MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.286: The Early Universe — October 26, 2005

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Lecture Notes 7

BLACK-BODY RADIATION AND THE EARLY HISTORY OF THE UNIVERSE

INTRODUCTION:

In Lecture Notes 4 and 5 we discussed the dynamics of Newtonian cosmology under the assumption that mass is conserved as the universe expands. In that case, since the physical volume is proportional to $R^3(t)$, the mass density $\rho(t)$ is proportional to $1/R^3(t)$.

According to special relativity, mass and energy are equivalent, with the conversion of units given by the famous formula,

$$E = mc^2 \; . \tag{7.1}$$

When one says that mass and energy are equivalent, one is saying that they are just two different ways of expressing precisely the same thing. The important quantity is the energy-momentum four-vector p^μ, the zeroth component of which is E/c. The value of E in the rest frame (*i.e.* the fame in which $\vec{p} = 0$) is given by mc^2. Thus, any form of energy shows up in the mass of the object which possesses that energy. For example, a hydrogen atom is made from a proton and an electron, but the mass of a hydrogen atom is *less* than the combined masses of the particles in isolation. If the two particles are started at infinite distance from each other, then as they are brought together they attract. They are therefore brought to a state of lower potential energy, and some energy ΔE is given off. This energy is called the binding energy of the hydrogen, and has a value of 13.6 eV. (Note: $1\,\text{eV} = 1$ electron volt $= 1.602 \times 10^{-12}\,\text{erg.}$) The energy is most commonly given off in the form of photons. In any case, the mass m_H of the resulting hydrogen atom is given by

$$\boxed{m_H = m_p + m_e - \Delta E/c^2 \; ,} \tag{7.2}$$

where m_p is the mass of the proton, and m_e is the mass of the electron. The negative potential energy of the system shows up as a (negative) contribution to its mass.

We are perhaps not used to thinking of electromagnetic radiation as having mass, but it is well-known that radiation has an energy density. If the energy density is denoted by u, then special relativity implies that the electromagnetic radiation has a mass density ρ given by

$$\boxed{\rho = u/c^2 \ .} \tag{7.3}$$

To my knowledge nobody has every actually "weighed" electromagnetic radiation in any way, but the theoretical evidence in favor of Eq. (7.3) is overwhelming — light does have mass. (Nonetheless, the photon has zero rest mass, meaning that it cannot be brought to rest. The general relation for the square of the four-momentum reads $p^2 = -(mc)^2$, and for the photon this becomes $p^2 = 0$. Writing out the square of the four-momentum leads to the following relation for photons:

$$\boxed{|\vec{p}|^2 - E^2/c^2 = 0 \ , \quad \text{or} \quad E = c|\vec{p}| \ .} \tag{7.4}$$

In this set of notes we will examine the role which the mass of electromagnetic radiation plays in the early stages of the universe.

RADIATION IN AN EXPANDING UNIVERSE

As the universe expands the number of photons is conserved, and thus the number density n is proportional to $1/R^3(t)$. However, we learned in Lecture Notes 3 that the frequency of each photon is redshifted as the universe expands, and that the ratio of the period at the time t_0 to the period at the time t_e is given by the redshift factor

$$1 + z = \frac{R(t_0)}{R(t_e)} \ . \tag{7.5}$$

Thus the frequency of each photon (frequency = 1/period) varies as $1/R(t)$ as the universe expands. According to elementary quantum mechanics, the energy of the photon is given by

$$E = h\nu \ , \tag{7.6}$$

where ν (Greek letter "nu") is the frequency and h is Planck's constant ($h = 4.136 \times 10^{-15}$ eV-sec). Thus the energy of the photon decreases as $1/R(t)$ as the universe expands. The energy density u_γ of the radiation is the number density n times E_γ, the mean energy per photon. (Note the Greek letter γ ("gamma") is often used to denote the photon.) Thus

$$\boxed{u_\gamma \propto \rho_\gamma \propto 1/R^4(t) \ .} \tag{7.7}$$

(Although I have justified this relation with quantum mechanical arguments, it can also be derived from classical electromagnetic theory. However, in this case the quantum argument is simpler.)

THE RADIATION–DOMINATED ERA

Today the energy density in the cosmic background radiation is given approximately by

$$u_r = 7.01 \times 10^{-13}\ \mathrm{erg/cm^3}\ . \tag{7.8}$$

(Here I have included the energy density of both the photons and the expected density of neutrinos — we'll talk about that later.) To find the corresponding mass density, use

$$\begin{aligned}\rho_r = \frac{u}{c^2} &= \frac{7.01 \times 10^{-13}\ \left(\mathrm{gm\text{-}cm^2\text{-}sec^{-2}}\right)\mathrm{cm^{-3}}}{\left(3 \times 10^{10}\ \mathrm{cm\text{-}sec^{-1}}\right)^2} \\ &= 7.80 \times 10^{-34}\ \mathrm{gm\text{-}cm^{-3}}\ .\end{aligned} \tag{7.9}$$

This can be compared with the critical mass density ρ_c, which was calculated in Eq. (4.26). One finds that the fraction of closure density in radiation ($\Omega_r \equiv \rho_r/\rho_c$) is given by

$$\Omega_r = \frac{7.80 \times 10^{-34}\ \mathrm{gm\text{-}cm^{-3}}}{1.88 h_0^2 \times 10^{-29}\ \mathrm{gm\text{-}cm^{-3}}} = 4.15 \times 10^{-5}\, h_0^{-2}\ , \tag{7.10}$$

where

$$H_0 = 100 h_0 \mathrm{km\text{-}sec^{-1}\text{-}Mpc^{-1}}\ .$$

For $h_0 = 0.72$, one finds $\Omega_r = 8.0 \times 10^{-5}$. This is only a very small fraction, but Ω_r was larger in the past. Since $\rho_r \propto 1/R^4$, while the mass density ρ_m of nonrelativistic matter behaves as $1/R^3$, it follows that

$$\rho_r/\rho_m \propto 1/R(t)\ . \tag{7.11}$$

If we assume for now that we live in an $\Omega_m = 0.33$ universe, then today $\rho_r/\rho_m \approx 8.0 \times 10^{-5}/0.33 \approx 2.4 \times 10^{-4}$. The constant of proportionality in Eq. (7.11) is then determined, giving

$$\frac{\rho_r(t)}{\rho_m(t)} = \frac{R(t_0)}{R(t)} \times 2.4 \times 10^{-4}\ . \tag{7.12}$$

Since $R(t) \to 0$ as $t \to 0$, the right-hand-side approaches infinity in this limit. Thus there was a time at which the value of the right-hand-side went through one, and this time is denoted by t_{eq}, the time of radiation-matter equality. We will assume that the universe is flat, and that for $t > t_{eq}$ we can make the crude approximation that the universe can be treated as if it were dominated by nonrelativistic matter. This approximation ignores the effect of radiation for times shortly after t_{eq}, and it also ignores the effect of dark energy (and the consequent acceleration) during the past

5 billion years or so. As discussed in Lecture Notes 4, during the matter-dominated era the scale factor behaves as $R(t) \propto t^{2/3}$. Thus, setting $\rho_r(t_{eq})/\rho_m(t_{eq}) = 1$ gives

$$\frac{R(t_{eq})}{R(t_0)} = \left(\frac{t_{eq}}{t_0}\right)^{2/3} = 2.4 \times 10^{-4} \ . \tag{7.13}$$

So $t_{eq} = 3.6 \times 10^{-6}\, t_0$, so for $t_0 = 13.7$ Gyr, $t_{eq} \approx 52,000$ years. Our approximations have been crude, but Barbara Ryden quotes a more precise numerical calculation (on p. 97), where she finds $t_{eq} \approx 47,000$ years.

DYNAMICS OF THE RADIATION–DOMINATED ERA

In order to understand the dynamics of the radiation-dominated era, one must understand the gravitational field created by the radiation. Since the radiation is highly relativistic, this problem is outside the scope of Newtonian dynamics. However, when the calculation is carried out using general relativity, one finds that Eq. (4.24) remains valid without any modifications. Repeating the equation here:

$$\boxed{\left[\frac{1}{R}\left(\frac{dR}{dt}\right)\right]^2 = \frac{8\pi}{3}G\rho - \frac{kc^2}{R^2} \ ,} \tag{7.14}$$

where ρ is the mass density. As you will show on Problem Set 4, Eq. (4.17) must then be modified, resulting in the equation

$$\boxed{\frac{d^2R}{dt^2} = -\frac{4\pi}{3}G\left(\rho + \frac{3p}{c^2}\right)R \ ,} \tag{7.15}$$

where p denotes the pressure. We will see later that the addition of the pressure term leads to dramatic consequences in the context of the inflationary universe model.

As a simple (but important) special case, consider the evolution of a radiation-dominated universe with $k = 0$. From Eqs. (7.7) and (7.14), one has

$$\frac{1}{R^2}\left(\frac{dR}{dt}\right)^2 = \frac{\text{const}}{R^4} \ , \tag{7.16}$$

which leads to

$$\frac{dR}{dt} = \frac{\sqrt{\text{const}}}{R} \ . \tag{7.17}$$

This equation can be solved by rewriting it as

$$R dR = \sqrt{\text{const}}\, dt \tag{7.18}$$

and then integrating both sides to obtain

$$\frac{1}{2}R^2 = \sqrt{\text{const}}\, t + \text{const}' \ . \tag{7.19}$$

The convention is to choose the zero of time so that $R(t) = 0$ for $t = 0$, which implies that $\text{const}' = 0$. Thus, the final result can be written as

$$\boxed{R(t) \propto \sqrt{t} \qquad \text{(radiation-dominated)} \ .} \tag{7.20}$$

The Hubble "constant" $H(t)$ is given by Eq. (3.7), which says that

$$H(t) = \dot{R}/R \ . \tag{7.21}$$

Combining this equation with Eq. (7.20), one has immediately that

$$\boxed{H(t) = \frac{1}{2t} \qquad \text{(radiation-dominated)} \ .} \tag{7.22}$$

The age of a radiation-dominated universe is therefore related to the Hubble constant by $t = \frac{1}{2}H^{-1}$. (Recall for comparison that for a matter-dominated flat universe with $R(t) \propto t^{2/3}$, the age is $\frac{2}{3}H^{-1}$.) The horizon distance is given by Eq. (5.7), and the result here is

$$\begin{aligned} \ell_{p,\text{horizon}}(t) &= R(t) \int_0^t \frac{c}{R(t')} dt' \\ &= \boxed{2ct \qquad \text{(radiation-dominated)} \ .} \end{aligned} \tag{7.23}$$

(Recall that this answer is to be compared with $3ct$ for the matter-dominated universe.) If one inserts Eq. (7.22) into Eq. (7.14) (with $k = 0$, still), one obtains a relation for the mass density as a function of time:

$$\rho = \frac{3}{32\pi G t^2} \ . \tag{7.24}$$

BLACK–BODY RADIATION

If a cavity is carved out of any material, and the walls of the cavity are kept at a uniform temperature T, then the cavity will fill with radiation. Assuming that the walls are thick enough so that no radiation can get through them, then the energy density (and also the entire spectrum of the radiation) is determined solely by the temperature T — the composition of the material is entirely irrelevant. The material is serving solely to keep the radiation at a uniform temperature. Radiation of this type is generally called either thermal radiation or black-body radiation.

The motivation for the name "black-body radiation" stems from the fact that a black body in empty space can be shown to emit radiation of exactly this intensity and spectrum. To see this, imagine a material inside the cavity which is genuinely black, in the sense that all light hitting it is absorbed. Since thermal equilibrium has been established, one concludes that the black body at temperature T must emit radiation which precisely matches the radiation which it is absorbing — otherwise it would either heat up or cool down, and that would violate the assumption of thermal equilibrium. Not only must the energy densities match, but the entire spectrum must match — otherwise one could imagine introducing a frequency selecting filter that would cause the black body to heat or cool. Note that the radiation emitted by the black body is ***not*** a reflection— we assumed that there was no reflection when we assumed that the body was black. Thus, the emitted radiation has to be attributed solely to thermal emission. Even if the black-body is removed from the cavity, it will continue to emit radiation of precisely this thermal spectrum.

The energy density and other properties of the radiation can be derived using the standard principles of statistical mechanics, but the derivation will not be included in this course. However, I will make a few comments about the underlying physics, and then I will state the results. The rule of thumb for classical statistical mechanics is the "equipartition theorem," which says that under certain circumstances (which I will not specify), each degree of freedom of a system at temperature T acquires a mean thermal energy of $\frac{1}{2}kT$. For example, in a gas of point particles each particle acquires a mean thermal energy of $\frac{3}{2}kT$, since motion in the x, y and z directions constitutes three degrees of freedom. For the system of radiation inside a cavity, each possible standing wave pattern corresponds to one degree of freedom. In a rectangular cavity, for example, a standing wave can be described in terms of a polarization, which has two linearly independent values, and a wave vector $\vec{k}$, with the wave amplitude proportional to $\text{Re}\{e^{i\vec{k}\cdot\vec{x}}\}$. For the standing wave to exist, each component of $\vec{k}$ must satisfy the condition that the wave amplitude must vary either an integral or half-integral number of cycles from one side of the cavity to the other. Thus a standing wave pattern exists only for a discrete set of frequencies. The discrete set of frequencies is, however, infinite, since there is no upper limit to the frequency of a standing wave. The number of degrees of freedom is therefore infinite, and the equipartition theorem cannot be applied. This problem is known as the "Jeans catastrophe," and represents an important failure of classical physics. The implications can be stated as follows: if classical physics were correct, then a region of electromagnetic field could never come into thermal equilibrium — instead it would continue indefinitely to absorb energy from its surroundings, and the energy absorbed would be used to excite higher and higher frequency standing waves of the field. The electromagnetic field would be an infinite heat sink, draining away all thermal energy.

Of course the electromagnetic field does not drain away all thermal energy, and the reason comes from quantum theory. Classically it would be possible to excite a standing wave by an arbitrary amount, but quantum theory requires that the excitations occur only by the addition of discrete photons, each with an energy $h\nu$, where ν is the frequency of the standing wave. For cases in which $h\nu \ll kT$, the classical answer is not changed — such standing waves acquire a mean energy of $\frac{1}{2}kT$ for each polarization. However, for those standing waves with $h\nu \gg kT$, the minimum excitation is much larger than the energy which is classically expected. These modes are then only rarely excited, and the total energy is convergent.

When the calculation is done quantum mechanically, one finds that black-body electromagnetic radiation has an energy density given by

$$\boxed{u = g\frac{\pi^2}{30}\,\frac{(kT)^4}{(\hbar c)^3}\ ,} \tag{7.25}$$

where

$$\begin{aligned} k = \text{Boltzmann's constant} &= 1.381\times 10^{-16}\ \text{erg/K} \\ &= 8.617\times 10^{-5}\ \text{eV/K}\ , \end{aligned} \tag{7.26}$$

$$\begin{aligned} \hbar = \frac{h}{2\pi} &= 1.055\times 10^{-27}\ \text{erg-sec} \\ &= 6.582\times 10^{-16}\ \text{eV-sec}\ , \end{aligned}$$

and

$$g = 2 \quad \text{(for photons)}\ . \tag{7.27}$$

The factor of g is introduced to prepare for the discussion below of black body radiation of particles other than photons. g is taken as 2 for photons because the photon has two possible polarization states.

One also finds that the radiation has a pressure, given by

$$\boxed{p = \frac{1}{3} u \ .} \tag{7.28}$$

The number density of photons is found to be

$$\boxed{n = g^* \frac{\zeta(3)}{\pi^2} \frac{(kT)^3}{(\hbar c)^3} \ ,} \tag{7.29}$$

where

$$\zeta(3) = \frac{1}{1^3} + \frac{1}{2^3} + \frac{1}{3^3} + \cdots \approx 1.202 \tag{7.30}$$

is the Riemann zeta function evaluated at 3, and

$$g^* = 2 \quad \text{(for photons)} \ . \tag{7.31}$$

Finally, the radiation has an entropy density given by

$$\boxed{s = g \frac{2\pi^2}{45} \frac{k^4 T^3}{(\hbar c)^3} \ .} \tag{7.32}$$

We will not need to know the precise meaning of entropy, but it will suffice to say that the entropy is a measure of the degree of disorder (or uncertainty) in the statistical system. Entropy is conserved if the system remains in thermal equilibrium, and this assumption appears to be quite accurate for most processes in the early universe. (The inflationary process, to be discussed later, is a colossal exception.) When departures from thermal equilibrium occur, the entropy is monotonically increasing.

In the laboratory the only kind of thermal radiation that can be achieved is that of photons. The radiation in the early universe, on the other hand, is believed to have also contained neutrinos. These neutrinos have zero rest mass like the photon, and they behave in many ways like photons (for example, they always travel at the speed of light). In thermal equilibrium they necessarily contribute to the radiation.

Neutrinos differ from photons, however, in one very important respect. The photon belongs to a class of particles called bosons, and these particles have the property that there is no limit to the number of particles that can exist simultaneously in a given quantum state. It is precisely because of this property that the photon can give rise to a classical electromagnetic field. The field behaves classically because it is composed of huge numbers of photons. The neutrino, on the other hand, belongs to a class of particles called fermions. For these particles it is impossible to have more than one particle in a given quantum state at one time. An electron is also a fermion, and the principle of one electron for a quantum state is sometimes called the "Pauli Exclusion principle."* Since fermions obey this exclusion principle, fewer of them are produced in thermal equilibrium — g is multiplied by 7/8, and g^* is multiplied by 3/4. Neutrinos exist as particles and antiparticles, which contributes a factor of 2 to both g and g^*. They appear to have only one spin state, so the factor of two that occurs for photons has no analogue for neutrinos. Neutrinos, on the other hand, are believed to exist as three different species of massless particles: the electron neutrino, the muon neutrino, and the tau neutrino. There is, however, considerable uncertainty — other species may also exist, and the known species may not really be massless.

[One might wonder why neutrinos are not produced when a piece of metal is heated until is glows. The answer is that neutrinos interact very weakly at these low energies, and their production rate is totally negligible. Thermal equilibrium neutrino radiation can in principle be seen at any temperature, but it is very difficult to produce. The radiation would reach thermal equilibrium only if it were confined to a box opaque to neutrinos, which means that the walls of the box would have to be much thicker than the diameter of the earth. In the early universe, however, the temperatures were much hotter. Neutrino interaction rates increase with energy, so in the early universe they interacted rapidly with the other particles, and were quickly brought to thermal equilibrium.]

As the temperature is increased, more and more types of particles contribute to the thermal radiation. Any particle with $mc^2 \ll kT$ will contribute in essentially the same way as a massless particle. In particular, when kT is much larger than the value of mc^2 for an electron (0.511 MeV), then electron-positron pairs contribute to the thermal radiation. Electrons and positrons each have two spin states, resulting in a factor of 4. They are again fermions, so

$$\left.\begin{aligned} g &= \frac{7}{8} \times 4 = \frac{7}{2} \\ g^* &= \frac{3}{4} \times 4 = 3 \;. \end{aligned}\right\} \qquad \text{(for } e^+\text{-}e^- \text{ pairs)} \tag{7.33}$$

* I recollect that some chemistry books talk about two electrons in each quantum state, one with its spin up and the other with its spin down. In the language of most physicists, however, this would be counted as two quantum states.

Including photons, three species of neutrinos, and the electron-positron pairs, the total value of g is given by $g_{\rm tot} = 10\frac{3}{4}$. This value is appropriate for values of kT which are larger than 0.511 MeV, but smaller than 106 MeV (where muons begin to be produced).

THERMAL HISTORY OF THE UNIVERSE

We now have all the ingredients necessary to calculate the temperature of the universe as a function of time. Eq. (7.24) gives the mass density as a function of time, and Eq. (7.25) relates the energy density to the temperature. Recalling the relationship of Eq. (7.3) between the energy and mass densities, one can solve for the temperature as a function of time:

$$kT = \left(\frac{45\hbar^3 c^5}{16\pi^3 g G}\right)^{1/4} \frac{1}{\sqrt{t}} \,. \tag{7.34}$$

To find the temperature at 1 sec after the big bang, we now need only plug in numbers:

$$\begin{aligned} kT &= \left[\frac{45\left(1.055\times 10^{-27}\right)^3 \text{ erg}^3\text{-sec}^3 \left(3\times 10^{10}\right)^5 \text{ cm}^5\text{-sec}^{-5}}{16\pi^3(10.75)\,(6.67\times 10^{-8})\,\text{cm}^3\text{-gm}^{-1}\text{-sec}^{-2}}\right]^{1/4} \\ &\qquad \times \frac{1}{(1\,\text{sec})^{1/2}} \times \left(\frac{1\,\text{erg}}{\text{gm-cm}^2\text{-sec}^{-2}}\right)^{1/4} \\ &= 1.378\times 10^{-6}\,\text{erg} \,. \end{aligned}$$

Using $1\,\text{eV} = 1.602\times 10^{-12}$ erg, one can convert this result if one wishes to

$$kT = 0.860 \text{ MeV} \,.$$

Since one knows that $T \propto t^{-1/2}$, one can write down a general expression for the time-temperature relation, for $0.511 \text{ MeV} \ll kT \ll 106 \text{ MeV}$, as

$$\boxed{kT = \frac{0.860 \text{ MeV}}{\sqrt{t \text{ (in sec)}}}} \,, \tag{7.35a}$$

or equivalently

$$\boxed{T = \frac{9.98\times 10^9 \text{ K}}{\sqrt{t \text{ (in sec)}}}} \,. \tag{7.35b}$$

As an example one can use Eq. (7.35b) to calculate the temperature of the universe at the end of the first seven days. (Here we are making a minor error, since the value $g_{\rm tot} = 10\frac{3}{4}$ is not appropriate when kT falls below 0.5 MeV.) One finds $T \approx 1.3 \times 10^7\,$K, which is roughly the temperature which is believed to exist in the core of a bright star.

RELATIONSHIP BETWEEN R AND T

When a gas of black-body radiation expands in thermal equilibrium, there is a simple relationship between the scale factor R and the temperature T. We have already seen that the energy density $\rho \propto 1/R^4$, and that $\rho \propto T^4$. It follows that the product RT remains constant as the universe expands. The constancy of RT is actually a direct consequence of statistical mechanics, and has nothing to do with the dynamics of the expanding universe. RT remains a constant provided that the expansion occurs slowly enough for the system to remain in thermal equilibrium, and provided that the effective value of g goes not change. When kT falls below 0.5 MeV and the electron-positron pairs disappear from the thermal equilibrium mix, it can be shown by using conservation of entropy that RT increases by a factor of $(11/4)^{1/3} = 1.40$.

RECOMBINATION AND DECOUPLING

The observed universe is about 80% hydrogen by mass. One can use statistical mechanics to understand the behavior of this hydrogen under the conditions prevalent in the early universe, but I will not attempt such a calculation in this course. As one might guess, hydrogen will ionize (*i.e.* break up into separate protons and electrons) if the temperature is hot enough. The temperature necessary to cause ionization depends on the density, but for the history of our universe one can say that the hydrogen is ionized when T is greater than about $4,000\,$K.

Thus, when the temperature falls below $4,000\,$K, the ionized hydrogen coalesces into neutral atoms. The process is usually called "recombination," although I am at a loss to explain the significance of the prefix "re-". When recombination occurs, the universe becomes essentially transparent to photons. The photons cease to interact with the other particles, and this process is called "decoupling". Decoupling occurs slightly later than recombination, at a temperature of about $3,000\,$K, since even a small residual density of free electrons is enough to keep the photons coupled to the other particles. The photons which we observe today in the cosmic background radiation are photons which for the most part have last scattered at the time of decoupling.

We can estimate the time of decoupling by using the constancy of RT. It is very accurate to assume that RT has remained constant from the time of decoupling to the present, since the effective value of g has not changed during this interval. Here T is interpreted as the temperature of the black-body photons, which in the early universe was identical to the temperature of the hydrogen. Using the subscript d to denote quantities evaluated at the time of decoupling, and subscript 0 to denote quantities evaluated at the present time, one has

$$R_d T_d = R_0 T_0 \ , \tag{7.36}$$

from which one has immediately that

$$\frac{R_d}{R_0} = \frac{T_0}{T_d} \ . \tag{7.37}$$

Assuming that the universe is flat, and making the crude approximation that it can be treated as matter-dominated from t_d to the present, one has $R(t) \propto t^{2/3}$ and

$$\left(\frac{t_d}{t_0}\right)^{2/3} = \frac{T_0}{T_d} \ . \tag{7.38}$$

Solving, one has

$$\begin{aligned} t_d &= \left(\frac{T_0}{T_d}\right)^{3/2} t_0 \\ &\approx \left(\frac{2.7\,\mathrm{K}}{3000\,\mathrm{K}}\right)^{3/2} \times \left(13.7 \times 10^9 \text{ yr}\right) \approx 370{,}000 \text{ yr} \ . \end{aligned} \tag{7.39}$$

On p. 159, Ryden quotes a more accurate numerical calculation, giving $t_d \approx 350{,}000$ yr.

THE SPECTRUM OF THE COSMIC BACKGROUND RADIATION

The cosmic background radiation was first discovered by Penzias and Wilson in 1965. They measured at one frequency only, but found that the radiation appeared to be coming uniformly from all directions in space. This radiation was quickly identified by Dicke, Peebles, Roll, and Wilkinson as the remnant radiation from the big bang. Since then the measurement of the cosmic background radiation has become a minor industry, and much data has been obtained about the spectrum of the radiation and about its angular distribution in the sky.

The prediction from big bang cosmology is that the spectrum should be thermal, corresponding to black-body radiation that has been redshifted from its initially very high temperature. It is a peculiar feature of the black-body spectrum that it maintains its thermal equilibrium form under uniform redshift, even though the photons in the radiation are noninteracting. That is, if each photon in the black-body probability distribution is redshifted by the same factor, the net effect is to produce a new probability distribution which is again of the black-body form, except that the temperature is modified by a factor of the redshift. Thus, the redshift reduces the temperature, but does not lead to departures from the thermal equilibrium spectrum.

The ideal Planck spectrum for such radiation has an energy density $\rho(\nu)d\nu$, for radiation in the wavelength interval between ν and $\nu + d\nu$, given by

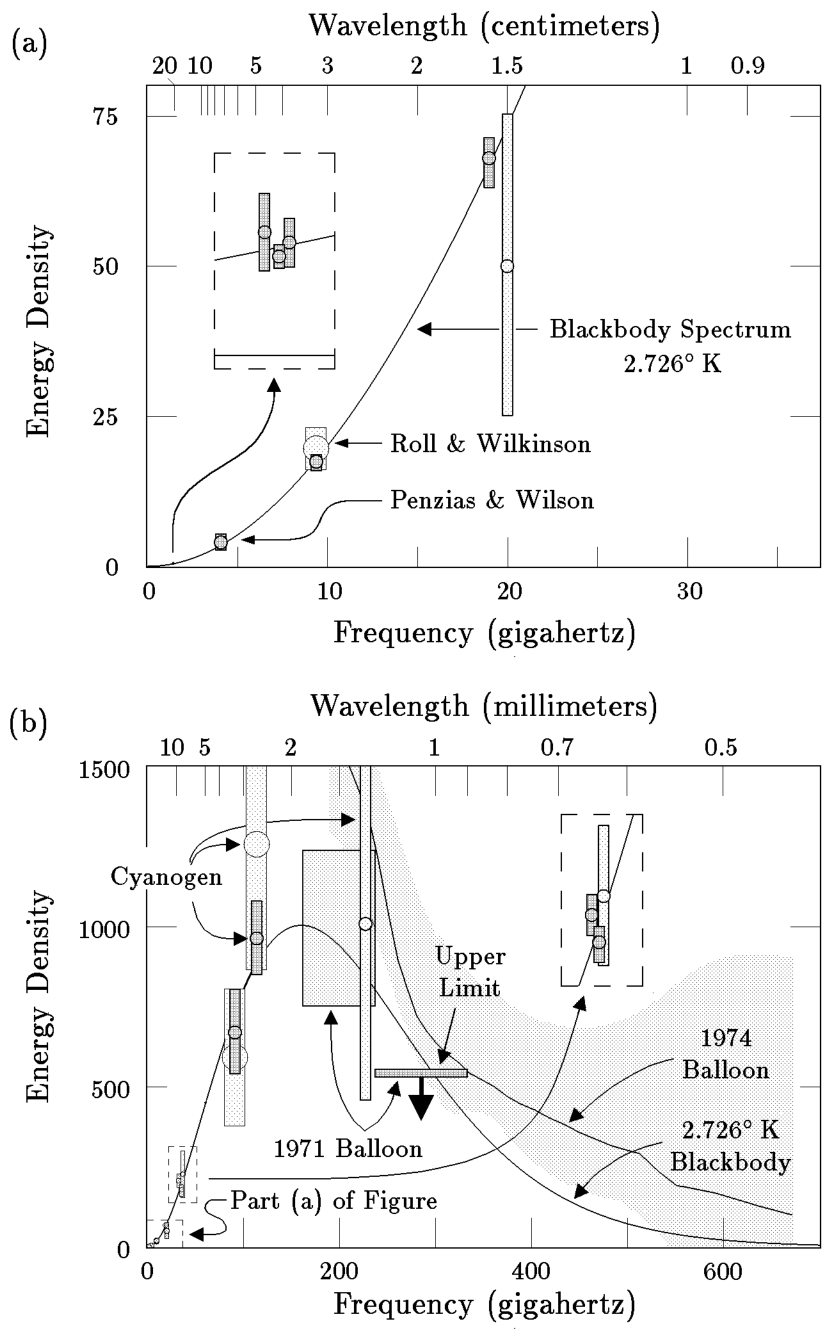
$$\rho(\nu)d\nu = \frac{16\pi^2\hbar\nu^3}{c^3}\frac{1}{e^{2\pi\hbar\nu/kT}-1}d\nu \; . \qquad (7.40)$$

(As with the other statistical mechanics results in this set of Lecture Notes, we will use Eq. (7.40) without derivation.) Observers usually do not directly measure the energy density, however, but instead measure the intensity of the radiation. It can be shown that the power hitting a detector per frequency interval per area of aperature per solid angle of aperture is given by

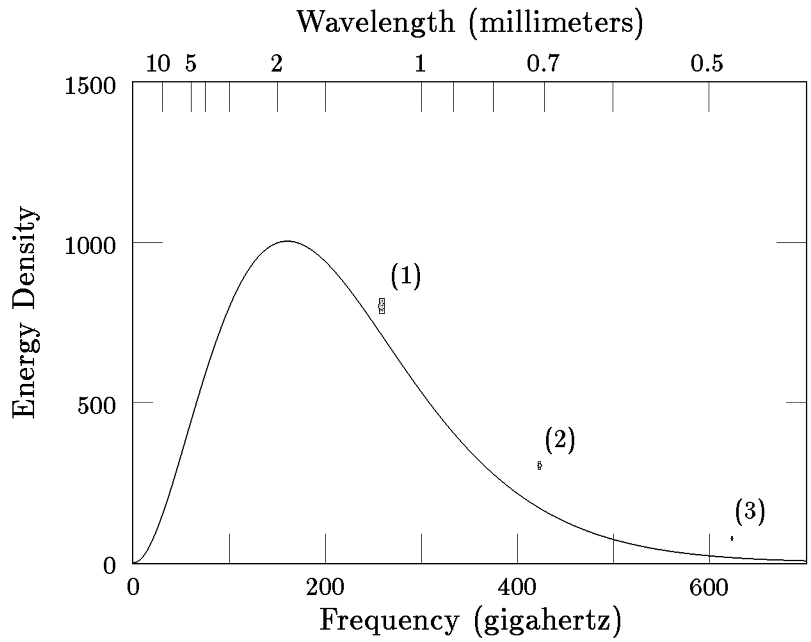
$$I_\nu(\nu) = \frac{c}{4\pi}\rho_\nu(\nu) = \frac{4\pi\hbar\nu^3}{c^2}\frac{1}{e^{2\pi\hbar\nu/kT}-1} \; . \qquad (7.41)$$

The data on the spectrum available in 1975 is summarized on the two graphs on the following page. The graphs show measurements of the energy density in the cosmic background radiation at different frequencies (or wavelengths). The lower horizontal axis shows the frequency in gigahertz (10^9 cycles per second), and the upper horizontal axis shows the corresponding wavelength. The solid line is the expected blackbody distribution, shown for the best current determination of the temperature, 2.726 K. Part (a) shows the low frequency measurements, including those of Penzias & Wilson and Roll & Wilkinson (which was published about 6 months after the Penzias & Wilson result). Part (b) includes the full range of interesting frequencies. The circles show the results of each measurement, and the bars indicate the range of the estimated uncertainty. The measurements with small uncertainties are shown with dark shading. A high-frequency broad-band measurement is shown on part (b), labeled "1974 Balloon"—the measured energy density is shown as a solid line, and the estimated uncertainty is indicated by gray shading. The 1971 balloon measurements were taken by the MIT team of Dirk Muehlner and Rainer Weiss. (The energy density on both graphs is measured in electron volts per cubic meter per gigahertz.)

The earth's atmosphere poses a serious problem for measuring the high frequency side of the curve, so the best measurements must be done from balloons,


(a)
Wavelength (centimeters)
20 10 5 3 2 1.5 1 0.9
75
50
25
0
Energy Density
Blackbody Spectrum
2.726° K
Roll & Wilkinson
Penzias & Wilson
0 10 20 30
Frequency (gigahertz)
(b)
Wavelength (millimeters)
10 5 2 1 0.7 0.5
1500
1000
500
0
Energy Density
Cyanogen
Upper
Limit
1974
Balloon
2.726° K
Blackbody
1971 Balloon
Part (a) of Figure
0 200 400 600
Frequency (gigahertz)

rockets, or satellites. In 1987 a rocket probe was launched by a collaboration between the University of California at Berkeley, and Nagoya University in Japan. The group consisted of T. Matsumoto, S. Hayakawa, H. Matsuo, H. Murakami, S. Sato, A.E. Lange, and P.L. Richards. Their paper, published in *The Astrophysical Journal*, vol. **329**, pp. 567-571 (1988), includes a graph of the following remarkable data:



Note that the points labeled 2 and 3 are much higher than the black body spectrum predicts. Fitting these points individually to a temperature, the authors find:

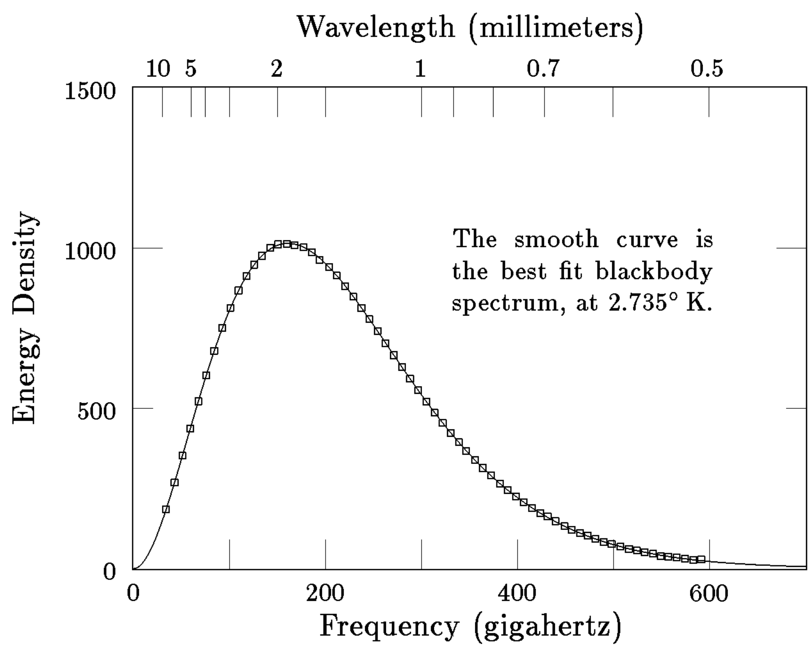
$$\text{Point 2: } T = 2.955 \pm 0.017\,\text{K}$$
$$\text{Point 3: } T = 3.175 \pm 0.027\,\text{K}$$

These numbers correspond to discrepancies of 12 and 16 standard deviations, respectively, from the temperature of $T = 2.74\,\text{K}$ that fits the lower frequency points. In terms of energy, the excess intensity seen at high frequencies in this experiment amounts to about 10% of the total energy in the cosmic background radiation. Cosmologists were stunned by the extremely significant disagreement with predictions. Some tried to develop theories to explain the radiation, without much success, while others banked on the theory that it would go away. The experiment looked like a very careful one, however, so it was difficult to dismiss. The most likely source of error in an experiment of this type is the possibility that the detectors were influenced by heat from the exhaust of the launch vehicle— but the experimenters very carefully tracked how the observed radiation varied with time as the detector moved away from the launch rocket, and it seemed clear that the rocket was not a factor.

The same group tried to check their results with a second flight a year later, but the rocket failed and no useful data was obtained.

In the fall of 1989 NASA launched the Cosmic Background Explorer, known as COBE (pronounced "koh-bee"). This marked the first time that a satellite was used to probe the background radiation. Within months, the COBE group announced their first results at a meeting of the American Astronomical Society in Washington, D.C., January 1990. The detailed preprint, with a cover sheet showing a sketch of the satellite, was released the same day.

The data showed a perfect fit to the blackbody spectrum, with a temperature of $2.735 \pm 0.06\,\mathrm{K}$, with no evidence whatever for the "submillimeter excess" that had been seen by Matsumoto *et al.* The data was shown with estimated error bars of 1% of the peak intensity, which the group regarded as very conservative. The graph is reproduced below.



Once again, the vertical axis is calibrated in electron volts per cubic meter per gigahertz.

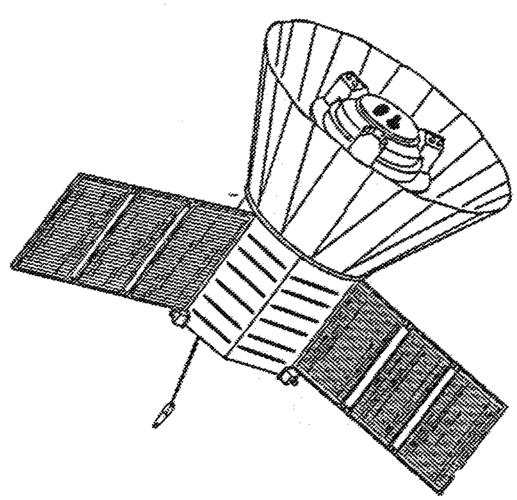
Since the COBE instrument is far more precise and has more internal consistency checks, there has been no doubt in the scientific community that the COBE result supercedes the previous one. Despite the 16σ discrepancy of 1988, the cosmic background radiation is now once again believed to have a nearly perfect black-body spectrum.

Preprint No. 90–01

COBE PREPRINT

A PRELIMINARY MEASUREMENT OF THE COSMIC MICROWAVE BACKGROUND SPECTRUM BY THE COSMIC BACKGROUND EXPLORER (COBE) SATELLITE

J.C. Mather, E. S. Cheng, R. E. Eplee, R. B. Isaacman, S. S. Meyer, R. A. Shafer, R. Weiss, E. L. Wright, C. L. Bennett, N. W. Boggess, E. Dwek, S. Gulkis, M. G. Hauser, M. Janssen, T. Kelsall, P. M. Lubin, S. H. Moseley, Jr., T. L. Murdock, R. F. Silverberg, G. F. Smoot, and D. T. Wilkinson.



COSMIC BACKGROUND EXPLORER

In January 1993, the COBE team released their final data on the cosmic background radiation spectrum. The first graph had come from just 9 minutes of data, but now the team had analyzed the data from the entire mission. The error boxes were shrunk beyond visibility to only 0.03%, and the background spectrum was still perfectly blackbody, just as the big bang theory predicted. The new value for the temperature was just a little colder, 2.726 K, with an uncertainty of less than 0.01 K.

The perfection of the spectrum means that the big bang must have been very simple. The COBE team estimated that no more than 0.03% of the energy in the background radiation could have been released anytime after the first year, since energy released after one year would not have had time to reach such a perfect state of thermal equilibrium. Theories that predict energy release from the decay of turbulent motions or exotic elementary particles, from a generation of exploding or massive stars preceding those already known, or from dozens of other interesting hypothetical objects, were all excluded at once.

Although a few advocates of the steady state universe have not yet given up, the COBE team announced that the theory is ruled out. A nearly perfect blackbody spectrum can be achieved in the steady state theory only by a thick fog of objects that could absorb and re-emit the microwave radiation, allowing the radiation to come to a uniform temperature. Steady state proponents have in the past suggested that interstellar space might be filled by a thin dust of iron whiskers that could create such a fog. However, a fog that is thick enough to explain the new data would be so opaque that distant sources would not be visible.

In this chapter we have discussed mainly the spectrum of the cosmic microwave background (CMB). Starting in 1992, however, with some preliminary results from the COBE satellite, astronomers have also been able to measure the anisotropies of the CMB. This is quite a tour de force, since the radiation is isotropic to an accuracy of about 1 part in 10^5. Since the photons of the CMB have been travelling essentially on straight lines since the time of decoupling, these anisotropies are interpreted as a direct measure of the degree of nonuniformity of the matter in the universe at the time of decoupling. These non-uniformities are crucially important, because they give us clues about how the universe originated, and because they are believed to be the seeds which led to the formation of the complicated structure that the universe has today. If all goes well, we will have one lecture at the end of the course devoted to these issues.