

Workshop: Using Visualization in Teaching Introductory E&M

AAPT National Summer Meeting, Edmonton, Alberta, Canada.

Organizers: John Belcher, Peter Dourmashkin, Carolann Koleci, Sahana Murthy

Faraday's Law Application

Learning objectives for workshop:

1. Understand how to incorporate a visualization activity into course materials.
2. Provide a model for designing new activities based on visualizations.
3. Understand how students experience a visualization activity.
4. Explore interesting physics using the application.
5. Participate in group learning and peer instruction.
6. Give feedback and suggestions for improving visualizations.

Learning Objectives for physics courses:

1. Understand magnetic flux.
2. Find different ways to induce current in a coil.
3. Find a relationship between flux, change in flux and induced current.
4. Test the relationship(s) found in 3.
5. Understand induced force using magnetic fields

1. Getting familiar with the application. (5 minutes)

Start the [Faraday's Law](#) application. From the "Actions" menu, you can choose between "Manual Mode" and "Generator Mode". In "Manual Mode", you can move the magnet or the ring (by clicking and dragging). You can use the buttons underneath the ring and magnet to start, pause and reset the simulation. You can vary the resistance of the ring and the strength of the magnetic dipole moment with the slide bars on the upper right. The "Generator Mode" simulates a simple AC (Alternating Current) generator. The physics is unchanged, but the magnet oscillates sinusoidally giving rise to a near-sinusoidal current in the ring.

You can choose between two different field visualizations: the iron filings or the field lines. The upper graph contains plots of external magnetic flux and total magnetic flux versus time. The lower graph plots the current in the ring as a function of time.

Play with the application until you are familiar with all the features.

2. Preliminary Questions: (10 minutes)

- a) What is the source of the external magnetic flux (red plot)? Over what area is the flux being measured?
- b) What choices determine the sign of the flux? Does it matter which way the magnet is oriented?
- c) Describe different ways that you can change the external flux
- d) Explain how the total magnetic flux (blue plot) is related to the external magnetic flux (red plot).
- e) How is the choice of sign for the current connected to the choice of sign for the flux?

3. Proposing a hypothesis. (5 minutes)

Your next goal is to propose a qualitative relationship between magnetic flux (seen in top graph) and current that flows in the ring (seen in bottom graph).

4. Testing hypotheses. (10 minutes)

Four groups utilizing the application came up with the following hypotheses.

Group A conjectured that the current through the ring is proportional to the total magnetic flux.

Group B proposed that the current through the ring is proportional to the *change* in the external magnetic flux.

Group C thought that the current through the ring is proportional to the *change* in the total magnetic flux.

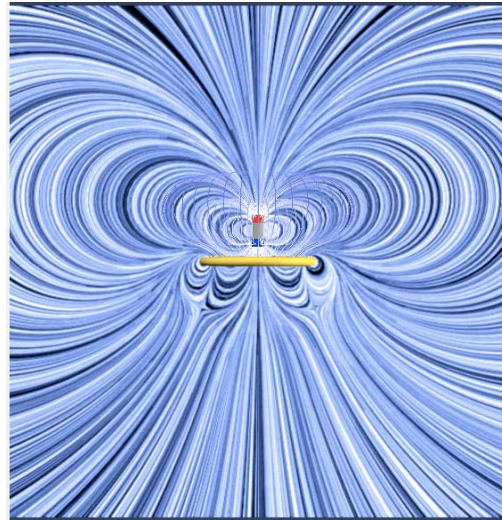
Group D proposed that the current is proportional to the difference between the total flux and external flux.

Use the application to test these various hypotheses. Design and run a virtual experiment that could rule out any of the hypotheses. Which did you rule out and why?

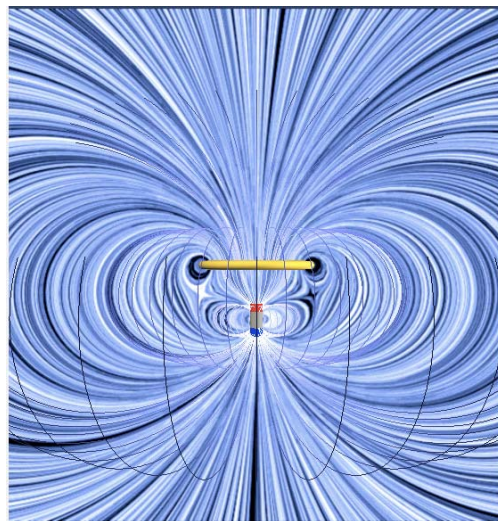
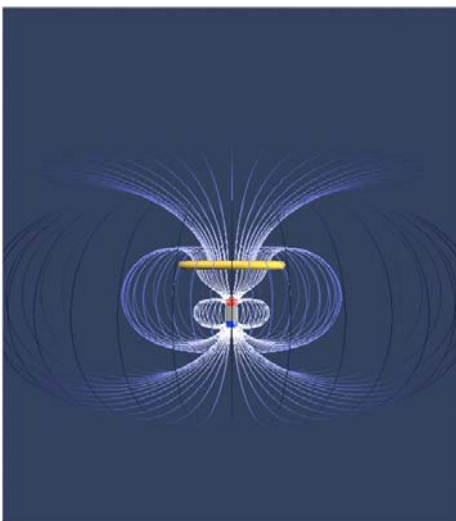
5. Visualize the force (10 minutes)

Open the following application: [Falling Magnet](#). Set the resistance in the ring to a small value, and set the magnetic strength to a high value. Start the application and observe the field lines as the magnet oscillates and eventually falls through the ring.

Restart the application and click on pause button when the magnet is being repelled upwards. You will see a field line pattern similar to the figure below on the left. Click on the iron filings bar and you will see a field pattern like the figure below on the right. Notice that the field lines are under compression pushing the magnet upwards.



Continue the application and allow the magnet to fall through the ring. When it is several magnet lengths below the ring pause the application. You will see a field line pattern similar to the figure below on the left. Click on the iron filings and you will see a field pattern like the figure below on the right. The field lines are now under tension attracting the magnet to the ring.



Exploration. (if you have time)

Try different values for resistance and magnetic strength. Stop the motion at various points where the magnitude of the induced current is largest and look at the field lines and iron filings visualizations. Discuss with your partner whether the fields lines are under tension or compression and whether this means that the force exerted on the magnet is attractive or repulsive.

Open Exercise:

What other physics question can you investigate using this application? Think of a question and try to explore the answer using the application.

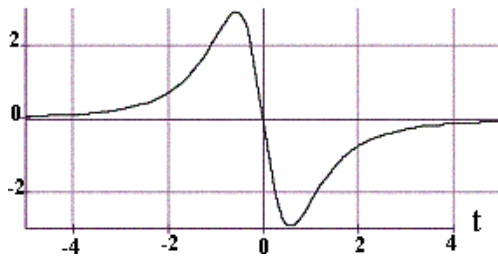
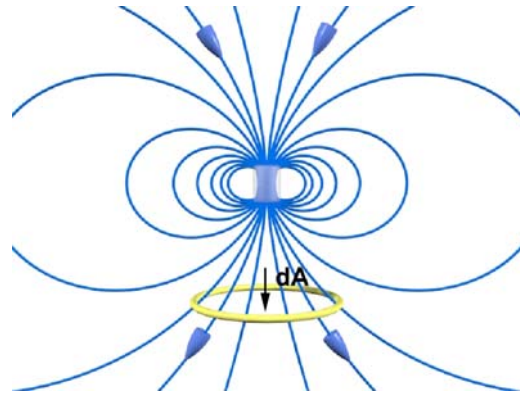
End of main activity. If you have finished with the activity, please go through the sample assessment questions (next page onwards) based on Faraday's law application.

Sample questions and problems based on the Faraday's law visualization.

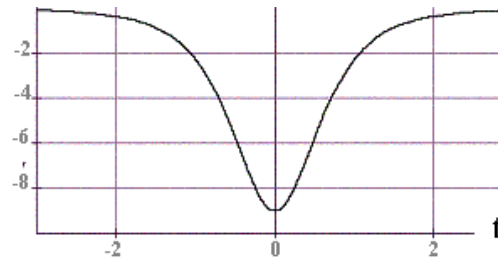
Example 1. ConcepTest.

To be discussed in this workshop. We typically use this question in class as a ConcepTest with clickers, after students have worked through the after the Faraday's law application activity.

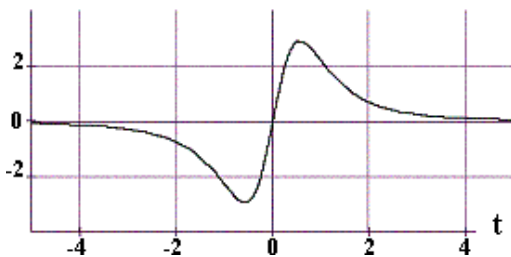
A coil of wire with resistance R defines an open surface whose normal $d\vec{A}$ points downward, as shown in the sketch. The coil is below a magnet whose south pole is on the top (see figure). We define positive current as clockwise as viewed from the top, and ignore any self-induced magnetic field generated by the induced current. As the coil moves from well below the magnet to well above that magnet, the induced current through the coil will look like:



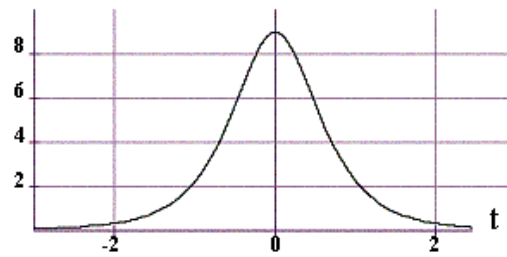
(1)



(2)



(3)



(4)

(5) I don't know.

Try to answer the ConcepTest using the application.

Solution to example 1: (3).

When the coil is moving up but before it reaches the center of the magnet, the external flux through the coil is positive (magnetic field and $d\vec{A}$ are both downward) and increasing. So the induced current will flow counterclockwise (as seen from above) in order to produce an induced flux upward through the coil. According to our choice of sign convention for the current, this induced current is negative.

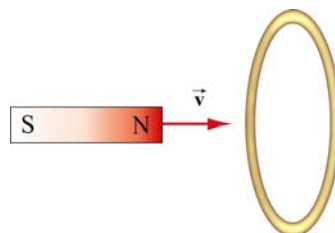
After the coil passes the center of the magnet, the external flux is still positive but now decreasing. So the induced current now flows clockwise (as seen from above) which produces an induced magnetic field downwards that opposes the change. According to our sign convention, the induced current is positive.

So the graph of the current vs. time as the coil moves from well below the magnet to well above that magnet is (3).

Example 2. Graphical representations.

This problem helps students work through multiple representations – pictures, equations, words, and graphs. We have used this question as an in-class collaborative group problem and as a homework problem.

A bar magnet is pulled through a stationary conducting coil of wire at constant speed, as shown in the figure to the right. It moves from a point far to the left of the coil, then through the center of the coil, to a point far to the right of the coil.



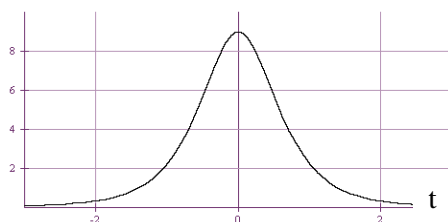
Assume that the north pole of the magnet enters the coil of wire first, and that the center of the magnet is at the center of the coil at time $t = 0$.

(a) Sketch qualitatively a graph of the magnetic flux due to the magnet Φ_B through the coil as a function of time.

Solution: The external flux through the coil due to the magnetic field of the magnet is:

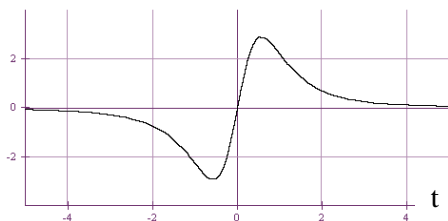
$$\Phi_{ext}^B = \iint_{\text{ring}} \vec{B}_{ext} \cdot d\vec{a}.$$

Choose the positive direction for unit normal to the coil pointing to the right. Then the dot product $\vec{B}_{ext} \cdot d\vec{a} > 0$ and hence the external flux is positive. The plot of flux vs. time looks like:



(b) Sketch qualitatively a graph of the current in the coil as a function of time. Take the direction of positive current to be clockwise as viewed from the left (note that this choice is consistent with the the choice of unit normal to the coil using the right hand rule).

Solution: The plot of current vs. time looks like:

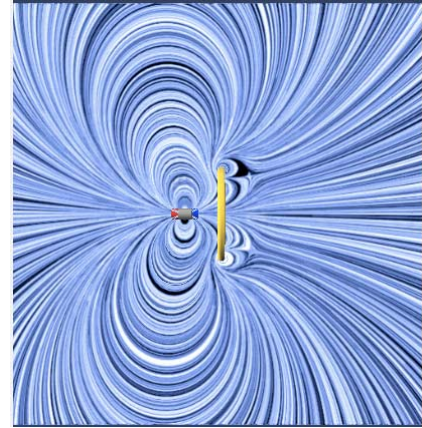


Remark: If you choose the opposite sign conventions for the unit normal and the direction of positive current, then your plots in (a) and (b) are mirror images with respect to the t -axis.

Example 3. Discussion Question.

We might use this question as an in-class discussion, after students have worked through the Faraday's law application activity, and after the instructor has discussed Lenz's law.

(a) A magnet enters a ring from the right. The iron filings representation of the total magnetic field is shown in the figure on the left. Determine the direction of the force exerted on the permanent magnet due to the current in the ring. Is your answer consistent with Lenz's law?

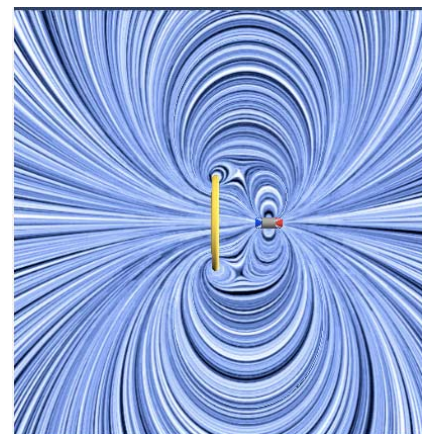


Solution: The magnetic field lines are pushing the magnet away from the ring indicating that the force is repulsive. Suppose the north pole of the magnet approaches the ring. Then the flux through the ring is increasing so an induced current flows counterclockwise (as seen from the left) generating magnetic flux through the ring that opposes the increasing flux due to the approaching magnet (Lenz' Law). We can model the current ring as a magnetic dipole with the dipole moment pointing to the left. Hence the permanent magnet and the current ring have north poles facing each other producing a repulsive force on the permanent magnet. This is consistent with Lenz' Law that is the statement that the system will oppose change. As the magnet approaches the ring, the force on the magnet must be resistive so the force on the magnet points to the left.

If it is the south pole of the magnet that approaches the ring, the force is still repulsive (can you see why?).

(b) What is the direction of the force on the magnet just after it has exited the ring?

Solution: The magnetic field lines are now pulling the ring and the magnet together so the force exerted on the magnet is attractive and hence to left. The induced current has changed direction and now flows clockwise (as seen from the left). Thus the dipole moment of the current ring points to the right. Again, we model the current ring as a magnet, with the north pole facing to the right, hence the permanent magnet and the dipole have opposite poles facing each other indicating that the magnetic force is now attractive.



Example 4. Quantitative Problem based on Faraday's law application.

This is an example where the Faraday's law application can be directly used to calculate properties of the ring. Students will learn to collect, analyze and interpret data from the application, just as they would with an experimental set-up. We might use this problem as an in-class (or lab) follow-up to the Faraday's law application activity, or as a homework problem.

Use the [Faraday's Law Application](#) to determine the self-induction of the ring. Your response should include a strategy for how you plan to solve this problem, any values of relevant quantities you obtained from the application, and a discussion of whether or not your result agrees with your expectations.

Solution:

The self-induction of the ring is defined as:

$$L \equiv \Phi_{ring}^B / I_{ring} .$$

When the magnet is moved relative to the ring, an induced current flows in the ring generating an induced flux through the ring. This induced flux through the ring is given by:

$$\Phi_{ring}^B = \Phi_{ind}^B = \Phi_{total}^B - \Phi_{ext}^B$$

The total flux and the external flux can be determined from the application. The self induction of the ring is therefore:

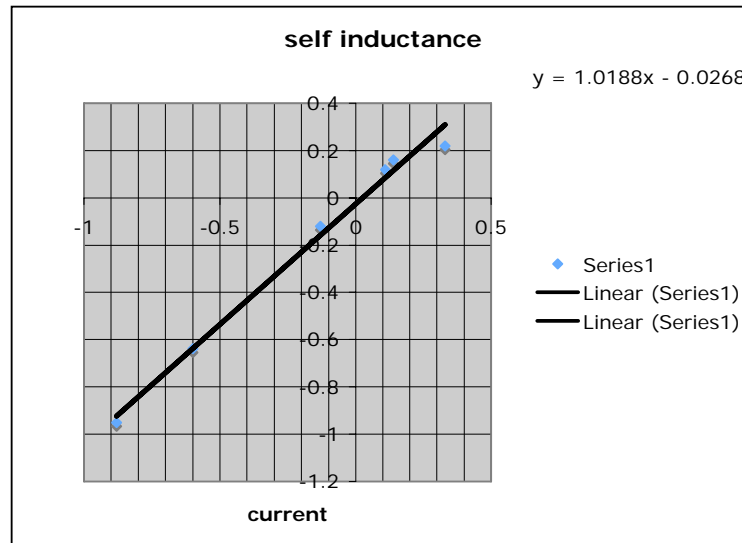
$$L = (\Phi_{total}^B - \Phi_{ext}^B) / I_{ring}$$

Data from Simulation:

$$\mu_B = 3.19 \text{ N-m/T}$$

| Total flux (W) | Ext flux (W) | Induced flux (W) | Current (A) | Inductance (H) |
|----------------|--------------|------------------|-------------|----------------|
| 0.34 | 1.22 | -0.88 | -0.95 | 0.93 |
| 0.05 | 0.18 | -0.13 | -0.12 | 1.08 |
| 0.16 | 0.05 | 0.11 | 0.12 | 0.92 |
| 0.2 | 0.06 | 0.14 | 0.16 | 0.88 |
| 0.2 | 0.8 | -0.6 | -0.64 | 0.94 |
| 0.4 | 0.07 | 0.33 | 0.22 | 1.50 |
| Ave (H) | | | | 1.04 |
| Best fit (H) | | | | 1.02 |

We then plot a graph of flux vs. current (see next page).



The average value and the best fit value are in reasonable agreement. Only one measurement deviated from the best straight line fit. That particular value of the current was the most error prone measurement.

Example 5. Discussion Question.

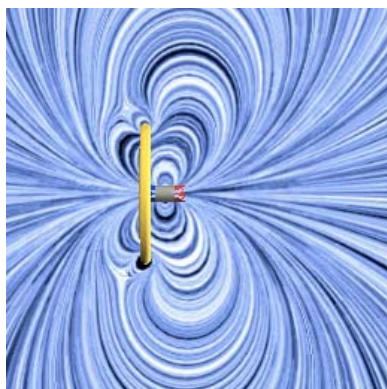
This question is can be used to generate an intense discussion between students working in a group, or between students and the instructor. We encourage you (and students) to work with the application while discussing these questions.

Start the [Faraday's Law Application](#) in the "Generator Mode". The following images (on the next page) were taken using the iron filings setting for various states in one cycle of motion. For each state, the current is listed below the figure.

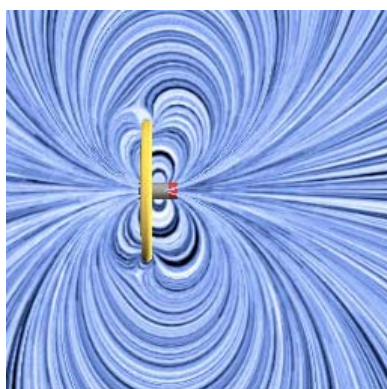
For each state, answer the following questions:

- a) Is the force exerted on the magnet to the left, zero, or to the right?
- b) Is the force on the magnet attractive, zero, or repulsive?
- c) What is direction of the velocity of the magnet?
- d) If red marks the north pole of the magnet which way is the current flowing in the ring?
- e) Are your answers to parts (a) and (d) consistent with Lenz' Law?

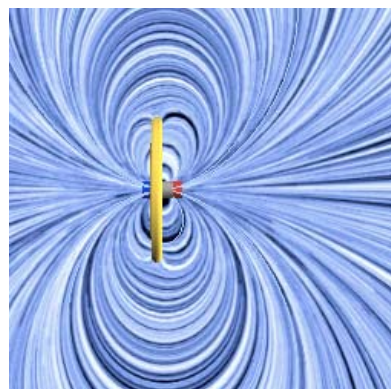
Example 5 (contd).



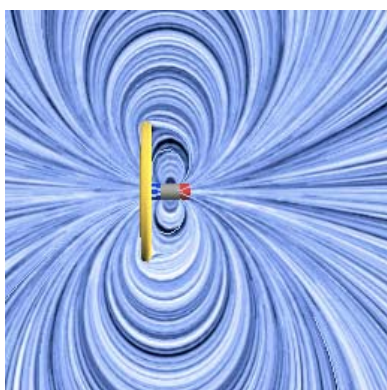
$I = -4 \text{ A}$



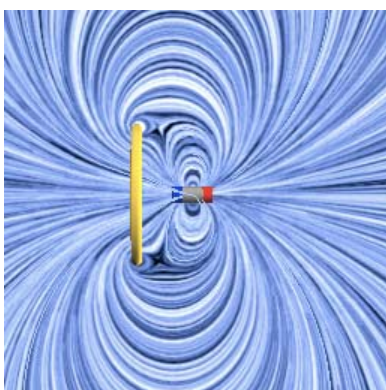
$I = -3 \text{ A}$



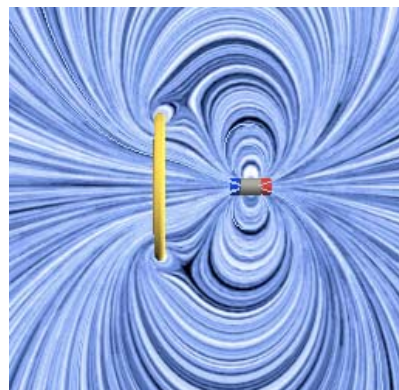
$I = -1.5 \text{ A}$



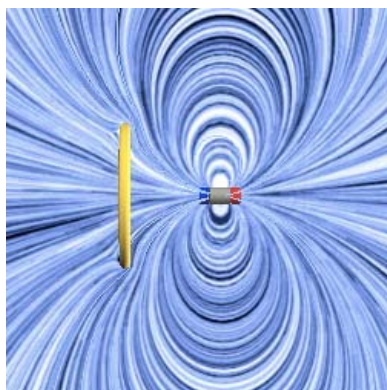
$I = 1.5 \text{ A}$



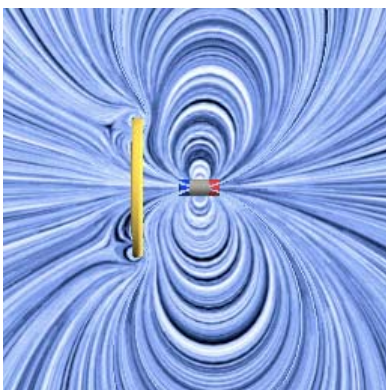
$I = 3.7 \text{ A}$



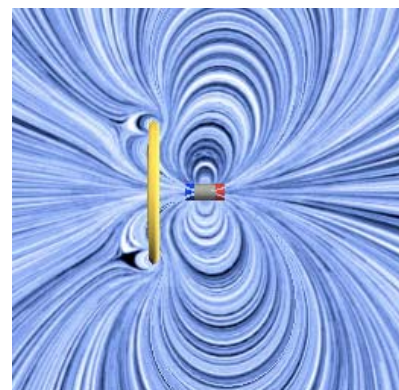
$I = 2.5 \text{ A}$



$I = -0.5 \text{ A}$



$I = -2.0 \text{ A}$



$I = -2.8 \text{ A}$

Example 6. Conceptual written-response questions.

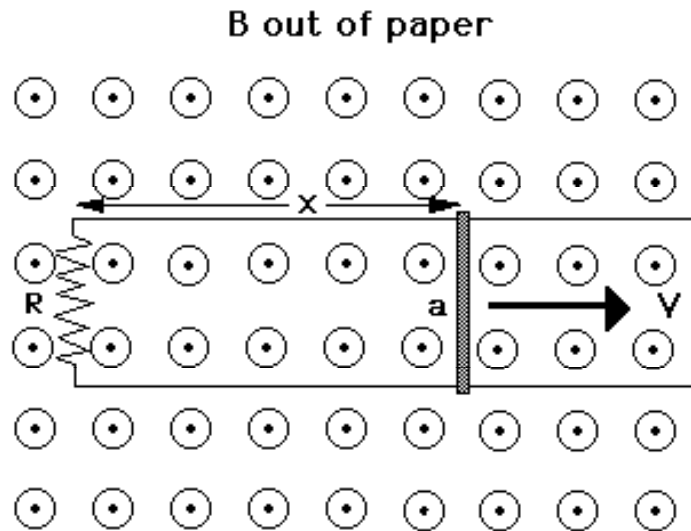
We have used these short conceptual questions in homework and on exams. Students learn how to explain physics ideas using words, and how to make scientific arguments based on physics concepts.

- (a) Suppose the north pole of a magnet is moved rapidly through a ring. An ammeter is connected in series with the loop. Using Faraday's Law, explain why the ammeter detects a current flowing through the loop. Which direction does the current flow?
- (b) If the north pole of a magnet is moved slowly through a ring, will the current induced in the loop be greater or smaller than if the north pole of the magnet were moved rapidly through the loop. Explain your reasoning.
- (c) List four different ways that magnetic flux can change in time. Specify how the flux is changing in each case.
- (d) Explain in your own words the difference between a magnetic field $\vec{\mathbf{B}}$ and the flux $\Phi_{mag} \equiv \iint_{\text{surface } S} \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}}$ of a magnetic field. Are they vectors or scalars? In what units may each be expressed? How are these units related? Are either or both (or neither) properties of a given point in space?

Example 6. Traditional Analytic Problem.

This is a traditional problem involving quantitative analysis of the described situation. We have used this as an in-class group problem, and on homework and exams. Note that while such problems have a role to play in an introductory physics class, they are not as effective in fulfilling other important goals such as generating discussion, exploring multiple representations, calculating physical quantities using visualizations and so on, as the previous examples did.

A conducting rod of mass m slides on parallel frictionless rails (on the page) in a uniform magnetic field \vec{B} . The magnetic field is directed out of the page. The rails are a distance a apart, and are connected by a resistor with resistance R . The conducting rod and the resistor are perpendicular to the rails, and on the page. At time t , the rod is a distance $x(t)$ from the resistor, moving with a speed $v(t) = dx(t)/dt$ (see figure).



(a) What is the magnetic flux Φ_B at time t through the circuit consisting of the resistor, the rod, and the intervening rails, in terms of the quantities given?

Solution: At time t , the flux through the loop is given by

$$\Phi_B = \iint \vec{B} \cdot d\vec{a} = Bxa$$

Note: That we have chosen the unit normal out of the page and thus the flux is positive.

(b) What is the emf in the circuit at time t , in terms of the quantities given? What is the current? Show on the sketch clearly (by means of arrows) the direction of this current.

Solution: The emf is equal to the negative of the time derivative of the flux:

$$\mathcal{E} = -\frac{d\Phi_B}{dt} = -Ba\frac{dx}{dt} = -Bav$$

The magnitude of the current is equal to:

$$I = \frac{|\mathcal{E}|}{R} = \frac{Bav}{R}.$$

The flux is out of the page and increasing so the induced flux must be into the page. By the right hand rule this correspond to a current flowing in the clockwise direction .

(c) What is the force \vec{F} on the rod at time t , in terms of the quantities given? Give its magnitude, and indicate its direction on the sketch.

Solution: The force on the rod is given by

$$\vec{F} = I\vec{a} \times \vec{B} = IaB(-\hat{i}) = \frac{B^2 a^2 v}{R}(-\hat{i})$$

The current is directed downwards, the magnetic field points out of the page, so the cross product points to the left, in the opposite direction of the motion of the bar (opposing force).

Note: The problem states that at time t the bar moves with a speed $v(t)$ however it doesn't explicitly mention that there are no other forces acting on the bar. There is no additional external force pulling the bar in the direction of the velocity of the rod. There are in fact two other forces acting on the bar, the gravitational force pointing downwards and the normal force of the rails on the bar acting upwards. Since these forces cancel vectorially, the net force on the rod at time t is only the force by the induced field.

(d) What is the rate of energy dissipation at time t due to Ohmic heating in the resistor, in terms of the quantities given?

Solution: The rate of energy dissipation is given by

$$P = I^2 R = \frac{B^2 a^2 v^2}{R}$$