

1. Distinguishing between ratchets and power strokes.

Consider a motor protein traveling in one dimension along a filament.

a. A power stroke with Brownian fluctuations.

In a power stroke, an external driving force causes the protein to move a fixed unit (Δx) per unit time (Δt). In addition, the protein makes some number (n) of independent random moves forward or backward with equal probability due to Brownian fluctuations. In other words, the average velocity of the protein is $\Delta x/\Delta t$, but the motion is stochastic. For $n = 10$, what is the free energy consumption per unit length in terms of $k_B T$?

The number of random moves per unit time is proportional to the diffusion coefficient

$$2D\Delta t = n(\Delta x)^2$$

From this, we can calculate the diffusion coefficient

$$D = \frac{10(\Delta x)^2}{2\Delta t}$$

From Einstein's relation, we can compute the drag coefficient:

$$\xi = \frac{k_B T}{D} = \frac{k_B T \Delta t}{5(\Delta x)^2}$$

The constant force driving the deterministic forward motion is

$$f = \xi \langle v \rangle$$

and the free energy consumption per unit length is

$$\xi \langle v \rangle \Delta x = \frac{k_B T \Delta t}{5(\Delta x)^2} \frac{\Delta x}{\Delta t} \Delta x = 0.2 k_B T$$

b. A Brownian ratchet that weakly biases fluctuations.

In a Brownian ratchet, there is no driving force, but instead internal barriers block backward fluctuations, which makes the protein more likely to fluctuate forward. Suppose a protein makes 10 random moves of 1 unit displacement (Δx) per unit Δt , and that each time the object passes a multiple of $5 * \Delta x$, a barrier of $1 k_B T$ is established at that location. At the barrier, what is the probability of forward and backward fluctuations? What is the free energy consumption per unit length? What is the average velocity of the protein as a fraction of $\Delta x/\Delta t$?

The probability of breaking a barrier depends on the strength of the barrier. Let p_f be the probability of forward fluctuation at the barrier and p_b be that of backward fluctuation. The free energy required to break the barrier in units of $k_B T$ is

$$\frac{\Delta G}{k_B T} = \log(p_f / p_b)$$

$$1 = \log(p_f / (1 - p_f))$$

$$e = p_f / (1 - p_f)$$

$$p_f = 0.73$$

For a barrier of 1 $k_B T$, $p_f = 0.73$ and $p_b = 0.27$.

Since the barriers are separated by $5 * \Delta x$, the free energy consumption per unit time is $1/5 = 0.2 k_B T$, the same as in part a.

The average velocity of the protein is

$$\langle v \rangle = 0.92 \frac{\Delta x}{\Delta t}$$

c. A Brownian ratchet that strongly biases fluctuations.

Again the protein makes 10 random moves of Δx per unit Δt , but now each time it passes a multiple of $100 * \Delta x$, a barrier of 20 $k_B T$ is established at that location. Now what is the probability of forward and backward fluctuations at the barrier? What is the free energy consumption per unit length? What is the average velocity of the protein as a fraction of $\Delta x / \Delta t$?

For a barrier of 20 $k_B T$, $p_f = 1 - 3.8e-11 \approx 1$ and $p_b = 3.8e-11 \approx 0$

The free energy consumption per unit time is $20/100 = 0.2 k_B T$, the same as above.

The average velocity of the protein is

$$\langle v \rangle = 0.1 \frac{\Delta x}{\Delta t}$$

With a strong barrier, backward fluctuations are almost completely blocked, and the average velocity is significantly lower than in parts a and b.

2. Binding on a cell surface.

A single receptor for ligand L is embedded in a cell membrane. The ligand is all around with a concentration of 1 μM , but any ligand at the surface of the receptor is consumed immediately. Fick's law tells us that there is a flux of ligand toward the receptor. Assume that the radius of the receptor, 30 \AA , is much larger than that of the ligand and that at 200nm away from the receptor, the ligand is at bulk concentration. Find the full concentration profile of and the maximum number of ligand molecules per time that the bacterium can consume.

At steady state,

$$\frac{\partial c}{\partial r} = 0$$

$$\nabla^2 c = \frac{1}{r} \frac{d^2 rc}{dr^2} = 0$$

$$\frac{drc}{dr} = C_1$$

$$rc = C_1 r + C_2$$

$$C(r) = B_1 + \frac{B_2}{r}$$

boundaries:

$$r = R_\infty, \quad C_L = 0$$

$$r = \infty, \quad C_L = C_0$$

$$B_1 = C_0 = 2uM$$

$$C_L(R_R) = 0 = C_0 + \frac{B_2}{R_R}$$

$$B_2 = -C_0 R_R = -2uM(50 \text{ \AA})$$

$$C_L(r) = C_0 + \frac{-C_0 R_R}{r}$$

$$J = -D_L \frac{\partial C_L}{\partial r}$$

$$J = -D_L \frac{R_R C_0}{r^2}$$

$$\text{Collision} = |J|A$$

Depending on the student's assumption for SA exposed for ligand binding, answers may vary for collision frequency.

