

The Instability of a South Polar Cap on Mars Composed of Carbon Dioxide

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If the martian south polar cap were composed of solid carbon dioxide, would it be sufficiently stable against collapse under its own weight? This question is examined in the light of new experiments on the creep of solid carbon dioxide and new high-resolution topographic mapping of the south polar cap. The conclusion from a simple model is that, with the strongest of three flow laws compatible with the measurements, CO₂ is strong enough to support a cap of this material with thickness about 1800 m for 10⁷ years or more, which is the age suggested by the cratering record. However, none of the three possible flow laws will allow a 3000-m cap, the thickness suggested by stereogrammetry, to survive for 10⁷ years, indicating that the south polar ice cap is probably not composed of pure CO₂ ice. © 2000 Academic Press

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1. THE PROBLEM

Spacecraft data indicate that the surface layers of the residual south polar ice deposit of Mars (Fig. 1a) probably consist of CO₂ ice (Paige *et al.* 1990) and are an important reservoir or sink for atmospheric CO₂ (or H₂O) over time. The extent to which the entire thickness of the south polar cap (and the surrounding layered deposits) could be composed of CO₂ ice is unknown. The hypothesis that the cap is made of solid carbon dioxide would have to be rejected if this material were so weak that a cap of the required thickness would quickly collapse under its own weight. Three factors stimulated a new look at this problem:

the launch of the Mars Polar Lander in January 1999 to search for water and carbon dioxide ice near the south pole, new creep experiments on solid carbon dioxide (Durham *et al.* 1999), and new high-resolution stereo topography of the south pole (Schenk and Moore 2000).

A simple model is examined here to estimate the order of magnitude of the collapse time of a putative carbon dioxide polar cap on Mars. The problem is different from the classical glaciological one that assumes a steady state, because it is thought that there has been no significant accumulation or ablation in the past 10⁷ years.

2. SHAPE, AGE, AND COMPOSITION OF THE SOUTH POLAR CAP

High-resolution Viking stereophotogrammetry measurements (Schenk and Moore 2000) show the south polar cap to be a distinct, approximately circular symmetric hump (Fig. 1b) 3000 ± 200 m high, with a radius of 225 ± 25 km. Crater densities suggest that the surface of the layered deposits, of which the polar cap is just a part, are at least 10⁸ years old (Plaut *et al.* 1988). Although no counts were performed on the polar cap itself, it is also very sparsely cratered and is inferred to be 10⁶ to 10⁸ years old.

Recent topographic measurements from the Mars Orbiter Laser Altimeter (MOLA) (Smith *et al.* 1999) confirm the overall stereo-derived measurements of the shape and dimensions of the south polar dome (Schenk and Moore 2000). The MOLA topography also suggests but does not demonstrate that the basal

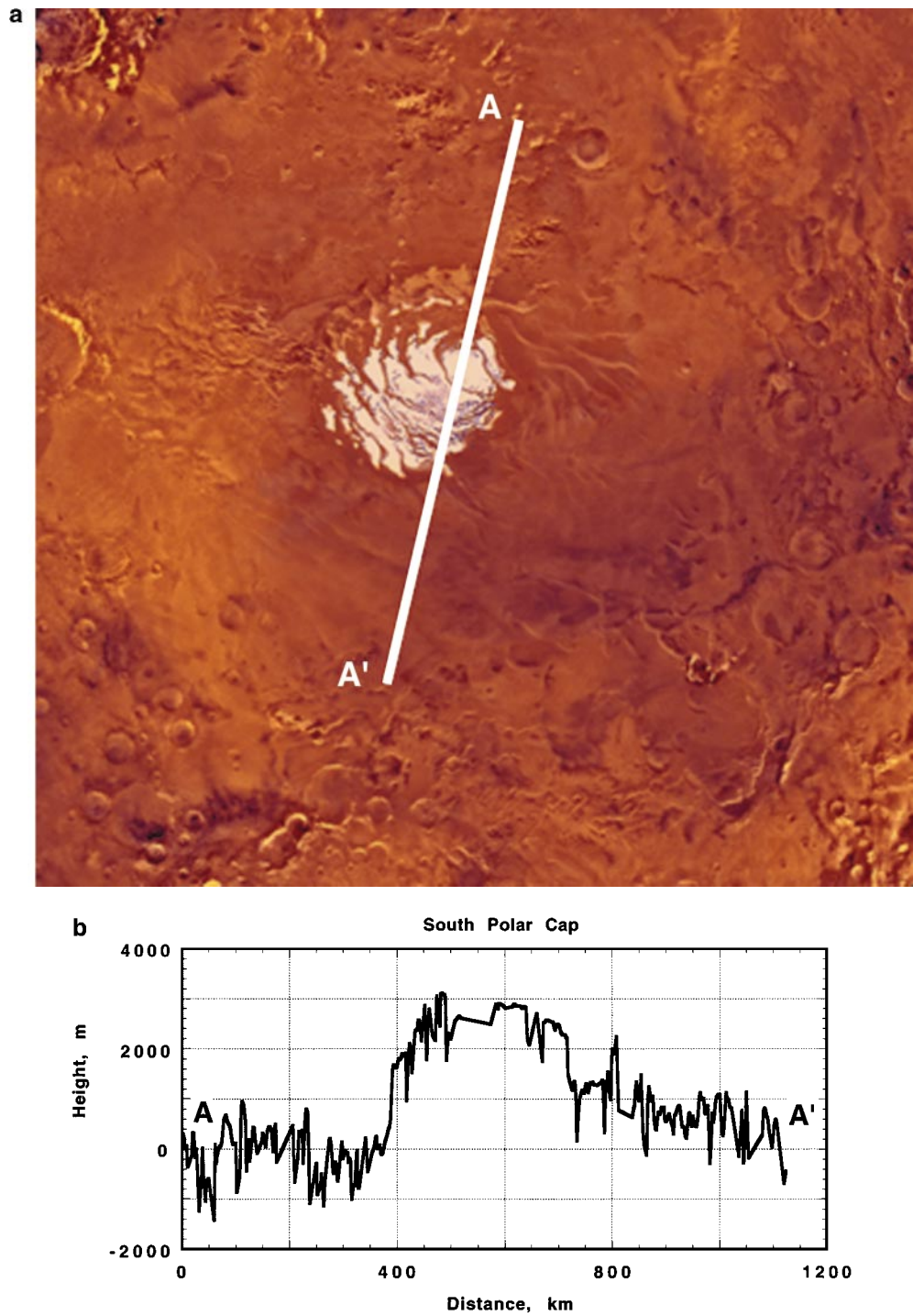


FIG. 1. (a) Color image mosaic of the south polar region of Mars, showing location of profile in (b) (line A–A'). Bright areas are covered by residual frost and ice deposits. South polar cap actually extends roughly 100 km beyond the edge of these deposits. The scene is centered on the south pole and is 2000 km across. (b) Topographic cross section across the south polar cap of Mars (from Schenk and Moore 1999). Profile begins in smooth deposits. The edge of the polar cap begins at ~375 km distance and extends to ~825 km. The right end of the profile is in layered deposits, which are elevated 1 to 2 km above the surrounding plains.

topography of the polar deposit may be deflected downward. The amount of flexure cannot be measured at present but could be as much as 2500 m or it could be negligible (Smith *et al.* 1999). A more detailed analysis of the role of basal topography in ice cap flow is beyond the scope of this work. For present purposes we shall assume a flat base for the cap. The consequences of a depressed basal topography are discussed at the end of this paper.

Although deep troughs and terraces cut into the surfaces of both domes and there is evidence that both deposits were once significantly larger in the distant past, the topographic profiles of the north and south polar ice caps have similar domed shapes (Zuber *et al.* 1998, Schenk and Moore 2000). The surface of the southern cap is scarred by a number of widely spaced concentric scarps, forming a series of descending terraces. These features suggest that the cap is currently undergoing erosion. The degree to which this shape is controlled by erosion is difficult to determine but the overall shape is suggestive of an equilibrium topographic profile of a mass deforming under its own weight. Future analyses may place improved constraints on the history of the south polar deposits. We assume that ablation and erosion has been volumetrically minor in the south polar deposit over the past 10^7 years.

The topographic center of the cap lies on the rim of an 800-km-wide impact basin (Schenk and Moore 2000), but this rim is narrow relative to the fairly flat top part of the dome. The broad symmetry and domed profile of the polar cap suggest that the upper surface does not significantly reflect this buried feature. Other than this rim, the terrains immediately surrounding the edge of the polar cap are relatively flat, suggesting that the local topography does not significantly affect flow within the ice cap itself.

3. THE FLOW LAW

Clark and Mullin (1976) looked at the question of a carbon dioxide polar cap on Mars but were hampered by a lack of reliable topographic data, which prevented them from knowing the true lateral and vertical extent of the south polar cap. They derived a flow law for CO₂ based on uniaxial compression experiments. They ran their tests without the benefit of confining pressure, so they were unable to guarantee suppression of processes involving change of volume, like fracturing and microfracturing. Therefore, their strength measurements must be considered lower bounds, leaving open the possibility that the strength under confining pressure is considerably greater.

New experiments by Durham *et al.* (1999) use similar uniaxial compression but in the presence of elevated confining pressure. They measure the differential stress σ required to maintain a constant displacement rate. The strain rate $\dot{\epsilon}_1$ is assumed to be given by

$$\dot{\epsilon}_1 = A_D \exp(-Q/R_G T) \sigma^n,$$

TABLE I
Computed Values for Different Flow Laws

	CO ₂ ice (a)	CO ₂ ice (b)	CO ₂ ice (c)	H ₂ O ice (d)
$A_D, s^{-1}(MPa)^{-n}$	9.55×10^{-3}	4.37×10^3	1.202×10^{11}	9.00×10^4
n	2	4.5	7	3
$Q, J/mol$	15,000	31,000	59,000	60,000
$C, yr^{-1} m^{-n}$	5.85×10^{-4}	3.79×10^{-8}	5.72×10^{-14}	7.94×10^{-13}
$\tau (r = 0.5 R_0), MPa$	0.111	0.126	0.129	0.071

where R_G is the gas constant, and most of the results fit this equation with the values of the material constants, A_D , n , and Q , shown in column (b) of Table I. However, as the authors discuss in their paper, there is considerable uncertainty in the value of n . Repeated anomalies in the experimental data indicate that n may be 7 or higher, while theoretical arguments, as well as experience with other crystalline materials in the geologic setting suggest that n could be much lower than 4.5 at the low stresses *in situ*. Deformation mechanisms such as grain boundary sliding (e.g., Goldsby and Kohlstedt 1997) are favored at low stresses and can have n values of 2 or less. To allow for all reasonable possibilities the alternative laws shown as (a) and (c) are also considered. Although the three laws look very different, because the values of A_D differ extremely, in fact they all agree at the lowest stresses used in the experiments; thus, the value of n decides the extrapolation from laboratory strain rates to the much slower ones in a martian cap. The experiments confirm that solid CO₂ is weaker than water ice by a factor of well over 10 and show that the measurements by Clark and Mullin (1976) underestimated the strength of CO₂ by a factor of only about 2.

The flow law thus measured by Durham *et al.* (1999) relates uniaxial compression rate $\dot{\epsilon}_1$ to uniaxial compressive stress σ , while for the cap model it is necessary to relate the rate of simple shear $\dot{\gamma}$ to the shear stress τ_0 . We write for the two kinds of flow law at a given temperature,

$$\dot{\epsilon}_1 = B \sigma^n \quad \text{and} \quad \dot{\gamma} = D \tau_0^n,$$

so that

$$B = A_D \exp(-Q/R_G T). \quad (1)$$

The transformation between them is made via the generalized flow law (Nye 1953, Paterson 1994) $\dot{\epsilon} = A \tau^n$, where $\dot{\epsilon}$ is the effective strain rate and τ is the effective shear stress, which is valid for all states of stress and strain rate. Here τ is constructed from the stress tensor σ_{ij} by defining the stress deviator $\sigma'_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}$ and writing $2\tau^2 = \sigma'_{ij}\sigma'_{ij}$ (τ positive). In a similar way $\dot{\epsilon}$ is constructed from the strain rate tensor $\dot{\epsilon}_{ij}$ by writing $2\dot{\epsilon}^2 = \dot{\epsilon}_{ij}\dot{\epsilon}_{ij}$ ($\dot{\epsilon}$ positive). It then follows that

$$D = 3^{\frac{1}{2}(n+1)} B. \quad (2)$$

4. THE ICE CAP MODEL

Because we are concerned only with an order of magnitude for the decay time we shall ignore the curvature of the planet's surface and assume a circular cap resting on a plane horizontal base (Fig. 2). With a mean annual surface temperature of 155 K and estimates for heat flow based on a terrestrial model, the temperature gradient may be high enough to cause basal melting (at 217 K) of a CO₂ cap (Ross and Kargel 1998). Nevertheless, so as not to overestimate the rate of thinning, we shall assume a uniform temperature of 180 K and that the base is below the melting point; the latter means that there is no sliding on the base (Paterson 1994). It is assumed, therefore, that the deformation is predominantly by horizontal shearing within the ice.

At radius r and time t the thickness is $h(r, t)$. The thickness at the center $h(0, t)$ is denoted by $h_0(t)$ and the radius of the base is $r_0(t)$. The required solution of the flow equations, a similarity solution, was introduced into glaciology by Halfar (1981) for two-dimensional flow and (Halfar 1983) for radial flow, as here. Hindmarsh (1990) has discussed Halfar's two-dimensional solution. Here we first deduce Halfar's radial flow solution from a similarity hypothesis and then apply the result to the martian situation.

Defining a scaled radial coordinate $\rho = r/r_0(t)$ and a scaled thickness,

$$\eta = \frac{h(r, t)}{h_0(t)} = \frac{h(r_0\rho, t)}{h_0(t)} = \eta(\rho, t),$$

we look for a solution of the flow equations in which $\eta = \eta(\rho)$, independent of t ; that is, the scaled shape always remains the same.

Let $q(r, t)$ be the radial flux (volume per unit time crossing an area of height h and unit circumferential length). Then conservation of volume requires that

$$\left(\frac{\partial(qr)}{\partial r}\right)_t + r\left(\frac{\partial h}{\partial t}\right)_r = 0, \quad (3)$$

the second term of which is

$$r\left(\frac{\partial h}{\partial t}\right)_r = r_0\rho\left(\frac{\partial(h_0\eta)}{\partial t}\right)_r = r_0\rho\left(\dot{h}_0\eta + h_0\frac{d\eta}{d\rho}\left(\frac{\partial\rho}{\partial t}\right)_r\right). \quad (4)$$

Because the overall volume is fixed and the scaled shape is fixed,

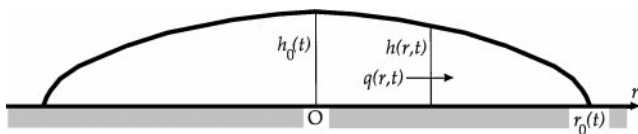


FIG. 2. The simplified model.

$d/dt(h_0r_0^2) = 0$, and hence $\dot{r}_0 = -(r_0/2h_0)\dot{h}_0$. Since $\rho = r/r_0$,

$$\left(\frac{\partial\rho}{\partial t}\right)_r = -\frac{r}{r_0^2}\dot{r}_0 = \frac{\rho}{2h_0}\dot{h}_0.$$

Thus, from (4),

$$r\left(\frac{\partial h}{\partial t}\right)_r = r_0\rho\dot{h}_0\left(\eta + \frac{1}{2}\rho\frac{d\eta}{d\rho}\right) = \frac{1}{2}r_0\dot{h}_0\frac{d}{d\rho}(\rho^2\eta), \quad (5)$$

and Eq. (3) becomes, in terms of ρ ,

$$\left(\frac{\partial(q\rho)}{\partial\rho}\right)_t + \frac{1}{2}r_0\dot{h}_0\frac{d}{d\rho}(\rho^2\eta) = 0. \quad (6)$$

The crucial step is in (5), where $(\partial/\partial t)_r$ is converted to $d/d\rho$. Equation (6) may now be integrated with respect to ρ at constant t ; thus

$$q\rho + \frac{1}{2}r_0\dot{h}_0 \cdot \rho^2\eta = \text{constant}.$$

Dividing by ρ and demanding that $q = 0$ at $\rho = 0$ sets the constant to zero.

We now use the flow relation (shallow ice approximation) (Nye 1952),

$$q = Ch^{n+2}(-\partial h/\partial r)_t^n = C\frac{h_0^{2n+2}}{r_0^n} \cdot \eta^{n+2}(-d\eta/d\rho)^n, \quad (7)$$

where

$$C = D(\rho'g)^n/(n+2). \quad (8)$$

ρ' is the density, and the full stop in (7) separates the time-dependent part from the ρ -dependent part. At the extreme edge the slope becomes infinite and the assumed relation (7) must break down; it should be replaced by different physics, but because the region affected is very narrow we shall ignore this defect. A similar problem arises in all models of this kind. The equation to be integrated is therefore

$$C\frac{h_0^{2n+2}}{r_0^n} \cdot \eta^{n+2}(-d\eta/d\rho)^n + \frac{1}{2}r_0\dot{h}_0 \cdot \rho\eta = 0. \quad (9)$$

The variables may now be separated to give

$$2C\eta^{n+1}(-d\eta/d\rho)^n = K\rho \quad (10)$$

and

$$\dot{h}_0 = -K(h_0^{2n+2}/r_0^{n+1}), \quad (11)$$

where K is a constant.

The profile follows from integrating (10). First, require that $\eta = 0$ at $\rho = 1$,

$$-\int_1^0 \eta^{1+1/n} d\eta = \left(\frac{K}{C}\right)^{1/n} \int_0^1 \rho^{1/n} d\rho,$$

from which

$$K = 2C \left(\frac{n+1}{2n+1}\right)^n. \quad (12)$$

Then the scaled profile is

$$\rho^{1+1/n} + \eta^{2+1/n} = 1, \quad (13)$$

as found by Halfar (1983). It is worth noting that, if the base were assumed to be not horizontal but depressed by a constant fraction f of the overlying height, the only difference is that h , h_0 , and η refer to the thickness rather than the elevation and C is replaced by $C(1-f)^n$ throughout. In particular, the relation between ρ and η in (13) is not changed.

Integration of (11) gives the time dependence. It is convenient to think of $t = 0$ as the present, when the central height is H_0 and the radius of the cap is R_0 say. Then $h_0 r_0^2 = \text{constant} = H_0 R_0^2$, and (11) becomes

$$\dot{h}_0 = -K (H_0 R_0^2)^{-(n+1)/2} h_0^{(5/2)(n+1)}.$$

This integrates to give the decay of the central height as

$$h_0(t) = H_0 \left(1 + \frac{t}{t_0}\right)^{-2/(5n+3)}, \quad (14)$$

with, using (12),

$$t_0 = \frac{1}{C(5n+3)} \cdot \left(\frac{2n+1}{n+1}\right)^n \cdot \frac{R_0^{n+1}}{H_0^{2n+1}}. \quad (15)$$

$t = -t_0$ is the time at which the central height is asymptotically infinite (Fig. 3). Since $H_0 R_0^2$ is constant the corresponding equation for the expansion of the outer rim is

$$r_0(t) = R_0 \left(1 + \frac{t}{t_0}\right)^{1/(5n+3)}, \quad (16)$$

and the aspect ratio diminishes according to

$$\frac{h_0(t)}{r_0(t)} = \frac{H_0}{R_0} \left(1 + \frac{t}{t_0}\right)^{-3/(5n+3)}. \quad (17)$$

In Eq. (15) C , R_0 , and H_0 are observed quantities, and so t_0 can be calculated. In this model the profile of the cap decays steadily from an initial δ -function at $t = -t_0$, but always retaining the same scaled shape. Of course, the real history of the martian

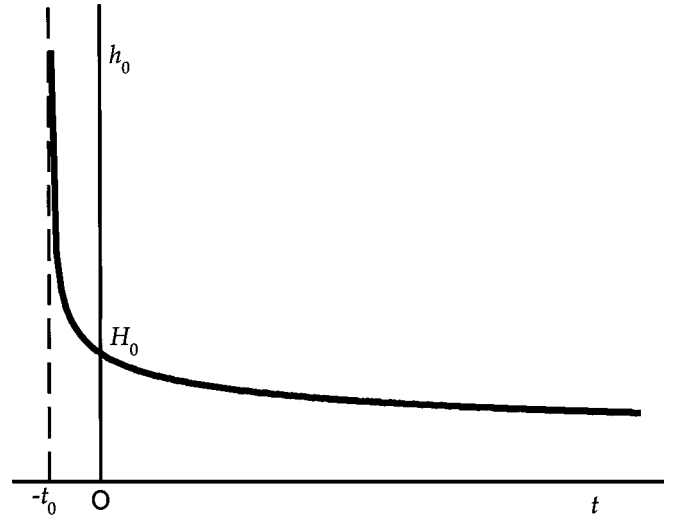


FIG. 3. Time dependence of the central height h_0 .

southern polar cap was not this, but the justification for using such a simple model is that, as Halfar (1983) has shown, the scaled profile is stable against all perturbations that preserve the total volume. Thus, whatever the actual genesis, the surface may be expected to have settled down quite quickly (in a time much less than t_0) to the steady profile calculated (much as molasses falling off a spoon onto a plate quickly forms a dome shape). We can say that, if the actual cap is more than 10^7 years old, it means that $t_0 > 10^7$ years.

5. NUMERICAL VALUES USED

The values used were $R_0 = 200$ km, $H_0 = 3000$ m, $\rho' = 1560$ kg m $^{-3}$, $g = 3.72$ m s $^{-2}$, $T = 180$ K. The first three columns of Table I show the constants for the three flow laws (a), (b), and (c) for solid CO $_2$. For comparison, column (d) of the table shows the corresponding constants for H $_2$ O ice at the same temperature. The flow law used for H $_2$ O ice is that recommended by Paterson (1994) (see below for alternatives). The fourth line of the table gives the values of C obtained by using Eqs. (1), (2), and (8).

6. PROFILE AND DISCUSSION

Figure 4 shows the scaled profiles computed from Eq. (13) for the four flow laws. The scaled profiles depend only on n and not on C . They are all quite similar and may be compared with the parabolic profile $\eta = (1 - \rho)^{1/2}$ corresponding to the assumption of perfect plasticity (Orowan 1949, Nye 1951). The approach to the limit $n \rightarrow \infty$ is clearly seen. The curves with finite n all have zero slope at the summit, in contrast with the parabola which does not. The parabola is then necessarily below the other curves. The present basal shear stress $\tau = \rho' gh(-\partial h/\partial r)$ is much the same for all the CO $_2$ flow laws, but it is smaller for water ice because of its lower density. The values of τ at $r = 0.5R_0$ are shown in the last line of the table.

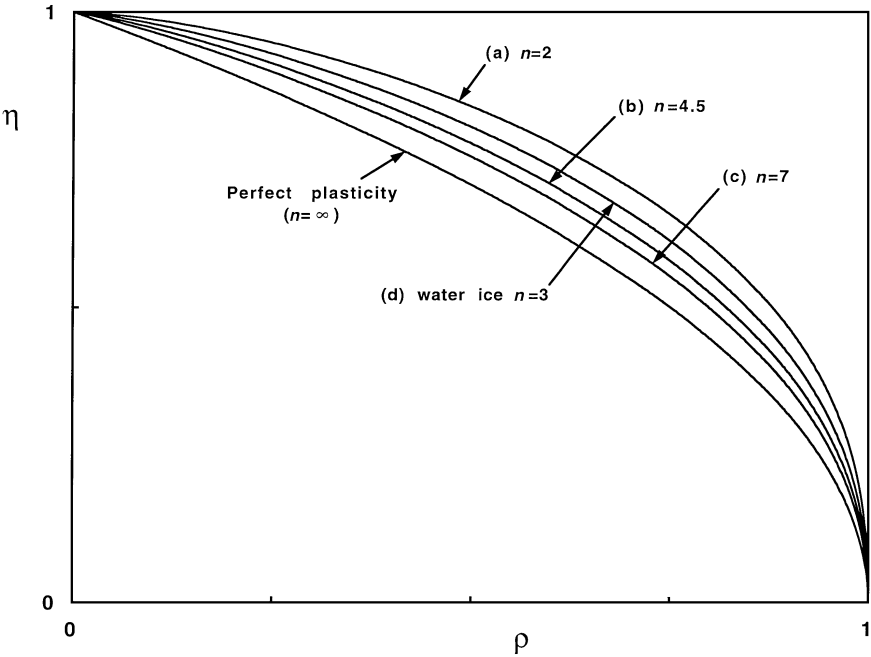


FIG. 4. Scaled profiles. The flow laws are as in Table I, with an additional parabolic curve for perfect plasticity.

Figure 5 plots t_0 against H_0 from (15) for these three different assumptions about the flow law for CO₂ and for water ice. The broken line drawn at $t_0 = 10^7$ years, the inferred minimum age, shows that even with the strongest of the three flow laws for CO₂

(law (c)) the height would be merely 1840 m after this length of time; alternatively, the broken line drawn at $H_0 = 3000$ m, the thickness suggested by stereogrammetry, shows that a cap of this height could have an age of 6700 years at most.

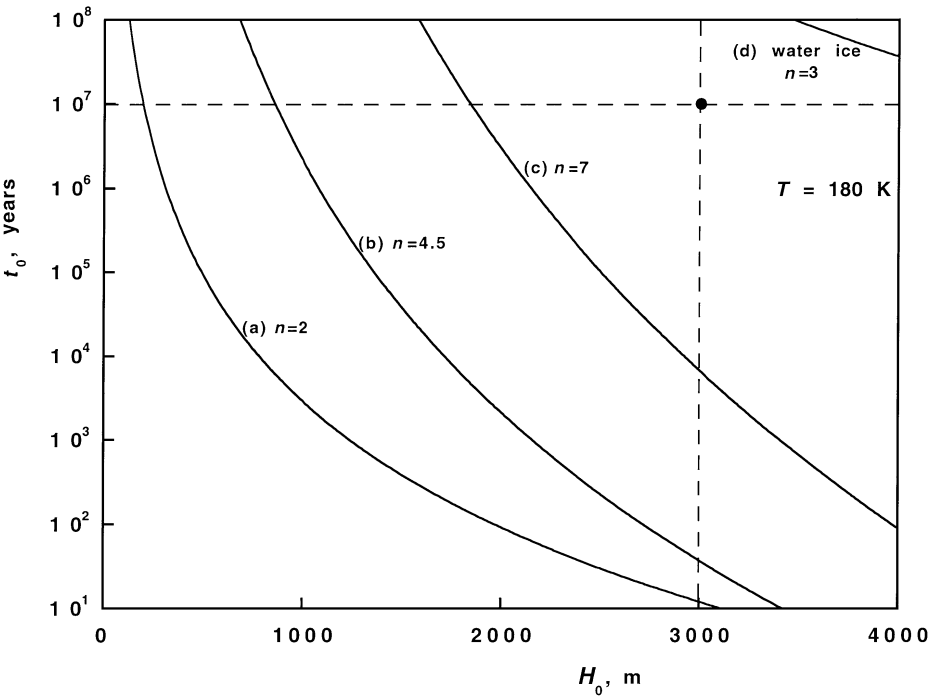


FIG. 5. The age t_0 versus the central height H_0 at the present time $t = 0$. (a), (b), and (c) refer to the three flow laws for solid CO₂, and for water ice (d), given in Table I. The observed height and the assumed age are indicated ($R_0 = 200$ km).

On the other hand, Fig. 5 also shows that H₂O ice is quite strong enough to support a 3000-m ice cap. H₂O ice at the same uniform temperature of 180 K could support a height of 4800 m after 10⁷ years. A similar conclusion follows for other ice flow laws. The flow law recommended by Durham *et al.* (1997) ($A_D = 1.26 \times 10^5$, $n = 4$, $Q = 61,000$, with units as in Table I) gives an even stronger ice cap, which could support a height of 5600 m after 10⁷ years. If grain boundary sliding operates in ice under martian conditions, the ice cap becomes weaker and falls to $H_0 = 3730$ m after 10⁷ years, based on the water ice flow law of Goldsby and Kohlstedt (1997) with grain size 1 mm ($A_D = 61.8$, $n = 1.8$, $Q = 49,000$, with units as in Table I). Thus the water-ice hypothesis is favored.

The model assumes a flat base. Figure 1b shows no evidence of any buried mountains, but positive elevation of the underlying terrain would mean that the surface of the cap is actually higher than is calculated here. The buried rim of Prometheus basin is only 10–20 km wide (Schenk and Moore 2000) and is not significant enough to affect our conclusions. Correspondingly, depression of the lower surface, as allowed in the recent MOLA observations (Smith *et al.* 1999), would lead to a lower elevation of the top surface. Viewed alternatively, holding the elevation of the modeled cap at the observed value of 3000 m, positive elevation of the underlying surface would mean a thinner ice layer is present and would therefore work against the conclusions presented here. Isostatic depression of the underlying surface, would indicate a thicker ice layer, reinforcing our conclusions.

We should also note that the experimental results for the flow law are for pure CO₂. Impurities, different grain sizes, preferred orientations, and so on would certainly have some effect (as they do in water ice), but they are perhaps unlikely to change the order of magnitude of the values computed for solid CO₂.

We conclude that the south polar cap probably consists of water ice, with an unknown admixture of dust.

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