Cilia and embrioogenesis

- Our bodies are asymmetric (e.g., heart on left, appendix on the right)
- Reversed in about 1:10,000 births (Kortagener's Syndrome; where dynein arms are missing from the microtubules of the motors that drive the cilia)
- Cilia are responsible for left-right symmetry
- Vortex tilt explains leftward fluid flow
- The flow distributes a signaling molecule
  - too small $\Rightarrow$ Pe $< 1$ diffuses so fast where advection is unimportant
  - too large $\Rightarrow$ Pe $> 1$ for both top and bottom flows / advection mixes in both directions

\[
\frac{\partial c(x,t)}{\partial t} = a(x,t) + \frac{D}{\text{diffusion}} \nabla^2 c - \frac{\nabla c}{\text{advection}} - \frac{\mu c}{\text{degradation}}
\]

- In inv mutant mouse, flow is slower and more meandering (more random tilting of cilia, disrupting flow)
Moving & Stirring

- Can a bacterium get more food by swimming?
- Can a cilium enhance its food uptake by sweeping fluid across its surface?

\[ T_m = \frac{d}{v} \quad \text{distance} \]

\[ T_m = \frac{V}{v} \quad \text{timescale that fluid is replaced} \]

\[ T_d = \frac{d^2}{D} \quad \text{timescale that diffusion will move a particle a distance} \]

Peclet Number

\[ \text{Pe} = \frac{T_d}{T_m} = \frac{v d}{D} \]

- Convection: internal movements of currents within fluids
- Advection: mass transfer due to convection

\[ \text{Pe} \ll 1 \quad \text{Diffusion is more rapid than convection} \]
\[ \text{Pe} \gg 1 \quad \text{Convection is more rapid} \]

- Diffusion is more rapid at short distances
- Convection is more rapid at long distances

- A bacterium is about 1 μm long, so \( v > 1000 \frac{\mu m}{s} \) to enhance food intake beyond diffusion (they only swim \( \sim 80 \frac{\mu m}{s} \))
- Bacteria do not move like a grazing cow (but they do move to greener pastures)
- Can bacteria move fast enough to outrun diffusion?

\[ \frac{v d}{D} > 1 \quad \Rightarrow \quad d > \frac{D}{v} \]

\[ \frac{D}{v} = 30 \mu m \]

So E. coli must "run" at least 30 μm to outrun diffusion (that's what they do)
Pe number and cell size

<table>
<thead>
<tr>
<th>Bacterium</th>
<th>Volvox</th>
<th>Cilia</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \nu \approx 10 \text{ mm} / \text{s} )</td>
<td>( \nu \approx 30 - 800 \text{ mm} / \text{s} )</td>
<td>( \nu \approx 50 \text{ mm} / \text{s} )</td>
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<tr>
<td>( \ell \approx 1 \mu \text{m} )</td>
<td>( \ell \approx 10 - 500 \mu \text{m} )</td>
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<tr>
<td>( \text{Pe} = 0.1 )</td>
<td>( \text{Pe} = 0.5 - 4000 )</td>
<td>( \text{Pe} = 2.5 )</td>
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Diffusion Dominant

Volvocine green algae
- for larger Volvox colony sizes, forced advection improves productivity
- multicellular forms evolve by active mixing (encourages differentiation into germ/reproductive and soma/flagellated cells)
- bottleneck occurs when spherical radius is too large for diffusion to meet metabolic demand
- requires advection from coordinated beating of surface flagella

\( I : \text{ inward molecules} / \text{s} \)

\[ I_p = 4\pi D R C_0 \]

\( D \): cell radius

\[ I_m = 4\pi R^2 B \]

\( B \): time-dependent nutrient demand rate per unit area

\( B \) critical point, \( I_o = I_m \)

\[ R_c = \frac{D C_0}{B} \]

Considering \( O_2 \)

\[ \frac{1}{(2 \times 10^{-5} \text{ cm}^2 / \text{s}) (10^{-17} \text{ cm}^3)} \sim 50 - 200 \mu \text{m} \]

\[ R_c = \frac{1}{(10^{-14} \text{ cm}^2 / \text{s})} \]
Cilia

- thin projections extending 5-10 μm from the eukaryotic cell surface
- like the Volvox flagella, they make an asymmetrical 'swimming' motion
- important for nutrient gathering via advection and the creation of flow during embryogenesis

\[ \text{Re} < 0.001 \] (laminar flow, hard to mix)

Mixing by Cilia

- the flow created by the cilia is chaotic and mixes by stretching and folding (kneading)
- mixing occurs via chaotic advection (advection + diffusion)
- for cilia \( \text{Pe} < 1-5 \) (small, but diffusion cannot be ignored)

\[ T_0 \sim \frac{L^2}{D} \] use mixing to make \( L \) smaller

Advection-Diffusion Eqn

\[ \frac{\partial c}{\partial t} + \nabla \cdot (\mathbf{V} c) = \nabla \cdot (D \nabla c) \]

- only relevant when \( \text{Pe} \ll 1 \)
- \( \text{Pe} \gg 1 \) \( \nabla^2 c \to 0 \)
- \( \text{Pe} \ll 1 \) \( \nabla c \to 0 \)

fairly rare where both terms are required