The Chaotic Obliquity of Mars

Jihad Touma and Jack Wisdom

Numerical integration of the rotation of Mars shows that the obliquity of Mars undergoes large chaotic variations. These variations occur as the system evolves in the chaotic zone associated with a secular spin-orbit resonance.

It has been understood for some time that the rotation of Mars is strongly affected by variations in its orbit (*1–3*). The large variations in the obliquity of Mars predicted in some models would have important consequences for the modulation of Mars' climate and other atmospheric behaviors such as dust storms. Variations of the obliquity have been suggested as a possible cause of the layered polar deposits on Mars (*4, 5*). However, a number of obstacles have prevented a reliable determination of the history of the obliquity of Mars. One obstacle is fundamental and unavoidable. Ward, Burns, and Toon (*6, 7*) pointed out that a variety of physical mechanisms can change the moments of inertia of Mars and that, as the moments of inertia change, secular spin-orbit resonances could be encountered that can drastically alter the obliquity of Mars. Secular spin-orbit resonances occur when the period of precession of the spin axis of Mars is commensurate with one of the periods in the variation of the orbit of Mars. The time scale for significant internal geophysical changes in the physical rotational parameters of Mars is estimated to be greater than 100 million years, perhaps even billions of years. On shorter time scales, that is, less than 100 million years, uncertainty in the orbital dynamics has been the most significant obstacle to a reliable determination of the history of the obliquity. The strongest secular spin-orbit resonance is associated with the near equality of the frequency of the precession of the spin axis with the frequency of the second inclination mode of the solar system, the mode that dominates the evolution of the orbit plane of Venus (*8*). As orbit theories have improved, the estimates of the difference between these frequencies have varied significantly (*9*). Ward and Rudy (*5*) used an improved Fourier decomposition of the evolution of the orbit of Mars from the secular semi-numerical theory of Laskar (*9*) to update the history of the obliquity of Mars. They found that on a time scale of several million years the obliquity reaches values as large as 45°, compared with a maximum of 35° found using earlier orbit theories. Laskar (*10, 11*) has found evidence that the evolution of the solar system is chaotic in a numerical integration of an averaged secular approximation of the equations of motion for the planets. Sussman and Wisdom (*12*) confirmed this result by direct numerical integration of the whole solar system. The time scale for exponential divergence is only about 4 to 5 million years. Thus the evolution of the planetary system is not quasiperiodic, and consequently not well described by Fourier series, an assumption made in all existing investigations of the obliquity of Mars. The discovery that the solar system is chaotic necessitates a new investigation of the evolution of the obliquity of Mars without Fourier approximations to the orbital variations. Here we present the results of a number of such direct numerical integrations of the rotation of Mars in the chaotically evolving planetary system.

Our physical model for the planetary evolution is the same as that used by Sussman and Wisdom (*12*), which is the same as Quinn, Tremaine, and Duncan (*13*) except for an unimportant difference in the treatment of the general relativistic corrections. In addition to the effects of general relativity, the model approximates the effect of the Earth-moon quadrupole on the evolution of the planetary orbits. We take Mars to be an axisymmetric body. We consider both the case in which only the solar torques are included and the case where all gravitational torques are taken into account. We ignore the effect of Mars' extended shape on the evolution of the planetary orbits. For the physical parameters of Mars, we assume $J_2 = 0.00196$ (*5, 14, 15*), and $\lambda = C/M R^2 = 0.366$ (*5, 15, 16*). We used two different sets of initial conditions (ICs). In set IC1 we took the initial conditions and masses from Quinn, Tremaine, and Duncan (*13*); in the other set, IC2, we took initial conditions and masses from JPL ephemeris DE202 (*17*). The rotational state of Mars was derived from Davies *et al.* (*18, 19*). We used the symplectic Lie-Poisson integration algorithm of Touma and Wisdom (*20*) which extends the symplectic *n*-body integration algorithm of Wisdom and Holman (*21, 22*) to include the rotational dynamics of extended bodies. The step size in our integrations is 7.2 days. Our integration extends backward in time.

The obliquity of Mars executes large irregular variations that range from about 11° to about 49° (Fig. 1). We have computed the Lyapunov exponent for the spin dynamics and found that the evolution is chaotic with an exponential divergence time scale of about 3 to 4 million years.

The sudden change in mean obliquity that occurs about 4 million years ago (Fig. 2) is not a sensitive feature of the orbit model. The same transition occurs with set IC2. Indeed, the initial segment of both integrations is remarkably similar to that found by Ward and Rudy (*5*), including the transition in mean obliquity. Beyond about 7 million years ago, the obliquity in our calculation begins to diverge from that found by Ward and Rudy. In light of the chaotic evolution of the spin axis, such divergence is to be expected. Ward and Rudy used Laskar's Fourier representation of the orbit of Mars, which approximates the

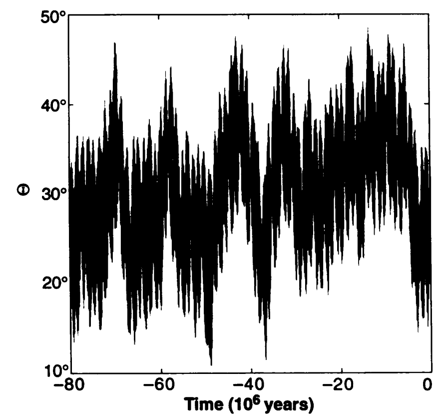

Fig. 1. The obliquity of Mars for initial conditions IC1 over the last 80 million years has large chaotic variations.

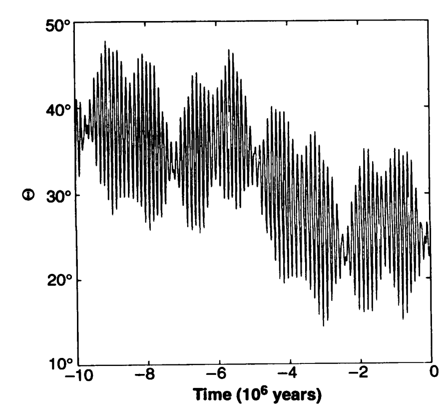

Fig. 2. The obliquity of Mars over the last 10 million years shows a relatively abrupt transition about 4 million years ago.

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chaotic evolution of the orbit over about 10 million years (5, 9). So if the chaotic orbit model of Laskar were used instead of the quasiperiodic representation of the orbit, we expect the obliquity of Mars would behave similarly. Thus the history of the obliquity of Mars over the last few million years is no longer sensitive to remaining differences in the models of the evolution of the orbit of Mars.

Ward and Rudy (5) attributed the transition in mean obliquity to passage through a secular spin-orbit resonance. As mentioned above, the most important secular spin-orbit resonance for the evolution of the rotation of Mars is associated with the near equality of the frequency of the precession of the spin axis with s_2', the frequency of the second inclination mode of the solar system. In Laskar's Fourier decomposition of the evolution of the orbit of Mars, the second inclination mode component is accompanied by a number of smaller components of similar frequency. A multiplet of components can be alternatively viewed as a single component with varying amplitude and frequency, the frequency of the variations being comparable to the frequency spread of the multiplet. Ward and Rudy pointed out that transitions across the resonance and capture into the resonance can be induced by this modulation. As the amplitude and frequency of the resonant perturbation vary, the amplitude of the resonance motion adjusts to approximately preserve the adiabatically invariant resonance action. If the modulation is great enough, transitions can be induced. A plot of the resonance angle $h_1 - \Omega_2^\circ$ versus time confirms the essential role this resonance plays in the dynamics (Fig. 3). Here, h_1 is the ascending node of the equator of Mars on the invariable plane and Ω_2° is the phase of the second proper mode of the solar system computed from our data with the transformation of Laskar (11). The transition in the mean obliquity that occurs about 4 million years ago coincides with a transition across the resonance associated with this combination of angles: the direction of rotation of the resonance angle changes as the mean obliquity changes (Fig. 4). The mean obliquity has been computed from the obliquity with a simple low-pass filter that removes frequencies above about 10 arc sec per year. As the resonance angle circulates the mean obliquity is relatively constant and stays on one side of the resonance; oscillation of the resonance variable is associated with an oscillation of the obliquity. This behavior is typical of nonlinear resonance. Transition from circulation in one direction to circulation in the other direction is associated with a sudden change in the mean obliquity as the resonance is crossed.

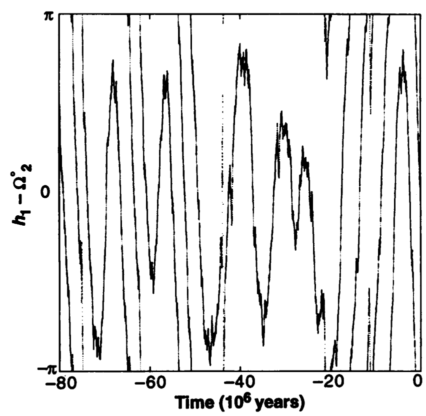

Fig. 3. The longitude of the equator of Mars minus the phase of the second inclination proper solar system mode alternates between circulation and libration.

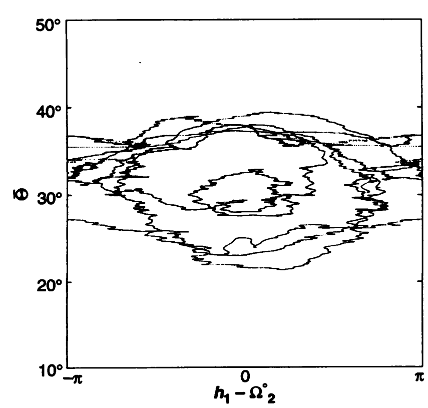

Fig. 4. The secular spin-orbit resonance is illustrated in this plot of the mean obliquity versus the principal secular spin-orbit resonance angle. For clarity, only the data for the most recent 40 million years are plotted.

It is amusing to note that the large change in mean obliquity about 4 million years ago does not happen in our model if the general relativistic corrections are not included. We have carried out a full direct integration spanning 20 million years without relativistic corrections and did not find large variations in the obliquity. This is perhaps not surprising because general relativity induces changes in the secular mode frequencies, particularly the first two frequencies, that are large compared to the frequency spread of the important multiplet near the second normal mode. Perhaps the geology of Mars will ultimately provide another test of the validity of general relativity.

Isolated spectral components in the Fourier decomposition of the orbital variations imply a region of libration of the corresponding nonlinear secular spin-orbit resonance. However, in Laskar's Fourier decomposition of the orbit of Mars, the resonances corresponding to the components of the multiplet near the second inclination mode are not isolated: the width of the libration zones for the principal components are much larger than the frequency separation of the components (23). The resonance overlap criterion (24) immediately suggests that the evolution of the obliquity of Mars is chaotic. However, the evolution of the obliquity of Mars is better understood in another way which takes advantage of the small frequency spread of the multiplet. Again, the multiplet of secular spin-orbit resonances can be combined into a single coherent resonance with parameters that vary with a frequency comparable to the frequency splitting of the multiplet. As the resonance parameters vary, the size of the libration region can vary drastically, but at the same time the amplitude of the motion adjusts to approximately preserve the resonance adiabatic invariant. Taken together these two facts imply that some trajectories can be forced across the time-varying separatrix. As the separatrix is crossed there is an irregular jump in the value of the adiabatic invariant, determined by the relatively rapidly rotating phase of the resonance variable. Wisdom (25) pointed out that repeated separatrix crossings during slow change of resonance parameters can give rise to chaotic behavior and that the size and shape of the chaotic zone can be well approximated by the set of trajectories that is forced to cross the separatrix. This mechanism of "two–time scale chaos" or "adiabatic chaos" was independently discovered by Escande (26). Wisdom was concerned with explaining the large chaotic zones near mean motion resonances in the asteroid belt and dealt with a coherent resonance representing a multiplet of mean motion resonances with time-evolving parameters governed by the slower secular evolution of the eccentricity and perihelion. Escande dealt with the more abstract problem of a mathematical pendulum with explicit time-varying parameters. The chaotic spin dynamics of Mars is an example of adiabatic chaos, but with an interesting twist. In the spin dynamics of Mars, the time variation of the resonance parameters that drive the spin system repeatedly across the resonance separatrix is also chaotically varying because of the chaotic evolution of the solar system. However, the evolution of the spin dynamics of Mars is separately chaotic and does not directly inherit its chaotic character from the chaotic character of the evolution of the solar system. The spin dynamics of Mars would still be chaotic even if the solar system was quasiperiodic rather than chaotic, provided the spectrum of the orbit variations contained sufficiently strong multiplets. We have verified this by computing the evolution of the spin of Mars over 100 million years, with the orbit of

Mars represented by the most important terms in Laskar's Fourier decomposition. With the assumption of principal axis rotation, and keeping only first-order terms in the orbital inclination, the evolution of the spin axis is governed by the simplified time-dependent one degree-of-freedom Hamiltonian

$$\overline{H} = -\frac{\alpha L_1}{2}\left[\frac{H_1^2}{L_1^2} + 2\frac{H_1}{L_1}\left(1 - \frac{H_1^2}{L_1^2}\right)^{1/2} \sum_i N_i \cos(h_1 - s_i' t - d_i)\right] \quad (1)$$

where h_1 is the ascending node of the equator plane on the invariable plane and is the coordinate canonically conjugate to the momentum H_1, the projection of the rotational angular momentum on the normal to the invariable plane. L_1 is the magnitude of rotational angular momentum of Mars; L_1 only enters the equations of motion through the ratio H_1/L_1, which is simply related to the obliquity. The precession constant $\alpha = (3\pi/P)(D/P)J_2/\lambda$, where P and D are the orbital and rotational periods of Mars, respectively, has the value α = 8.26 arc sec per year for our adopted physical parameters. N_i, s_i', and d_i are the amplitudes, frequencies, and phases, respectively, of the terms describing the variation of the orbit plane. The included terms are those used by Ward and Rudy (*5*). We find that the evolution of the obliquity in this simplified model is still chaotic with an exponential divergence time scale of about 5 million years, comparable to that found in our full numerical integrations, which integrate the rotation of the axisymmetric Mars without approximation and automatically include the chaotic evolution of the solar system.

The mechanism of adiabatic chaos in the spin dynamics of Mars can be illustrated by considering the simplified model with only the two strongest components of the orbit variation near the secular spin-orbit resonance. The system has a single degree of freedom with quasiperiodic forcing. One of the two frequencies can be eliminated by transformation to a frame rotating with that frequency, leaving a single degree of freedom with periodic forcing at the difference frequency. A stroboscopic surface of section for this model is shown in Fig. 5. Points are plotted once per cycle of the periodic forcing. At the chosen phase of the section the coherent combination of the two resonances is at maximum amplitude, and the maximum variation of mean obliquity is represented on the section. We see that only two components are sufficient to give a large region of chaotic behavior in the spin dynamics of Mars. The range of obliquity variation in the simple model matches rather well the range of mean obliquity observed in Fig. 4. The coherent resonance that models the two components varies in amplitude and phase. Trajectories inside the separatrix when the amplitude is maximum are forced to cross the separatrix as the amplitude decreases and the separatrix shrinks. Thus the chaotic zone is nearly identical to the region enclosed by the separatrix at the maximum amplitude. The size and shape of the chaotic zone are accurately predicted (Fig. 6). Keep in mind though that the quasiperiodic representation of the orbit variations is only valid for a few million years: The actual coherent resonance parameters vary chaotically as the orbit varies chaotically.

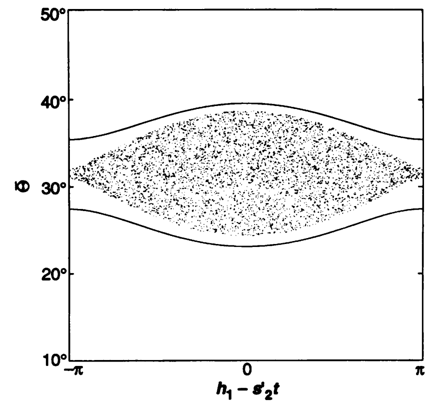

Fig. 5. A surface of section for the simple two-term quasiperiodic orbit model shows a large chaotic zone that completely engulfs the secular spin-orbit resonance. Quasiperiodic trajectories bound the chaotic zone.

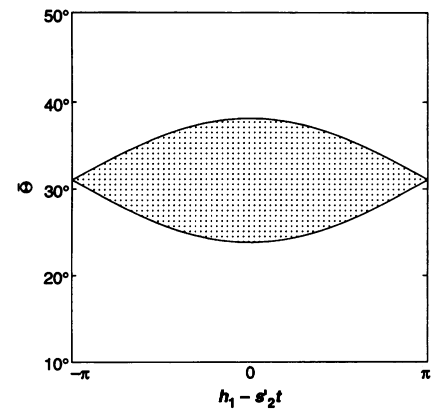

Fig. 6. The predicted region of chaotic behavior in the two-resonance model is in excellent agreement with the chaotic zone found on the surface of section in Fig. 5.

We emphasize that the orbital dynamics is no longer a limiting factor in determining the obliquity of Mars over approximately the last 10 million years. All of the orbit models give qualitatively the same results, with nearly identical transitions in mean obliquity about 4 million years ago. However, if the physical parameters of Mars were determined to be significantly different from the nominal values we have used, the calculated evolution of the obliquity of Mars could be radically altered. To evaluate the sensitivity to physical parameters we have integrated the quasiperiodic model over a range of physical parameters, keeping the initial conditions of Mars fixed. The crucial parameter is the precession constant; for our adopted physical parameters α = 8.26 arc sec per year. We find that in the quasiperiodic model the spin dynamics is chaotic in the range $7.4 < \alpha < 8.6$ arc sec per year. We note that the exponential divergence time for the spin dynamics varied in the range of 1 to 5 million years as α was varied. The range of obliquity also varies with α, with larger mean obliquities associated with larger values of α. Of course, the model is approximate. The full problem should be reexamined if the estimates of the physical parameters are significantly revised.

The history of the obliquity of Mars over longer intervals must be considered to be inherently unpredictable. The evolution of both the orbit and spin of Mars is chaotic and it is inherently impossible to make detailed forecasts of chaotic systems. Also, there are the unpredictable long-term changes in the physical parameters of Mars. Thus any calculation of the remote history of the obliquity of Mars could only be considered to be representative of a range of possibilities. These difficulties are only compounded by the possibility that over the age of the solar system the obliquity of Mars may increase secularly as a result of delayed postglacial rebound associated with climate changes induced by obliquity variations (*27*). Thus, little constraint can be placed on the primordial obliquity of Mars.

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Lidar Observations of the Meteoric Deposition of Mesospheric Metals

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The mesospheric sodium and iron layers at an altitude between about 80 and 110 kilometers are routinely monitored by atmospheric physicists using resonance fluorescence lidar techniques because these constituents are excellent tracers of mesopause chemistry and dynamics. The mesospheric metals are the products of meteoric ablation. Existing ablation profiles are model calculations based in part on radar observations of the ionized background atmosphere left in the wake of high-speed (>20 kilometers per second) meteoroids. Thin trails of neutral metal atoms, ablated from individual meteoroids, are occasionally observed with high-power lidars. The vertical distribution of 101 sodium and 5 iron meteor trails observed during the past 4 years at Urbana, Illinois; Arecibo, Puerto Rico; and near Hawaii is approximately Gaussian in shape with a centroid height of 89.0 (±0.3) kilometers and a root-mean-square width of 3.3 (±0.2) kilometers. This directly measured ablation profile is nearly the same as the mean iron layer profile but is considerably different from existing models and the distribution of ionized meteor trails observed by radars. A lower limit on the influx to the mesopause region from the lidar meteors is approximately 1.6 × 10^3 sodium and 2.7 × 10^4 iron atoms per second per square centimeter, which corresponds to an annual flux of meteoric debris into the mesosphere of about 2.0 (±0.6) gigagrams. Because the lidars can detect only the ablation trails left by the larger meteors, the observations suggest that the actual meteoric influx may be larger than the more recently reported values, which range between 16 and 78 gigagrams per year.

For more than 20 years resonance fluorescence lidar techniques have been used to study the chemistry and dynamics of the atomic metal layers in the height range between 80 and 110 km. Meteoric ablation is the major source of these layers, and photo-ionization in the lower thermosphere and chemical reactions involving O, O_2, O_3, and CO_2 below the mesopause are the major sinks. The seasonal variations in Na and Fe abundances, which are both minima in summer, are believed to be related to the temperature dependencies of the sink reactions, which are more efficient at the lower summertime temperatures of the mesopause. The predicted increases of atmospheric CO_2 during the next century and the accompanying increases in radiative cooling (*1*) are likely to have major effects on the concentrations and vertical distributions of the mesospheric metal layers. In fact, Clemesha *et al.* (*2*) recently reported a long-term decrease in the height of the Na layer at Sao Jose dos Campos, Brazil, which averaged 49 (±12) m $year^{-1}$ from 1972 to 1987. These researchers suggested that this change is consistent with long-term cooling trends detected by other techniques and predicted by models that incorporate the expected increases in middle atmosphere CO_2 and CH_4 concentrations.

The metal layers are also of interest because they serve as tracers of the thermal and dynamic state of the upper atmosphere. Narrow band Na lidars are now being used to measure mesopause temperature and Doppler wind profiles (*3*) and the technology exists to make similar observations by probing other metal species. The Na layer is also of considerable current interest to optical astronomers who are now developing laser guide star technology for adaptive image compensation in ground-based telescopes (*4*, *5*). In this technology, powerful lasers are used to create bright artificial stars in the Na layer, which provide the reference wave front for the adaptive imaging systems.

Knowledge of the meteoric source distribution of the mesospheric metals is essential for understanding their chemical evolution. The high vertical and temporal resolution achieved with modern high-power lidars facilitates the observation of what are apparently trails left by ablating meteors. These trails are very thin [~100 m full width at half maximum (FWHM)] and short-lived, remaining in the lidar system's 1-mrad field of view (FOV) for typically less than 30 s. The peak atomic densities in these trails can be quite large, reaching 10^4 cm^{-3} for Na and 10^5 cm^{-3} for Fe. The most likely source of these events is the recent ablation of a metal-rich meteoroid. To date, various lidars operated by our group have observed 101 Na and 5 Fe meteor trails. The altitude distribution of these events is considerably different from that of theoretical models of the meteoric ablation profile and the distribution of ionized meteor trails observed by radars. Trails observed by the lidars contribute at least 10% of the commonly accepted value for the total Na and Fe influx to the mesosphere.

Most of the data reported here were obtained at the Urbana Atmospheric Observatory (40°N, 88°W), with a tunable dye laser that was capable of making nighttime measurements of the altitude distributions of mesospheric Na, Fe, Ca, and Ca^+ (*6*, *7*). This system typically operates with a vertical resolution of 37.5 or 48 m (depending on computer configuration) and a temporal resolution of 30 s for Na observations and 3 min for Fe, Ca, and Ca^+ observations. This lidar was used to study the mesospheric Na layer at Urbana in 1988 and 1989 and at Arecibo, Puerto Rico, in the spring of 1989. This system was also used to make airborne Na lidar observations in Hawaii in March and April 1990. During the past year the system has been used for Fe, Ca, and Ca^+ observations at Urbana. The Na Doppler-temperature lidar is a narrow band system that can be used to measure line-of-sight winds and temperatures at mesopause heights by probing the fine structure of the Na D_2 resonance line (*3*). Only the Na density profiles obtained by this system, with its resolution of 48 m and 1 min, are reported here. All of the University of Illinois lidars have a full-width beam divergence of about 1 mrad. The lidars are thus probing a patch of the atmosphere about 100 m in diameter at an

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