Cilia and embriogenesis

- Our bodies are asymmetric (eg, heart on left, appendix on the right)
- Reversed in about 1:10,000 births (Kartagener's Syndrom, where dyein arms are missing from the microtubules of the motors that drive the cilia)
- cilia are responsible for left-right symmetry
- vortex tilt explains leftward fluid flow
- the flow distributes a signaling molecule

<u>too small</u> => Pe<1 diffuses so fast where advection is unimportant

<u>too large</u> => Pe>1 for both top and bottom flows / advection mixes in both directions

$$\frac{\partial c(x,t)}{\partial t} = \underbrace{a(x,t)}_{\text{production}} + \underbrace{D\nabla^2 c}_{\text{diffusion}} - \underbrace{\vec{v}\vec{\nabla} c}_{\text{advection}} - \underbrace{\mu c}_{\text{degradation}}$$

- in inv mutant mouse, flow is slower and more meandering (more random tilting of cilia, disrupting flow)

Moving & Stirring

- Can a bacterium get more food by swimming?
- Can a cilia enhance its food uptake by sweeping fluid across its surface?

$\tau_m \sim \frac{d}{v}$ ← distance, ← speed   timescale that fluid is replaced

$\tau_D \sim \frac{d^2}{D}$ timescale that diffusion will move a particle a distance

Peclet Number

convection: internal movements of currents within fluids

$$P_e = \frac{\tau_D}{\tau_m} = \frac{vd}{D}$$

advection: mass transfer due to convection

$P_e \ll 1$ Diffusion is more rapid than convection

$P_e \gg 1$ Convection is more rapid

Diffusion is more rapid at short distances

Convection is more rapid at long distances

- A bacterium is $d \sim 1\mu m$ long so $v > 1000 \frac{\mu m}{s}$ to enhance food intake beyond diffusion (they only swim $\sim 30 \frac{\mu m}{s}$)
- bacteria do not move like a grazing cow (but they do move to greener pastures)
- Can bacteria move fast enough to outrun diffusion?

$$\frac{vd}{D} > 1 \Rightarrow d > \frac{D}{v}$$

$$\frac{D}{v} = 30\mu m$$

So E. coli must 'run' at least 30 μm to outrun diffusion (that's what they do)

Pe number and cell size

Bacterium	Volvox	Cilia
$v \sim 10 \frac{\mu m}{s}$	$v \sim 30-800 \frac{\mu m}{s}$	$v \sim 50 \frac{\mu m}{s}$
$l \sim 1 \mu m$	$l \sim 10-500 \mu m$	$l \sim 50 \mu m$
$D \sim 100 \frac{\mu m^2}{s}$	$D \sim 100 \frac{\mu m^2}{s}$	$D \sim 100 \frac{\mu m^2}{s}$
$Pe \sim 0.1$	$Pe \sim 0.5-4000$	$Pe \sim 2.5$
Diffusion Dominant		Advection important (more so over longer length scales)

Volvocine green algae

- for larger Volvox colony sizes, forced advection improves productivity
- multicellular forms evolve by active mixing (encourages

$\Rightarrow$ differentiation into germ/reproductive and soma/flagellated cells)

- bottleneck occurs when spherical radius is too large for diffusion to meet metabolic demand
- requires advection from coordinated beating of surface flagella

I : inward $\frac{\text{molecules}}{s}$

$$I_D = 4\pi D R \overset{\text{bulk concentration}}{C_0}$$

$$\underset{\text{cell radius}}{R}$$

$$I_M = 4\pi R^2 \underset{\text{time-dependent nutrient demand rate per unit area}}{\beta}$$

@ critical point, $I_D = I_M$

$$\boxed{R_c = \frac{D C_0}{\beta}}$$

Considering O_2,

$$R_c = \frac{(2 \times 10^{-5} \, cm^2/s)(10^{17} \, \frac{1}{cm^3})}{(10^{14} \frac{1}{cm^2 s})} \sim 50-200 \, \mu m$$

Cilia

- thin projections extending 5-10 um from the eukaryotic cell surface
- like the Volvox flagella, they make an asymmetrical 'swimming' motion
- important for nutrient gathering via advection and the creation of flow during embryogenesis

$Re \sim 0.001$ (laminar flow, hard to mix)

Mixing by Cilia

- the flow created by the cilia is chaotic and mixes by stretching and folding (kneading)
- mixing occurs via chaotic advection (advection + diffusion)
- for cilia $Pe \sim 1\text{-}5$ (small, but diffusion cannot be ignored)

$\tau_D \sim \frac{L^2}{D} \leftarrow$ use mixing to make L smaller

Advection-Diffusion Eqn

$$\frac{\partial c}{\partial t} + \underbrace{\overset{\text{fluid velocity (vector)}}{\vec{v}} \cdot \vec{\nabla} c}_{\text{Advection}} = \underbrace{D \nabla^2 c}_{\text{Diffusion -}}$$

only relevent when $Pe \sim 1$

$Pe \gg 1 \quad D\nabla^2 c \to 0$

$Pe \ll 1 \quad \vec{v} \cdot \vec{\nabla} c \to 0$

fairly rare where both terms are required