatively: $\quad I_{DS} + i_{ds} = I_{DS} + g_{ds}\, v_c$

Biases Cancel

MOSFET Small-Signal Model

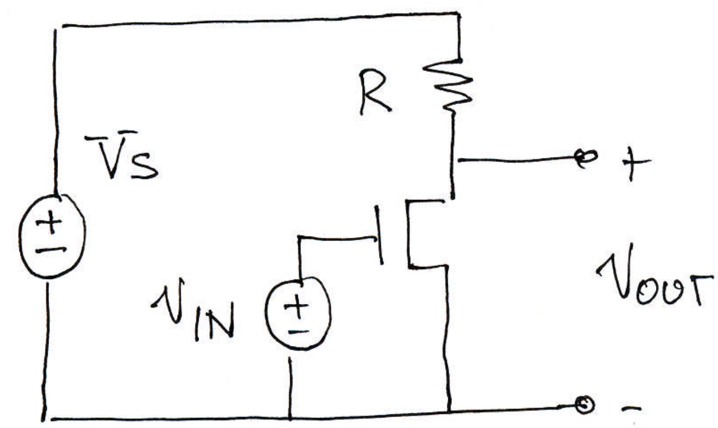


Small-Signal Model

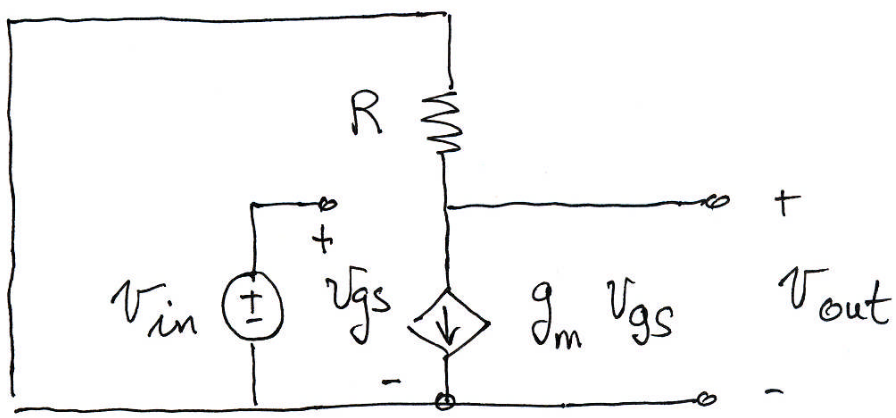


$$g_m = K(V_{GS}-V_T)$$

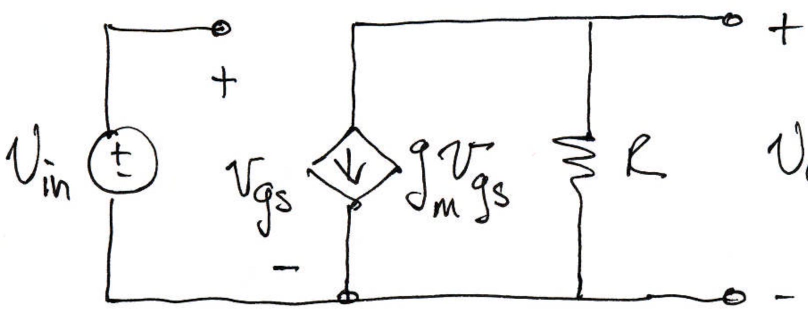
MOSFET Amplifier S-S Model



Use Small-Signal Device Models



Re-draw



$$v_{out} = \overbrace{-g_m R}^{\text{Small-Signal Gain}} v_{in}$$

$K(V_{IN}-V_T)$