



MIT 15.S50 LECTURE 4

Wednesday, January 23rd, 2013

REVIEW OF EXPECTATION AND EQUITY: DO YOU GO ALL-IN OR FOLD?



OBVIOUSLY IT DEPENDS ON THE PLAYERS...

- Is the BB a gambly player? Is it likely that he or she will call with a wide range of hands?
- How have I been playing? Do I look like a crazy all-in monkey, or will my all-in get the credibility of a very good hand?
- (Suppose it is far from the final table.)



ANALYSIS

- Let's suppose the BB calls a “reasonable” range:

AA	AKs	AQs	AJs	ATs	A9s	A8s	A7s	A6s	A5s	A4s	A3s	A2s	
AKo	KK	KQs	KJs	KTs	K9s	K8s	K7s	K6s	K5s	K4s	K3s	K2s	
AQo	KQo	QQ	QJs	QTs	Q9s	Q8s	Q7s	Q6s	Q5s	Q4s	Q3s	Q2s	
AJo	KJo	QJo	JJ	JTs	J9s	J8s	J7s	J6s	J5s	J4s	J3s	J2s	
ATo	KTo	QTo	JTo	TT	T9s	T8s	T7s	T6s	T5s	T4s	T3s	T2s	
A9o	K9o	Q9o	J9o	T9o	99	98s	97s	96s	95s	94s	93s	92s	
A8o	K8o	Q8o	J8o	T8o	98o	88	87s	86s	85s	84s	83s	82s	
A7o	K7o	Q7o	J7o	T7o	97o	87o	77	76s	75s	74s	73s	72s	
A6o	K6o	Q6o	J6o	T6o	96o	86o	76o	66	65s	64s	63s	62s	
A5o	K5o	Q5o	J5o	T5o	95o	85o	75o	65o	55	54s	53s	52s	♣♣
A4o	K4o	Q4o	J4o	T4o	94o	84o	74o	64o	54o	44	43s	42s	♦♣
A3o	K3o	Q3o	J3o	T3o	93o	83o	73o	63o	53o	43o	33	32s	♥♣
A2o	K2o	Q2o	J2o	T2o	92o	82o	72o	62o	52o	42o	32o	22	♠♣

25.2%

- Your equity against this range:

Player 1	R/D	22+, A2s+, K8s+, QTs+, A2o+, KTo+	63.545%
Player 2	R/D	T8o	36.455%



DO THE MATH

- 75% of the time, he folds, and we win 2.5BB's
- 25% of the time, he calls,
 - 36% of the time, we win the all-in and win 16.5BB's
 - 64% of the time, we lose the all-in and lose 14.5BB's
- $0.75(+2.5) + 0.25(0.36(+16.5) + 0.64(-14.5))$
- $=0.75(+2.5) + 0.25(-3.34)$
- Clearly positive.
- $=1.04\text{BB}$
- You're making an entire big blind by shoving instead of folding! We really are maximizing our expected # of chips by shoving!



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DO THE MATH AGAIN

- Now we have 28% equity, instead of 36% equity, against his calling range
- Expectation when he calls:
 - $0.28(+16.5) + 0.72(-14.5) = -5.82$
- Overall expectation:
 - $0.75(+2.5) + 0.25(-5.82)$
- Still positive! This shove is an EXCELLENT play that earns you 0.42BB's, not a “crazy bad” play. We are still earning positive chips/money!



THE POINT?

- If you're only calling this 25% from the Big Blind in this situation, then you're allowing the Small Blind to shove any 2 cards profitably.
- Let's suppose we consider adding QJo to our calling range. We need to call 14BB to win a total pot of 31BB, so we need 45% equity.
- Nonetheless, since most Small Blinds don't shove enough, we might not actually have this!



MORE MATH

- Let's assume that the Small Blind only shoves the top 25% of hands (instead of 100%, which includes 32o).

- Your equity with QJo:

Player 1	$\frac{R}{D}$	66+, A2s+, K6s+, Q8s+, J8s+, T8s+, A7o+	57.774%
Player 2	$\frac{R}{D}$	QJo	42.226%

- You only have 42% equity. Not enough to call the all-in!



LET'S TAKE A LOOK AT THE NASH RANGES

- SB **66.8%**, 22+ Kx+ Q2s+ Q5o+ J2s+ J7o+ T2s+ T7o+ 93s+ 96o+ 84s+ 86o+ 73s+ 76o 63s+ 65o 53s+ 43s
- BB **38.5%**, 22+ Ax+ K2s+ K5o+ Q6s+ Q8o+ J8s+ JTo T9s
- This is a potential formula to follow, when figuring out which hands to go all-in with from the SB, and which hands to call with from the BB.
- However, if the BB is calling way fewer than 38.5% of hands, then as SB you can go all-in with a lot more than 66.8% of hands.
- If the SB is going all-in with way fewer than 66.8% of hands, then as BB it is a huge mistake to call with as much as 38.5% of hands.



BUT IT'S AN EQUILIBRIUM...

- SB **66.8%**, 22+ Kx+ Q2s+ Q5o+ J2s+ J7o+ T2s+ T7o+ 93s+ 96o+ 84s+ 86o+ 73s+ 76o 63s+ 65o 53s+ 43s
- BB **38.5%**, 22+ Ax+ K2s+ K5o+ Q6s+ Q8o+ J8s+ JTo T9s
- If the SB is going all-in with 66.8% of hands, then the optimal strategy for the BB is go call with 38.5% of hands.
- If the BB would call an all-in with 38.5% of hands, then the optimal strategy for the SB is to go all-in with 66.8% of hands.



A FEW MORE NASH RANGES

○ BTN, 10BB

- **43.9%**, 22+ Ax+ K2s+ K6o+ Q5s+ Q9o+ J7s+ J9o+ T6s+ T9o 96s+ 98o 85s+ 75s+ 65s 54s

○ CO, 7BB

- **38.8%**, 22+ Ax+ K2s+ K9o+ Q5s+ QTo+ J7s+ J9o+ T7s+ T9o 96s+ 86s+ 75s+ 65s 54s

○ LJ, 10BB

- **23.4%**, 22+ A2s+ A8o+ K7s+ KJo+ Q8s+ QJo J8s+ JTo T8s+ 98s

○ UTG (9 handed), 3BB

- **24.1%**, 33+ A2s+ A8o+ K6s+ KTo+ Q8s+ QTo+ J9s+ JTo T9s



SUPPOSE I AM TOO SCARED TO GO ALL-IN...
CAN I RAISE TO 2BB AS AN ALTERNATIVE?



LET'S PUT OURSELF IN BB'S SHOES...

- If we go all-in, BU will need to call 6 to win a pot of 17.5, so he will call so long as he thinks he has at least 34% equity against our range.
 - So he will pretty much always call. (“When given 2-to-1 to call a pre-flop all-in, you can pretty much always call unless you are guaranteed to be a 70-30 or worse.”)
- When we go all-in, assuming BU calls, we risked 7 to win a pot of 17.5, so we need 40% equity for this to be profitable.



ANALYSIS AS THE BB

- If our hand has 40% equity vs. BU's range, then going all-in is +EV.
- If our hand doesn't have 40% equity vs. BU's range, calling could still be +EV, since we are given 4.5-to-1 to see the flop.
- Even if our hand has 40% equity vs. BU's range, calling could be more +EV than going all-in.



STRATEGY FOR BB

- If our hand has more than 50% equity vs. BU's range, then it is most likely in our interest to get all-in instead of just calling, since it is in our favor to put money in 1-to-1.
- If our hand has less than 50% equity vs. BU's range, then it is most likely in our interest to just call, even though we have Reverse Implied Odds postflop from being out of position.
- Even if our hand has less than 40% equity vs. BU's range, as long as it has more than ~30% equity, it is probably +EV to call given 4.5-to-1 odds to see the flop.



BIG LESSON

- None of these opportunities for the BB would've arisen had the BU just went all-in in the first place!
- The BB would've just had to call the hands with 40% equity, and fold the rest.
- But now, we're allowing the BB to still get the money in when he has 50%+ equity, play any hand with ~30% equity, etc.



WHAT ABOUT RAISING TO 4X?



ANALYSIS

- If the BB were to fold, then raising 4x is equivalent to going all-in.
- If the BB were to re-raise all-in himself, then raising 4x is equivalent to going all-in (since we are going to call his re-raise).
- Are there situations where the BB is better off calling, and folding some flops? Probably not, since the pot will be 1880 on the flop where each player has 800 left. So the BB calling is equivalent to the BB re-raising all-in, since no player is ever folding postflop.



SUCKER RAISE

- However, the analysis for the last bullet is hard. Maybe there are some opportunities for the BB to squeeze extra EV with a few of the hands in his range, but just calling preflop.
- Going all-in gives the BB strictly fewer options, so it is the “right” play.
- For raising to 2x or 3x or 4x to be a better play than going all-in, the BB has to be **SO STUPID** that having more options will somehow cause him to make a worse decision. This is the same concept as what Bill Chen called a “sucker bet” in his lecture.



CHECKPOINT

- We just:
 - Reviewed how to do an Expectation / Equity calculation.
 - Explained why going all-in is so profitable when both
 - Everyone in front of you has folded
 - You are risking less than 15BB
 - Explained why raising to 2.25BB as a cowardly alternative, when the above two factors are present, is nowhere near as profitable
 - Give the other plays more options
 - Allow them to play a larger % of hands, instead of having to fold everything



AN IN-DEPTH COMBINATORIAL HAND ANALYSIS



CALLING FOR IMPLIED ODDS IN POSITION



BB ALSO CALLS.



CO CONTINUATION BETS



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ANALYSIS

- Definitely not folding, and raising is suicidal when we have a medium-strength hand that could play well in position on a lot of turns. We have a back door flush draw, and our straight draw is very legitimate when there is no flush draw out there.
- We must be wary of a Big Blind check-raise, although this is less likely when a flush draw isn't out.



BB FOLDS



HE BETS 30 ON TURN



ANALYSIS

○ Benefits of Calling:

- See the river for sure, instead of having to fold to his turn 3-bet all-in. We really want to see the river because we have plenty of opportunities to improve, and already have a pair to start with, so will win the hand by checking sometimes.

○ Benefits of Raising:

- Can get him to fold better hands, say JJ.
- Can bet the river and win a bigger pot when hit a club or an 8. (It is hard to do this when we hit a 9 or 7, since in both cases there is 4-to-a-higher-straight.)
- I like calling in this situation. With a draw that cannot win showdown like 95cc, I would raise. Or with a draw that is good enough to call his turn 3-bet all-in, like AJcc, I would raise.



WE CALL.



HE CHECKS RIVER.



WE BLUFF THE RIVER!



ANALYSIS

- Yes, we do have some chance of winning with our pair of 8's, so there may be no need to bluff. But:
 - There are a lot of higher cards on the board now. The pair of 8's that looked pretty good on the flop is a lot worse after a Q turn and A river.
 - The pot is big; money went in on both the flop and turn. If our opponent has a Ten or Queen or even weak Ace, he will fold to our bet.
 - The Ace is always a scary card.
 - The advantage of being in position is that we already know our opponent checked the river. So he is unlikely to have a great hand, and more likely to fold to our bluff.



CO FOLDS.



EXACT ANALYSIS

- What range of hands do we put our opponent on exactly?
- Obviously this is impossible to answer exactly, the entire purpose of poker is to approximate this as well as possible.
- To do this, we must put ourselves in our opponent's shoes.



WHAT IS OUR OPPONENT'S RANGE HERE?



ANALYSIS

- Usually, trying to put him on exact hands preflop is difficult, especially when he opens as late as the Cutoff.
- Estimating that he opens somewhere around 33% is probably reasonable. This includes pair, any suited ace, any two Broadway cards, but not unsuited hands, and stuff like T8o.



WHAT IS OUR OPPONENT'S RANGE HERE?



ANALYSIS

- Our opponent continuation bet.
- What do we put him on? Let's look at what he would and wouldn't continuation bet.
- Let's review the factors of when a player would continuation bet. We talked about this when we introduced continuation betting, and also when we talked about playing draws postflop.



RULES FOR CONTINUATION BETTING

- Incentives for Continuation Betting:
 - Your hand is good enough that it beats even most of his calling hands.
 - Your showdown value is poor, but you have some equity (or back door equity).
 - You are out of position and cannot see a free turn by checking.
 - Few opponents (1 or 2).
- Incentives against Continuation Betting:
 - Your hand is so dominant that you need to give him a turn card to hope that he improves.
 - You have decent showdown value.
 - You have zero equity because your hand is just so terrible.
 - You are in position and can see a free turn by checking.
 - There are too many people in the pot to get them all to fold.



THE SITUATION AGAIN...



ANALYSIS

- Hands that he checks to trap or check-raise:
 - 97, TT, maybe 88 (unlikely because we have an 8)
- Hands that he value bets:
 - AA-JJ, 66, T8, 86, T6, AT, KT, etc.
- Hands of medium-strength that he check-calls:
 - JT, T9, 99, 77, T7, 76, etc.
- Hands that he “bluffs”:
 - A6, K8, AK, AJ, Q9, QJ, J7, etc.
- Hands that he check-folds because they have zero equity:
 - Q5, 33, A2ss, etc.



“BAYESIAN UPDATING”, OR MORE “BAYESIAN ELIMINATING”

- ~~○ Hands that he checks to trap or check raise:~~
 - ~~• 97, TT, maybe 88 (unlikely because we have an 8)~~
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- Hands that he “bluffs”:
 - A6, K8, AK, AJ, Q9, QJ, J7, etc.
- ~~○ Hands that he check folds because they have zero equity:~~
 - ~~• Q5, 33, A2ss, etc.~~



THE SITUATION ON THE TURN



HIS INCENTIVES FOR BETTING AGAIN ON THE TURN

- You bet a good hand for value on the flop, and your hand is still good on the turn. You're betting for more value.
- You bluffed a speculative hand on the flop, and now either hit the turn or improved your draw.
- An overcard to the board came, so you thought bluffing the turn could be good.



A screenshot of a Texas Hold'em poker game interface. The game is in progress on a green felt table. The community cards are 10 of Clubs, 8 of Hearts, 6 of Diamonds, and Queen of Clubs. The pot is \$47. The players and their stacks are: BB (\$194), UTG (\$200), SB (\$199), HJ (\$200), CO (\$150), and BTN (\$180). The BTN player is showing 8 of Clubs and 7 of Clubs. The CO player has bet \$30. The interface includes buttons for Chat, Notes, Stats, and Info at the bottom left, and buttons for LEAVE TABLE and VIEW LOBBY at the top right. The last action log shows "CO bets 30\$".

BB
\$194

UTG
\$200

SB
\$199

HJ
\$200

CO
\$150

BTN
\$180

Community Cards: 10♣, 8♥, 6♦, Q♣

Pot: \$47

CO bets 30\$

Chat Notes Stats Info

LEAVE TABLE
VIEW LOBBY

Last Action



RIVER SITUATION



HE CHECKS RIVER...

- It seems improbable that our opponent would be trying to trap us with his check:
 - He's already been showing plenty of aggression, so it's not like he can pretend he has a bad hand by checking.
 - With the pot already so big, he can get a large % of the remaining stacks by just betting (instead of check-raising).
 - The Ace is not a good card for our range, so not a card we're likely to bet.
- So it's unlikely that he has a nutty hand.
- However, he could very easily still have a hand that beats us and calls our potential bluff.



ANALYSIS ON BOARD...

- We conclude that his range on the river will consist of
 - AKs, AJs, KQs (that had backdoor flush draw on flop)
 - QJ, Q9s, KK
 - A9, T8s, 86s
- Assuming that he will call with exactly top pair or better.
- We are risking 70 to win 100.
- Since he calls about 50% of the time, the bluff is profitable.
- Note that we are only winning once in a blue moon by checking, in the rare case he has K9 or something weird like J7s, 74s and decided to not bluff the river.

