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Meteorite transport—Revisited II

Jack WISDOM*

Massachusetts Institute of Technology, Cambridge, Massachusetts 02139, USA
*Corresponding author. Email: wisdom@mit.edu

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Abstract–In Wisdom (2017), I presented new simulations of meteorite transport from the chaotic zones associated with major resonances in the asteroid belt: the ν_6 secular resonance, the 3:1 mean motion resonance with Jupiter, and the 5:2 mean motion resonance with Jupiter. I found that the observed afternoon excess (the fact that approximately twice as many meteorites fall in the afternoon as in the morning) of the ordinary chondrites is consistent with chaotic transport from the 3:1 resonance, contradicting prior reports. Here I report an additional study of the transport of meteorites from ν_6 secular resonance and the 3:1 mean motion resonance. I use an improved integration algorithm, and study the evolution of more particles. I confirm that the afternoon excess of the ordinary chondrites is consistent with transport from the 3:1 resonance.

INTRODUCTION

In Wisdom (2017), I reported a new study of meteorite transport to Earth from the asteroid belt by way of the chaotic zones associated with the ν_6 secular resonance, the 3:1 mean motion resonance with Jupiter, and the 5:2 mean motion resonance with Jupiter. I studied the evolution of test particles in two models: the first included only perturbations from the outer planets and the second included all the planets, with relativistic corrections. I performed the integrations with the Wisdom and Holman (1991) method, supplemented with an algorithm to handle close encounters (Wisdom 2016). In these integrations, I used a stepsize of 0.05 days. In Wisdom (2016), I compared integrations using that algorithm to various popular integration methods: RMVS3 (Levison and Duncan 1994), SYMBA (Duncan et al. 1998), and MERCURY (Chambers 1999). I found that the new algorithm performed well. For the larger stepsizes studied, the new method was stable when the other methods failed.

Among the conclusions of Wisdom (2017), I found that the afternoon excess of the ordinary chondrites is consistent with the afternoon excess from the simulations of meteorites transported from the 3:1 resonance. The afternoon excess statistic is defined as the number of meteorites falling in the afternoon (between noon and 6 P.M.) divided by the number of meteorites falling during the day (6 A.M. to 6 P.M.). The afternoon excess of the ordinary chondrites as a whole was estimated as 0.630 ± 0.019; for H chondrites, the afternoon excess was estimated as 0.600 ± 0.037, for L chondrites it was 0.674 ± 0.033, and for LL chondrites it was 0.686 ± 0.083. By comparison, the afternoon excess for simulated meteorites originating in the ν_6 resonance was 0.550 ± 0.004, for simulated meteorites originating in the 3:1 resonance was 0.627 ± 0.008, and for simulated meteorites originating in the 5:2 resonance was 0.679 ± 0.026.

These results did not confirm a number of prior results of Gladman et al. (1997) and Morbidelli and Gladman (1998). For instance, Gladman et al. (1997) state that the primary role of the inner planets is to slow down the transport process and that without the inner planets the development of sun-grazing orbits typically occurs on a million year timescale. Carrying out integrations with and without the inner planets, I showed that the presence of the inner planets actually speeds up the process leading to sun-grazing and ejection. The integrations of Gladman et al. (1997) did not include Mercury, and they did not report the stepsize they used in their integrations. Morbidelli and Gladman (1998) found that the afternoon excess for meteorites originating in the ν_6 resonance was 0.52, and for the 3:1 resonance was 0.48, both much lower than the observed statistic. They concluded that the afternoon excess was an observational artifact. Indeed, they state "The fact that no reasonable choice of the parameters of our model allows us to recover the observed P.M. ratio of falls leads us to believe that the latter is erroneously high, perhaps due to the effect of social biases combined with small number statistics." In Wisdom (2017), I traced the problem to the fact that they used an incorrect algorithm to compute the afternoon excess statistic from their simulations. In fact, the afternoon excess of the observations and my simulations were consistent with one another.

Since this investigation, I have been studying the transport of ejecta from both Mercury and Mars to Earth in order to get a new estimate of the relative likelihood of there being meteorites from Mercury and Mars in the meteorite collections. This study will be reported elsewhere. In this study, I tested the sensitivity of the transport rate to the stepsize used in the integrations. These integrations again used the Wisdom and Holman (1991) method, supplemented with the encounter algorithm of Wisdom (2016). These integrations used stepsizes that ranged from 0.1 to 1.0 days. Unfortunately, I found that the simulated transport rate from Mercury depended on the stepsize, over this range. The problem was traced to the fact that some encounters were being missed because of the high velocities and small Hill spheres of the inner planets. A new algorithm has been developed and tested. With the new algorithm, I found that the transport rate from Mercury to Earth was now statistically independent of stepsize over the range of 0.1–1.0 days. Although the stepsize used in Wisdom (2017) was small, it seemed wise to repeat those calculations with the new algorithm. I report the results of these new calculations in this paper. The new algorithm differs from Wisdom (2016) only in the method used to predict close encounters.

In the next section, I describe the new encounter detection algorithm. Then I report the new results. The new results are different in detail from those of Wisdom (2017), but the main conclusions still hold. This is followed by some concluding remarks.

REVISED ENCOUNTER ALGORITHM

The algebraic details of the algorithm were described in Wisdom (2016), and will not be repeated here. The Hamiltonian governing the evolution of test particles is a sum of two parts

$$H^T = H^T_K + H^T_I \tag{1}$$

where K^T_K is a Kepler Hamiltonian, and

$$H^T_I = \sum_{j>1} H^T_j \tag{2}$$

is the interaction Hamiltonian, which is written as a sum of interactions of the test particle with each planet with index j. A switching function is introduced so that the terms are regrouped if there is a close encounter between the test particle and a planet. Define

$$H^T_A = H^T_K + \sum_{j>1} H^T_j (1 - K_{R_j}(r_{1j})) \tag{3}$$

and

$$H^T_B = \sum_{j>1} (H^T_j K_{R_j}(r_{1j})) \tag{4}$$

where r_{1j} is the distance between the test particle (with index 1) and the planet with index j, and K is a switching function. The function K has value one if r_{1j} is large compared to the Hill radius R_j of planet j, and it is equal to zero if r_{1j} is small compared to the Hill radius. If $K = 1$, then the terms are grouped as before; if $K = 0$ for some j, then some terms are moved from H^T_B to H^T_A. There is a smooth transition between the two cases. Define

$$F(x) = x^3/(1 - 3x + 3x^2), \tag{5}$$

$0 < x < 1$. For $x < 0$, $F(x) = 0$, and for $x > 1$, $F(x) = 1$.

Then define

$$K_R(r) = F((r - h_1 R)/(h_2 R)), \tag{6}$$

where h_1 and h_2 are constants. I used $h_1 = h_2 = 1.5$. I use the method Wisdom and Hernandez (2015) to advance the Kepler orbit if $r > h_3 R$ and otherwise I use numerical integration (the Bulirsch–Stoer method). Here I used $h_3 = 4$. The strategy was that the system would switch to numerical integration before the switching function began to switch. If the spatial steps are small compared to the Hill spheres, then the switch to numerical integration begins before the switching function begins to turn on. But for small Hill spheres and large step sizes, the switching region can be entered without first switching to numerical integration. Indeed, for very large steps, the encounter region can be missed altogether.

To solve this problem, I chose to always first evolve the orbits as if there were no encounter. Then I use the beginning and ending positions and velocities of this step to interpolate to see if there was an encounter during the step. Define $f_j(t)$ to be the square of the distance between the test particle and planet j at time t. The data $f_j(t)$, $df_j(t)/dt$, $f_j(t+h)$, $df_j(t+h)/dt$ define a unique cubic interpolating polynomial. If this polynomial has a minimum within the step between t

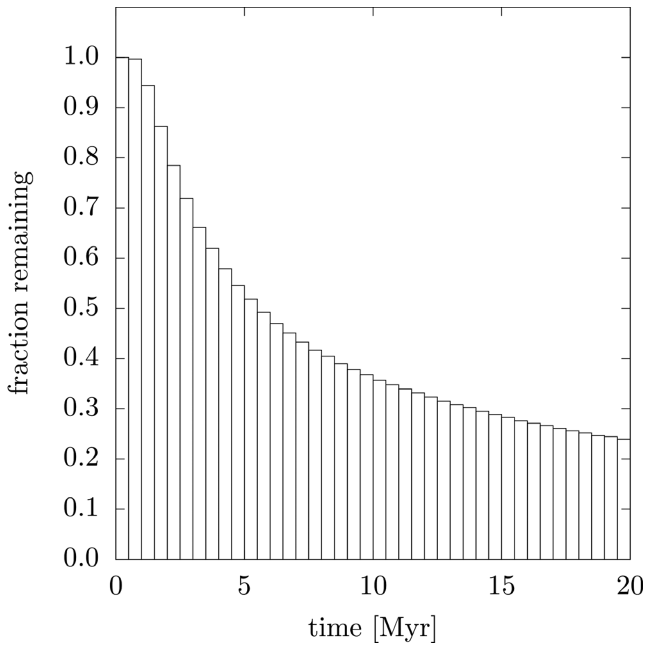

Fig. 1. Decay of the number of particles initially in the ν_6 resonance over 20 Myr. All the planets were included.

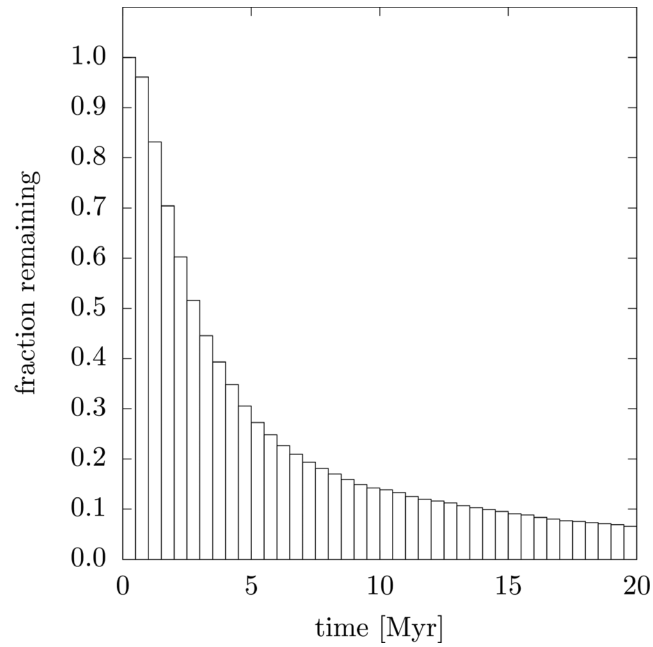

Fig. 2. Decay of the number of particles initially in the 3:1 resonance over 20 Myr. All the planets were included.

and $t + h$, and if the minimum distance is less than $h_3 R$, then I switch to numerical integration. As an extra precaution, I chose to do two numerical integration steps: the first step integrates to the time of the predicted minimum distance, and the second from there to the end of the timestep. I found that this procedure very accurately found and handled encounters, even with relatively large timesteps.

RESULTS

With the revised algorithm, I studied the evolution of test particles in both the ν_6 resonance and the 3:1 resonance. The procedure for selecting the initial conditions of the test particles was the same as in Wisdom (2017). The integrations spanned 20 Myr. For these integrations, I used a stepsize of 0.5 days, within the range of stepsizes that was found to give results independent of stepsize. A test particle was considered "active" if its orbit crossed a planetary orbit, whether or not it actually had an encounter with a planet. In my simulations, there were 5670 active particles that began in the ν_6 resonance, and 5450 active particles that began in the 3:1 resonance. Of the active particles in the ν_6 resonance, 3306 had encounters with the Sun (58%), 744 became hyperbolic (13%), and 284 had collisions with one of the planets (5%). Of the active particles in the 3:1 resonance, 3006 had encounters with the Sun (55%), 1470 became hyperbolic (27%), and 76 had collisions with one of the planets (1%).

Next, I compare the decay of active particles in the ν_6 and 3:1 resonances. Figure 1 shows the fraction remaining of those test particles initially in the ν_6 resonance as a function of time. It compares very well with the results of Wisdom (2017). (Note that the scale on the ordinate in Fig. 7 in Wisdom [2017] is incorrect. With the way the data are plotted the scale should have ranged from 0.0 to 1.0.)

Figure 2 shows the fraction remaining of those active particles initially in the 3:1 resonance as a function of time. It again compares very well with the results of Wisdom (2017), although there are somewhat more particles remaining in the new integrations.

The time of fall statistic has played an important role in meteorite delivery studies. It has been used to rule out a variety of sources of meteorites, and it pointed to the existence of a new dynamical route for the delivery of meteorites in the asteroid belt (Wetherill 1968), which was later found (Wisdom 1985). Wisdom (1985) found that meteorites can evolve in chaotic zones from asteroidal type orbits to Earth crossing orbits; Wisdom (1985) specifically studied the 3:1 resonance with Jupiter. Subsequent studies by Scholl and Froeschle (1991) suggested that meteorites could also be transported to Earth by way of a chaotic zone associated with ν_6 secular resonance. I estimated the time of fall histogram and the afternoon excess statistic from the simulations by the method described in Wisdom (2017). The simulated time of fall histogram for the ν_6 resonance is shown in Fig. 3. The afternoon

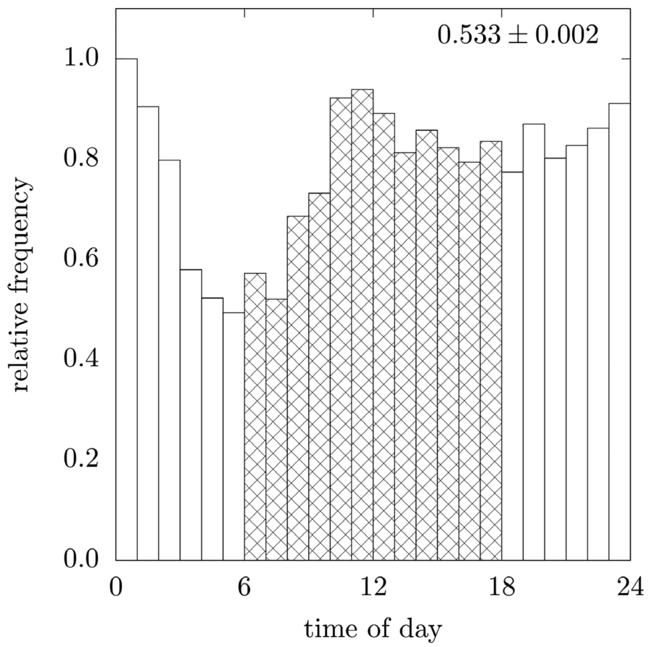

Fig. 3. Simulated time of fall from the ν_6 resonance over 20 Myr. The number in the upper right corner is the afternoon excess statistic.

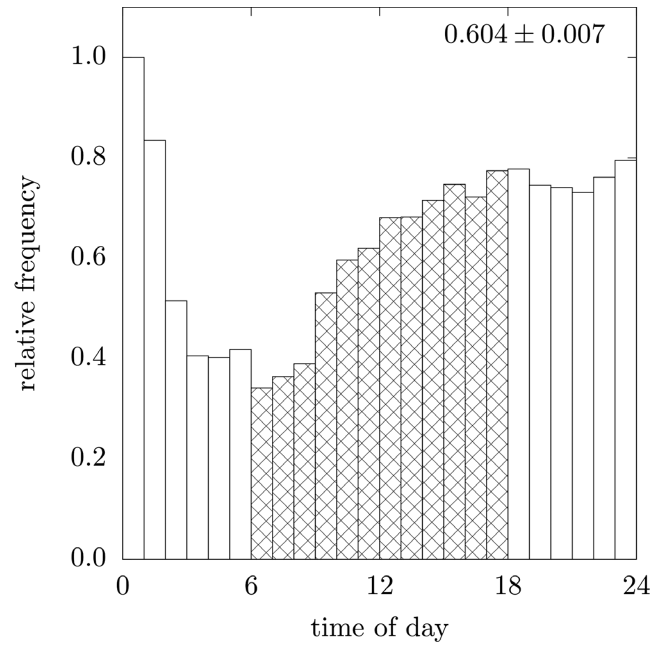

Fig. 4. Simulated time of fall from the 3:1 resonance over 20 Myr. The number in the upper right corner is the afternoon excess statistic.

excess statistic is 0.533 ± 0.002. Errors were estimated using the bootstrapping method (Efron and Tibshirani 1993; Wisdom 2017). This is close to the value 0.550 ± 0.004 that was obtained in Wisdom (2017). Because the algorithm has been improved, the new value is to be considered more reliable. The simulated time of fall histogram for the 3:1 resonance is shown in Fig. 4. The afternoon excess statistic is 0.604 ± 0.007. This is a little lower than the value 0.627 ± 0.008 that was obtained in Wisdom (2017). Again, because the algorithm has been improved, the new value is to be considered more reliable.

One might wonder if the afternoon excess statistic would be substantially lowered if the integrations were extended in time. The reason this might be the case is that a small number of the active particles that remain after 20 Myr have small semimajor axes and would contribute to a smaller afternoon excess. To test this, the 3:1 integrations were extended to 40 Myr. The afternoon excess statistic was found to be 0.587 ± 0.007, only marginally lower than the value obtained over 20 Myr. So this does not explain the fact that Morbidelli and Gladman (1998) found substantially lower values.

As stated in the Introduction, the problem in Morbidelli and Gladman (1998) was that they used an incorrect algorithm for computing the afternoon excess statistic from their simulations. The derivation of their method assumes that the orbits can be precessed to a collision. For this to be valid, the angular orbital elements must have a uniform distribution. But, in fact, they do not. In particular, the argument of pericenter evolves in a distinctly nonuniform manner. So that we are comparing apples to apples, I applied the method used in Morbidelli and Gladman (1998; a simplified version was kindly provided by Allesandro Morbidelli) to estimate the afternoon excess statistic for my 3:1 simulations. Over 20 Myr I found 0.45 and over 40 Myr I found 0.43. This is to be compared to the value they found, without collisional destruction, of 0.48. This confirms that the low values they find in their study for the afternoon excess statistic is an artifact of using an incorrect method.

The method I use makes fewer assumptions, and in particular does not assume the orbits precess uniformly. I let the dynamical evolution take care of all precession. Of course, ultimately one would like to repeat these calculations without any approximation, with just a brute force simulation of a much larger number of test particles.

The afternoon excess statistics from the simulations should be compared to the observed afternoon excess statistics for the ordinary chondrites. As mentioned above, the afternoon excess of the ordinary chondrites as a whole was estimated as 0.630 ± 0.019; for H chondrites, the afternoon excess was estimated as 0.600 ± 0.037, for L chondrites it was 0.674 ± 0.033, and for LL chondrites it was 0.686 ± 0.083. The afternoon excess results suggest that the primary source for the H chondrites is the 3:1 resonance, and, perhaps, the L and LL chondrites come from farther out in the asteroid belt. Although material launched in the v_6 resonance is readily transported to Earth, the afternoon excess statistic suggests that this is not the dominant route through which the ordinary chondrites have reached Earth from the asteroid belt.

SUMMARY

I have studied the transport of meteorites from the v_6 and 3:1 resonances in the asteroid belt to Earth. This study was motivated by a revision in the integration algorithm. The revision improved the detection of close encounters.

Although there are differences in detail, overall the results are in good agreement with the results of Wisdom (2017). The afternoon excess statistic for the 3:1 simulations is in surprisingly good agreement with the observed afternoon excess statistic for the H chondrites. The statistic for the v_6 simulations is not consistent with that of the H chondrites. This suggests that most H chondrites are transported to Earth by the 3:1 resonance rather than the v_6 resonance. The afternoon excess statistic for the L and LL chondrites is larger, which suggests that they originate further out in the asteroid belt.

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