Jen is dealt a random card from a deck with the cards 1,2,...,10. Bill is dealt a random card from a separate deck with the cards 1,2,...,10. Both players are forced to ante $50 into the pot at the start of the hand.

The betting works as follows: only Jen has the option to either Bet $100 or Check. If she checks, then showdown happens immediately and the player with the higher card wins the pot (in the case of a tie, the pot goes to Bill). If she bets, then Bill can either Call or Fold. If Bill Calls, then he also wagers an additional $100 and showdown occurs (now for a larger pot of $300). If Bill Folds, then Jen collects the pot of $100 without showdown.

For each Bill that Jen is playing against, outline her most profitable exploitative strategy. Circle the hands that she should Bet $100 with, and also calculate how much she expects to profit per game using this strategy.

Example: vs. Bill Cosby who calls every card, we only want to "value-bet". The best strategy is to bet with cards 6's or better (it actually doesn't matter whether we bet a 6). The amount we make per game is: half the time, we get dealt 6+, in which case both players wager $150 (since we will bet and he will call) and we will win 70% of the time (we will win 50% of the time if we have a 6, and 90% of the time if we have a 10, so this averages out to 70%); the other half, we get dealt 1-5, in which case both players wager $50 (since we will check) and we will win 20% of the time. The total EV is:

\[ 0.5 \times (0.7 \times 150 + 0.3 \times (-150)) + 0.5 \times (0.2 \times 50 + 0.8 \times (-50)) = 15 \]

1. Bill Gates: very tight, he will only call with a 10+.
   1 2 3 4 5 6 7 8 9 10
2. Bill Nye: pretty loose; he will call with 4+.
   1 2 3 4 5 6 7 8 9 10
3. Bill Clinton: pretty tight; he will call with 8+.
   1 2 3 4 5 6 7 8 9 10
4. (bonus) Bill Chen: he will call with the optimal strategy. Namely, he will call with a range of hands that minimizes how much you expect to win per game. This could be a randomized strategy, eg. "Always call 8+, and call a 7 60% of the time." Outline what his optimal strategy is, and circle the hands you would bet against this strategy.
   1 2 3 4 5 6 7 8 9 10

(corollary to bonus) If Jen Shahade (who plays the optimal strategy for Jen vs. Bill's optimal strategy) plays vs. Bill Chen, who is expected to win? The player who gets to bet (Jen) or the player who wins ties (Bill)?