Quantum Mechanics of Alpha Decay

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In this experiment, a liquid scintillator was used to measure the half-lives of Rn^{222} and Po^{214}. These were measured to be (4.1 ± 0.7) days and (157 ± 3) μs, respectively, as compared to their theoretical values of 3.82 days and 163.7 μs. A combination of plastic scintillation counter and silicon barrier detector was used to obtain a value of (0.26 ± 0.03) μs for the half-life of Po^{212}, while the accepted value is 0.298 μs. A silicon barrer detector was used to measure the half-lives of Po^{218} and Pb^{211}. These were measured to be (3.13±0.03) min and (37.4±0.5) min, respectively, compared to the accepted values of 3.1 min and 36.1 min. The half-lives of three isotopes of polonium were found to accurately follow the Geiger-Nuttal relation for half-lives and alpha particle energies.

1. INTRODUCTION

The study of radioactive elements and their applications as a source of high energy helium nuclei and electrons has been integral to the development of quantum physics [1]. X-rays were discovered by Wilhelm Rontgen in 1895, through studying the fluorescing glass of a cathode-ray tube. Immediately thereafter, Antoine Becquerel set about studying the radiation emitted by other fluorescent materials, using uranium salts sprinkled onto photographic paper that had been wrapped in light-tight paper. Thus was radioactivity discovered. Marie Curie was the one who gave radioactivity its name during the course of her studies of radioactive thorium and her discovery of radium, an element that was a million times more radioactive than uranium.

In 1899, Rutherford showed that the radiation emitted by uranium can be separated into two types based on their energy. He named the higher energy form of radiation "beta rays" and the lower energy form "alpha rays". Becquerel subsequently found that beta rays were actually the equivalent of electrons by observing how beta rays were deflected by electric and magnetic fields. In 1900, Rutherford and Soddy made the important revelation that unstable nuclei, in decaying, actually transmuted into other elements. Two years later, the exponential law of radioactive decay was formulated, characterizing the decay of radioactive nuclei [2].

In addition to allowing the investigation of nuclear structure, radioactive elements have many applications outside of physics. In medicine, radioactive elements are used as radiotracers to see how much of a certain chemical is being absorbed by the human body. Also, small radioactive sources can be used in house smoke detectors.

2. THEORY

2.1. Instability of Large Nuclei

Most natural elements that have atomic numbers ranging from $Z = 1$ to $Z = 82$ are stable, while elements that have $Z > 82$ tend to decay. Light nuclei can achieve stability with equal numbers of protons and neutrons. In heavier nuclei, however, more neutrons are needed, proportionally, in order to overcome the Coulombic repulsion of protons. When there is an imbalance of neutrons and protons that causes a nucleus to be unstable, the atom can transmute into another element in order to alleviate such instability [2].

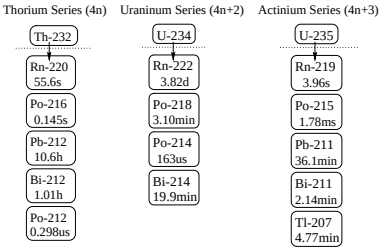

FIG. 1: The three decay chains studied in this experiment, with their naturally-occuring parent isotopes placed at the very top of each chain. These parent isotopes each emit a different isotope of radioactive radon gas, which subsequently decays into other nuclides.

There are three primary kinds of radioactive transmutation. The first, which is most common in elements heavier than lead, is alpha decay, which occurs when a radioactive nucleus emits a charged He nucleus consisting of 2 neutrons and 2 protons. There are four possible sequences of alpha decay, labeled by the atomic weight A_w of the nuclides contained within each sequence: $A_w = 4n, 4n+1, 4n+2$, or $4n+3$, where n is a positive integer. The four different alpha sequences are named, respectively, the Thorium, Neptunium, Uranium, and Actinium series. Only three out of these four series can be observed in nature — in the Neptunium series, the longest-lived isotope has a half-life on the order of 10^6 years. As the Earth's age is on the order of 10^8 years, most of the longest-lived isotope in the Neptunium series has already decayed [3]. The decay chains pertinent in this experiment are shown in Figure 1.

The second kind of radioactive transmutation is called beta decay, and consists of the conversion of a neutron into a proton via the emission of a positron and an electron neutrino, or the conversion of a proton into a neutron via the emission of an electron and anti-neutrino. This process is used to correct unstable proportions of

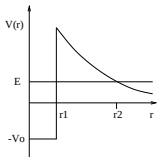

FIG. 2: Adapted from [4]. The square well represents the potential well around the nucleus, which is created by strong and Coulombic forces. Note that, outside of r_1, the potential follows a $1/r$ curve. E represents the energy of a given alpha particle.

neutrons and protons. The third kind of radioactive transmutation is called spontaneous fission, and involves the rapid decay of a single nucleus into two smaller nuclei. This process is used by elements heavier than uranium, and in particular the artifical elements with $Z > 100$ [2]. We are primarily concerned with alpha decay in this experiment (we measure the lifetime of one beta decay indirectly) and we do not study spontaneous fission.

2.2. Gamow Theory of α-Decay & Geiger-Nuttall Relation

One of the initial puzzles of radioactive alpha decay was that it seemed as though the alpha particle emerged from a region of the nucleus where its kinetic energy was negative (note Figure 2, which shows the potential seen by the alpha particle). Because the alpha particle has a positive charge of $2e$, it will be repelled by the leftover nucleus once it escapes from the nuclear binding force. Before it can do so, however, it must pass through a potential barrier that, in uranium, is twice the kinetic energy of the emitted alpha particle. At first, scientists speculated that perhaps the Coulomb law broke down within some distance from the center of the nucleus. However, Rutherford and Chadwick discovered that this was not the case — the Coulomb field was found to hold inside the radius from which the uranium alpha-particle emerged. In due time, scientists realized that this demonstrated not a paradox, but direct experimental evidence that supported the quantum mechanical phenomenon of tunneling through a potential barrier [1]. Gamow was the first to successfully explain the phenomenon of alpha decay [4].

Following Gamow's derivation, as given in [4], we analyze the movement of an alpha particle in the potential of an atomic nucleus and use the WKB approximation to obtain the transmission amplitude, which describes the likelihood that the alpha particle will tunnel out of the square well surrounding the nucleus:

$$\begin{aligned} T &= e^{-2\gamma}, \\ \gamma &\equiv \frac{1}{\hbar}\int_{r_1}^{r_2} |p(x)|dx, \\ p(x) &\equiv \sqrt{2m(E-V(r))} \end{aligned} \tag{1}$$

We can think about the alpha particle bouncing back and forth within the potential well of the nucleus with an average velocity v. The average time between its collisions with the "wall" of the potential is approximately $2r_1/v$, so the frequency of collisions is $v/2r_1$. The probability that the alpha particle will escape during any particular collision is $e^{-2\gamma}$, so the probability of emission at any given time is $(v/2r_1)e^{-2\gamma}$. We can then deduce that the lifetime of the parent nucleus is about $\tau = (2r_1/v)e^{2\gamma}$. We do not actually know the value of v, but the variation in v is insignificant compared to variation in the exponential factor. For any given element, if we plot the logarithm of the lifetime of its various isotopes against $1/\sqrt{E}$, where E is the energy of the alpha particle emitted by any particular isotope, we obtain a straight line [4]. This agrees closely with the experimentally determined Gieger-Nuttall Relation [2]:

$$ln\lambda = a_1 ln E_\alpha + a_2 \tag{2}$$

where λ to the reciprocal of the isotope's lifetime, E_α is the energy of the α-particle, and a_1 and a_2 are constants related to the dimensions of the specific nucleus. An important note is that the half-life of a given isotope, $\tau_{1/2} = ln(2) \times \tau$, is often cited instead of its full lifetime.

2.3. The Bateman Equations

Another way of determining the characteristic of alpha decay is through solving a system of differential equations called the Bateman Equations. We examine the decay of two radioactive isotopes, A and B, where B is formed out of the decay of isotope A. We first examine the decay of A by using the fundamental law of radioactivity, which states that the change in any given time, dA/dt, is proportional to the amount of isotope A that is present at any given time: $\frac{dA}{dt} = -A/\tau_A$, where τ_A is the lifetime of A. Integrating this equation, one obtains the exponential decay law for the declining alpha activity of A: $A(t) = A_0 e^{(-t/\tau_A)}$, where A_0 is the amount of A present at $t = 0$ [2].

Examining the behavior of B is somewhat more complex, since the amount of B at any given time depends both on the decay rate of B and the decay rate of A: $\frac{dB}{dt} = A/\tau_A - B/\tau_B$, where B is the amount of isotope B present at any given time and τ_B is the lifetime of isotope B. We must solve this differential equation by using variation of parameters. When we do so, we find that the solution to $B(t)$ will be equal to the sum of a homogenous solution, which has the same form as $A(t)$, and a particular solution. The solution is given below, where B_0 is the amount of B present at $t = 0$:

$$B(t) = B_0 e^{(-t/\tau_B)} + A_0 \frac{\tau_B}{\tau_A - \tau_B}[e^{(-t/\tau_A)} - e^{(-t/\tau_B)}] \tag{3}$$

Equation (3) being a somewhat complicated expression, it is of use to obtain some limiting cases. For example,

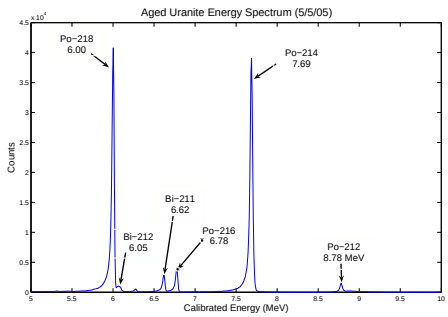

FIG. 3: Sample MCA calibration data with peaks labelled.

of interest in this experiment is the case where $\tau_A \gg \tau_B$ and $t \gg \tau_B$:

$$B(t) \approx A_0 \frac{\tau_B}{\tau_A}[1 - e^{(-t/\tau_B)}] \quad (4)$$

Equation (4) tells us that under these limiting conditions, the rate of decay of B begins at zero and increases asymptotically to the value of A's decay. This implies that if we have an isotope A that has a lifetime much longer than its daugher B, we can observe A's lifetime by waiting for B's decay rate to equal that of A's, then watch the decay of B. τ_B, in this case, will then be equal to τ_A. In this experiment, we take advantage of this fact in measuring the half-life of Pb^{211} through watching the decay of its daughter, Bi^{211}. Naturally, we can also watch the decay of any given radioactive isotope B by removing its source, A, and watching how its alpha activity declines.

3. EXPERIMENT

3.1. Measuring the Half-lives of Po^{218} and Pb^{211}

The radioactive source for this portion of the experiment consisted of uraninite rocks that constantly give off radioactive radon gas. The radon subsequently decays into polonium, and so much recoil energy results from this decay that the polonium atoms are stripped of an electron and become positively charged polonium ions. We apply a high voltage between the sealed can and the ground, causing the polonium ions to move towards and stick to the silicon barrier detector's surface, thus allowing us to record the alpha decay of the polonium ions. Before determining any half-lives, the Multi-Channel Analyzer (MCA) had to be calibrated, in order to determine the channel number to energy relation. This was done by taking 600second spectra of the sealed can, which had high voltage applied to it. A sample calibration run is shown in Figure 3.

To measure the half-life of Po^{218}, a region of interest (ROI) encompassing the proper decay energies was selected within the calibrated MCA spectrum and consecutive spectrum were recorded over a 10-20 minute total time interval. Two runs were done on different days, one with consecutive 60sec intervals and another with consecutive 30sec intervals. The total counts recorded in the selected ROI at each interval was recorded, and the data was fit to an exponential curve. The results of both runs were averaged to obtain a final value for the half-life of Po^{218}.

To measure the half-life of Pb^{211}, we had to use a slightly different strategy, since the decay of this nuclide is actually a beta decay and cannot be observed in the MCA spectrum. Noting that the daughter of Pb^{211}, Bi^{211}, has a much shorter half-life than its parent, we took advantage of Equation (4) and instead watched the alpha decay of the daughter, Bi^{211}, which we could observe in the MCA. We followed a similar procedure, this time using consecutive 500sec runs over a period of several hours, since the half-life of Pb^{211} was longer than that of Po^{218}. Again, the data was fit to a decaying exponential, and the half-life was deduced.

3.2. Measuring the Half-life of Rn^{222}

A liquid scintillator was used in this portion of the experiment. The Beckman scintillation detector consists of two photomultiplier tubes (PMT's) that record flashes of light in the liquid sample placed between them. The radioactive sample of radon we used was dissolved into a liquid scintillation cocktail and placed into a hermetically sealed container. Liquid scintillation cocktails typically consist of xylene or other organic liquids with a small amount of complex aromatic hydrocarbons dissolved in them.

Radon-222, the nuclide of interest, has a half-life which is on the order of a few days, making it the longest-lived nuclide that we observe in this experiment. Thus, to measure its decay, we measured the total counts observed in a 900-second time interval using a simple counter. We then repeated this 900-second measurement several times over a period of about two week in order to measure the slow decay of Rn^{222} and deduce its half-life.

3.3. Measuring the Lifetimes of Po^{212} and Po^{214}

To measure the short half-life of Po^{214} (shorter than 300 μs), we use a Time-to-Amplitude Converter (TAC), which does exactly what its name claims it does. Po^{214} is produced by the beta decay of Bi^{214}, and the lifetime of the Po^{214} nucleus can be determined by measuring the distribution of time intervals between the scintillation pulse produced by the beta decay of Bi^{214} and the subsequent scintillation pulse produced by the alpha particle emitted in the decay of Po^{214}. An aged radon source that was immersed in the scintillation cocktail was used as a radioactive source. The two photomultiplier tubes of the scintillation counter were set on coincidence mode, and the +5 VDC logic pulses produced from the coinci-

dence output was input to the stop output of the TAC and the delayed gate generator. The pulses of the delayed gate generator were input to the start input of the TAC, whose range was set to several multiples of the expected nuclide lifetime. Together, the output of the TAC and the MCA recorded the time intervals between the delayed start pulse and the next stop pulse–the start pulse was delayed so that the TAC would not be triggered to stop by the same pulse that started it. To determine the relation between MCA channel numbers and lifetimes of the Po^{214} nuclei, it was necessary to first calibrate the TAC's output by inputting a signal with a known time period and range.

To measure the extremely short half-life of Po^{212}, it was necessary to use the fact that the birth of a Po^{212} atom is accompanied by the beta decay of the parent Bi nucleus. The resulting energetic electron can be detected by a scintillation detector covered with thin aluminum foil, which can be penetrated by most of the decay electrons. To perform this part of the experiment, it was necessary to first plate the silicon barrier detector used in the uraninite setup (by sealing the can and applying high voltage) with about 2 days worth of radioactive polonium atoms, in order to generate an observable decay of the Po^{212} nuclei. The can was then unsealed, and the silicon barrier detector placed into the well of the scintillation counter so that electrons emitted downward, signalling the birth of a Po^{212} nucleus, could be used to start the timing sequence of a time-to-amplitude converter (TAC). The alpha particle emitted in the subsequent decay of the Po^{212} nucleus was then detected by the silicon detector, and this pulse was used to stop the TAC's timing sequence. The MCA then recorded a pulse whose channel number corresponded to the lifetime of the single Po^{212} nucleus. This part of the experiment was run overnight, in order to gather statistically significant data. Again, the TAC was set to a range appropriate to the nuclide's expected half-life, and to determine how channel numbers corresponded to lifetimes, the TAC was calibrated by inputting a known pulse of fixed period and observing how it was recorded by the MCA.

4. RESULTS AND ERROR ANALYSIS

To perform the calibration for this part of the experiment, the decay activity of the sealed can was recorded for a 600 second interval before each run, and the peaks in the MCA spectrum were labeled according to expected nuclides and their energies. All accepted values for nuclide energies and half-lives used in this paper were obtained from [5]. Two sample calibration runs are shown above, from two different days — the same can was used in each case, and it was sealed approximately three days before the first spectrum was recorded. Observing the changes from one calibration run to the other demonstrates the properties of the Rn decay chains. For example, we see that both Po^{212} and Bi^{212} emerge in the

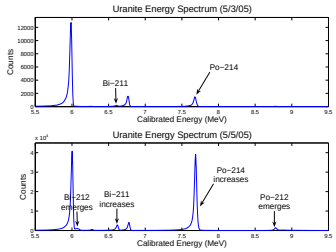

FIG. 4: Evolution of the energy spectrum for a can of uraninite. Reasons for the changes observed are outlined in the paper.

second spectrum, which was taken two days after the first. This can be explained by the fact that both nuclides have Pb^{212} as their parent, and Pb^{212} has a half life of about 10 hours; thus, waiting a few days allows enough of the parent to decay that we can see the activity of its daughters. Similar reasoning explains other features of the plots. Po^{214}'s parent is Rn^{222}, which has a half-life of nearly 4 days, while Bi^{211}'s parent is Pb^{211}, which has a half-life of approximately 0.5 h — in both these cases, waiting two days causes the activity of the daughter nuclides to build up.

All half-lives were measured by fitting the decay curves to falling exponentials, then multiplying the obtained time constants (which represent the full lifetime of a nuclide) by ln(2). The half-lives of Po^{218} and Pb^{211} were measured to be (3.13 ± 0.03) min and (37.4 ± 0.5) min, respectively. Error was determined by using the Poisson distribution to account for random error in countrates, and an additional constant error of 5% was added for the uncertainty in the ROI determinations in the MCA spectra.

The half-lives of Rn^{222} and Po^{214} were measured to be (4.1 ± 0.7) days and (157 ± 3) μs, respectively. A value of (0.26 ± 0.03) μs for the half-life of Po^{212}. Error was determined by using the Poisson distribution to account for random error, and a percentage error was added to account for error in the scintillator detector. To determine if our data corresponded with the Geiger-Nuttall relation, we fit the lifetimes and energies of the three Po isotopes to the empirical Geiger-Nuttall relation explained in the Theory section of this paper and observed a reasonable agreement with theory (Figure 8).

5. CONCLUSIONS

We were able to observe the decay of different isotopes of Rn by watching the evolution of the alpha spectra of a sealed can of uraninite. The half-life of Po^{218} was measured to be (3.13 ± 0.03) min, which was within one standard deviation of the accepted value of 3.1 min. The half-life of Pb^{211} was measured to be (37.4 ± 0.5) min, which was within three standard deviations of the accepted value, 36.1 min. These two values, obtained with

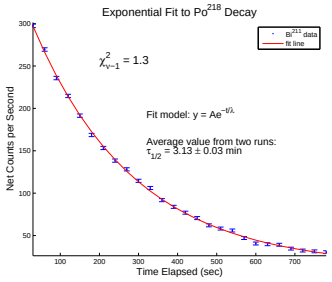

FIG. 5: An example run of Po-218 decay.

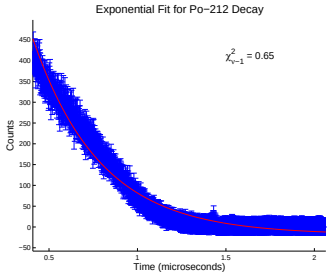

FIG. 6: The decay of Po-212 with fit.

the solid barrier detector and the sealed can of uranite, show reasonable agreement with accepted values. Improvements that could be made to this part of the experiment would be to repeat runs of the same nuclide for greater accuracy, investigate the half-lives of other nuclides observable in the decay chain, observe the build-up of alpha decay activity rather than its decline, and to characterize the systematic error of the silicon barrier detector.

The half-lives of Rn^{222} and Po^{214} were measured to be (4.1 ± 0.7) days and (157 ± 3) μs, respectively, as compared to their theoretical values of 3.82 days and 163.7 μs. A value of (0.26 ± 0.03) μs was obtained for the half-life of Po^{212}, which is within two standard deviations of the accepted value is 0.298 μs, although the fit to experimental data looks less exponential than other data obtained in this experiment — there may be some source of noise in the system. The half-lives of the three isotopes of polonium were found to accurately follow the Geiger-Nuttal relation for half-lives and alpha particle energies. Improvements that could be made to this second half of the experiment would be to repeat runs of various nuclides, or to record spectra for longer times to increase accuracy.

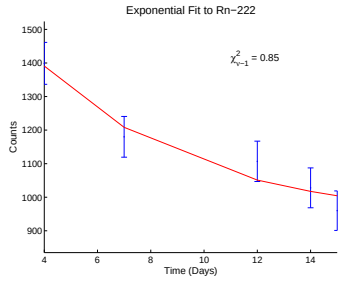

FIG. 7: The decay of Rn-222 with fit — note that the fit line looks jagged because the fit script only rendered points with x-values equal to the those of the 5 input datapoints.

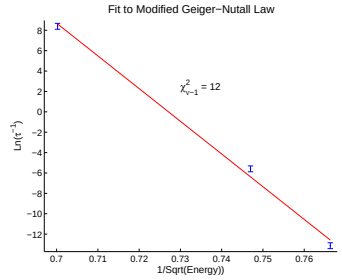

FIG. 8: The decay of Po-212 with fit.

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