

# **Boundary Element and Finite Element Methods for Moving-Boundary Electrochemical Problems**

by

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## **Abstract**

A tertiary current distribution model was developed for predicting two-dimensional shape change during electro-deposition through polymeric masks. The cathode geometry consisted initially of a parallel array of microscopic, rectangular trenches. Concentration polarization, activation polarization, and a stagnant diffusion layer treatment of convective mass transport were incorporated into the nonlinear model equations which were discretized using the boundary element method (BEM). Calculations of the deposit shape history were made for different values of the geometrical parameters, the level of convection, and the degree of polarization.

Three formulations of the BEM and one of the Galerkin finite element method (FEM) were used for the spatial discretization of a two-dimensional, moving boundary test problem based on Laplace's equation. Euler-predictor, trapezoid-corrected integration was used in time. The method was applied to a finite domain or semi-infinite strip by the use of infinite elements with reciprocal decay functions or a Green's function that met particular boundary conditions.

The accuracy and computation costs of the calculations were compared for three sets of linear boundary conditions, and various initial conditions for the moving surface. The BEM calculations using piecewise quadratic interpolating functions were more accurate and more efficient than those using linear interpolation. At high accuracy levels, they were also preferred to the FEM calculations that used biquadratic interpolation, although the FEM was found to be advantageous when accuracy requirements were relaxed. For the semi-infinite domain, the BEM based on a specialized Green's function (GBEM) was the only method that reliably satisfied the far field condition. The GBEM offered superior accuracy and efficiency in a more compact representation.

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