The Physics of Frisbees

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Accepted April 6, 2005; submitted in original form March 6, 2005

Abstract. Frisbees are a common source of entertainment and sport, although the physics behind these flying discs is often taken for granted. Frisbees operate under two main physical concepts, aerodynamic lift and gyroscopic stability. When flying through the air, a Frisbee can be viewed as a wing, with Bernoulli's Principle governing the magnitude of the lift force which keeps it aloft. The various forces applied are not centered on the disc though, so to keep the Frisbee from flipping over a high angular momentum is needed. This angular momentum resists the torque caused by the various forces. A computer program using the numerical technique Euler's method was written to model the trajectory of a flying Frisbee. Different trials were made with different angles of attack and the various distances and heights that the Frisbee reached were observed.

Keywords: Frisbee, Bernoulli, Gyroscopic Stability, Angle of Attack, Lift, Drag

1. Introduction

For decades, Frisbees[1] have been a widely used source of amusement for people of all ages. They have spawned numerous new sports (Ultimate Frisbee, disc golf and others) and each year more of them are sold than baseballs, basketballs and footballs combined[2]. These simple plastic discs can travel large distances and seem to defy gravity as they hover in the air before they finally touch down. Most people take for granted the science behind these entertaining aspects of Frisbees, but they can be explained in general using just two physical concepts, aerodynamic lift and gyroscopic stability.

The history of the modern "Frisbee" dates to 1871 in Bridgeport, Connecticut where William Russell Frisbie opened a small bakery, The Frisbie Pie Company. Frisbie's pies were popular at the nearby Yale University and students began to enjoy tossing the empty pie tins around. This became more popular and students began to call the tins "Frisbies" and the act of throwing them "Frisbie-ing". The first production of actual plastic flying discs was in 1958 when Fred Morrison bought the patent for the "flying disc" but they didn't actually

[1] "Frisbee" is a registered trade mark of Wham-O Inc.
[2] Wham-0.com

become popular until 1958 when Wham-O released their trademarked "Frisbee".

Most recently, research has been conducted into determining the coefficients that determine that magnitude of all forces acting on a Frisbee as well as the biomechanics involved in throwing a Frisbee (Hummel, 2003). Also, there is further unpublished research looking into other aspects of a Frisbee's flight including methods of taking data directly from the Frisbee's flight.

2. The Theory of Frisbee Flight

The two main physical concepts behind the Frisbee are aerodynamic lift (or the Bernoulli Principle) and gyroscopic inertia . A spinning frisbee can be viewed as a wing in free flight with the Bernoulli Principle being the cause of the lift and the angular momentum of the disc providing its stability.

2.1. Aerodynamic Forces

The two main aerodynamic forces acting on a Frisbee are the drag and lift forces. To determine magnitude of these forces two very common physical relationships are used.

To calculate the drag force, we first must find the Reynolds number of the system so as to know which drag relationship to apply. The Reynolds number, $\Re$, is given by,

$$\Re = \frac{\rho v d}{\eta}, \tag{1}$$

where ρ is the density of the fluid (in our case air), v is the velocity of the fluid (or the velocity of the frisbee relative to the fluid), d is the characteristic dimension of the object (for a Frisbee, the characteristic dimension is it's diameter), and η is the viscosity of the fluid. For a standard Frisbee thrown at sea level, the density of air is approximately 1.23 kg/m^3, the velocity of an average Frisbee throw is 14 m/s. The diameter of a standard Frisbee (accepted by the National Ultimate Association) is 0.260 m and the viscosity of air is $1.73 \times 10^{-}5$ Ns/m^2. This gives $\Re$ equal to 2.59×10^5. For a Reynolds number of this magnitude, the Prandtl relationship is used to calculate the drag force, F_d, and is given by,

$$F_d = -\frac{C_D \rho \pi r^2 v^2}{2} = -\frac{C_D \rho A v^2}{2}. \tag{2}$$

The coefficient C_D is a drag coefficient that varies with the object, and is given in Hummel (2003) as being a quadratic function solely dependent on the angle of attack α. α is the angle formed between the plane of the frisbee and the relative velocity vector.

$$C_D = C_{D0} + C_{D\alpha}(\alpha - \alpha_0)^2. \tag{3}$$

The coefficients C_{D0}, α_0 and $C_{D\alpha}$ are constants and and depend on the physical aspects of the Frisbee.

The lift force felt by a Frisbee is very similar to the lift force on airplane wings and is calculated using the Bernoulli principle. The Bernoulli Principle is a well known principle that states that there is a relationship between the velocity, pressure and height of a fluid at any point on the same stream line. Fluids flowing at a fast velocity have a lower pressure than fluids flowing at a slower velocity. This can be written mathematically as,

$$\frac{v_1^2}{2} + \frac{p_1}{\rho} + gh_1 = \frac{v_2^2}{2} + \frac{p_2}{\rho} + gh_2, \tag{4}$$

where v is the velocity of the fluid, p is the pressure of the fluid, ρ is the density of the fluid, g is the acceleration of gravity and h is the height of the fluid. The subscripts 1 and 2 refer to different points in the fluid along the same streamline. This equation is commonly referred to as Bernoulli's equation. For our purposes, the height difference between the air flowing above and the air flowing below the Frisbee is negligible, therefore the two height dependent terms cancel out. We will also assume that the velocity of the air flowing above is directly proportional to the velocity of the air below because the difference in path length is constant(i.e. $v_1 = Cv_2$). We now have the equation,

$$\frac{C^2 {v_2}^2}{2} + \frac{p_1}{\rho} = \frac{{v_2}^2}{2} + \frac{p_2}{\rho}. \tag{5}$$

Setting $F_L/A = p_1 - p_2$ (where F_L is the lift force and A is the area of the Frisbee) and solving for F_L gives,

$$F_L = \frac{1}{2}\rho v^2 A C_L. \tag{6}$$

Throughout the steps needed to determine (6), the coefficient C was incorporated into the coefficient C_L. C_L is given in Hummel (2003) as being a linear function of the angle of attack, α.

$$C_L = C_{L0} + C_{L\alpha}\alpha, \tag{7}$$

where C_{L0} and $C_{L\alpha}$ are constants that depend on the physical properties of the Frisbee.

2.2. Gyroscopic Stability

The rotation of a Frisbee is a necessary component in the mechanics of how a Frisbee flies. Without rotation, a Frisbee would just flutter to the ground like a falling leaf and fail to produce the long distance, stable flights that people find so entertaining. This is caused by the fact the aerodynamic forces described in the previous section are not directly centered on the frisbee. In general, the lift on the front half of the disc is slightly larger than the lift on the back half which causes a torque on the Frisbee (See Figure 1).

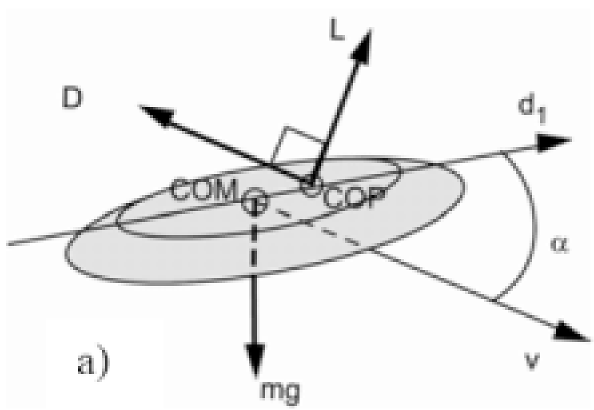

Figure 1. Diagram of the off-center center of pressure (COP) and the center of mass (COM) that results in a torque exerted on the Frisbee

When a Frisbee isn't spinning, this small torque flips the front of the disc up, and any chance for a stable flight is lost. When a Frisbee is thrown with a large spin, it has a large amount of angular momentum that has a vector in either the positive or negative vertical direction. When the small torque is exerted, the torque vector points to the right side of the frisbee (when viewed from behind.) This can be determined using the righthad rule with:

$$\vec{\tau} = \vec{\mathbf{r}} \times \vec{\mathbf{F}} \tag{8}$$

Since,

$$\vec{\tau} = \frac{d\vec{\mathbf{L}}}{dt}, \tag{9}$$

the angular momentum vector will begin to precess to the right. This phenomenon can easily be viewed when throwing a Frisbee, this is the reason that many thrown Frisbees bank to either the left of the right. Due to this, the greater the initial angular momentum given to the Frisbee, the more stable it's flight will be.

3. Numerical Modelling of a Frisbee in Flight

To model the flight of a Frisbee, a Java program was written that used the numerical technique Euler's method applied to the forces described in the previous section (see code in Appendix). To accomplish this, the different forces were separated into horizontal and vertical components, and Euler's method was applied each component. It should be noted that in the model it is assumed that the Frisbee is given enough initial spin so as to maintain a stable flight. In applying Euler's method, the trajectory of the Frisbee is divided into discrete time steps, Δt, and at each step a new horizontal velocity, v, and horizontal position, x, is defined:

$$v_{i+1} = v_i + \Delta v, \tag{10}$$

$$x_{i+1} = x_i + \Delta x, \tag{11}$$

where Δv and Δx are the changes in velocity and position respectively. A similar equation to equation (11) can be used with the vertical position, y, used instead of x. The Δv's are obtained by solving the following relationships.

$$F_x = F_D, \tag{12}$$

$$m\frac{\Delta v_x}{\Delta t} = \frac{1}{2}\rho v_x^2 A C_D, \tag{13}$$

$$\Delta v_x = \frac{1}{2m}\rho v_x^2 A C_D \Delta t, \tag{14}$$

where F_D is the drag force on the Frisbee. Also,

$$F_y = F_g + F_L \tag{15}$$

$$m\frac{\Delta v_y}{\Delta t} = mg + \frac{1}{2}\rho v_x^2 A C_L \tag{16}$$

$$\Delta v_y = \left(g + \frac{1}{2m}\rho v_x^2 A C_L\right)\Delta t \tag{17}$$

where the subscripts x and y denote the horizontal and vertical velocity respectively and F_g is the force of gravity. Δx and Δy are simply stated as,

$$\Delta x = v_x \Delta t \tag{18}$$
$$\Delta y = v_y \Delta t \tag{19}$$

The program written contains a method *simulate* which takes five input parameters, initial y position and velocity, initial x velocity (the initial x position always set to zero), the angle of attack (in degrees) and the Δt. All units other than that of angle of attack are in SI units. In all of the trials, a $\Delta t = 0.001s$ was used. Trials with $\Delta t = 0.001s$ and $\Delta t = 0.002s$ were both tested and the difference between the results was unnoticeable. (Note: In the simulation the values of the coefficients used were: $C_{D0} = 0.08$, $C_{D\alpha} = 2.72$, $C_{L0} = .15$, $C_{L\alpha} = 1.4$.)

4. Results

When conducting the simulations, all trials had an initial height of 1 m, an initial x velocity of 14 m/s which is considered that standard velocity of a thrown frisbee, and an initial y velocity of 0 m/s. Trials were conducted using angles of attack ranging from 0° to 45°. This was the only parameter that was changed because the coefficients of lift and drag depend solely on angle of attack. It can be seen from figures 2, 3 and 4 that the angle of attack has a large effect on the trajectory of the Frisbee. With low angles of attack (generally less than 5 degrees) the lift force was very small and the frisbee dropped quickly to the ground after a short distance, usually less than 20 m. With larger angles of attack, a larger lift force was apparent and the frisbee reached greater heights and travelled much further, up to 40 m. The maximum distance travelled was obtained with an angle of attack of approximately 12° and it travelled 40 m with a maximum height of 7.7 m. At larger angles of attack the Frisbee went significantly higher, but due to the much larger drag force travelled a smaller distance. Trials that were conducted with different initial velocities followed a trend similar to those with an initial velocity of 14 m/s. At lower velocities the lift force was greatly reduced and the Frisbees just dropped to the ground faster. At higher velocities the lift force was greater and their trajectories were higher and longer.

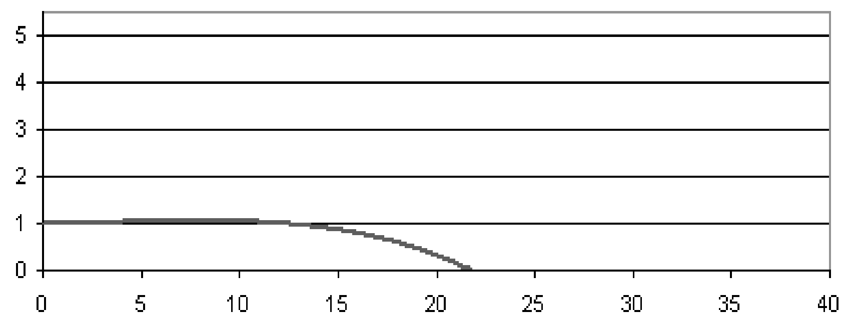

Figure 2. Plot of height(m) versus distance(m) for a Frisbee with initial velocity 14 m/s and angle of attack 5°.

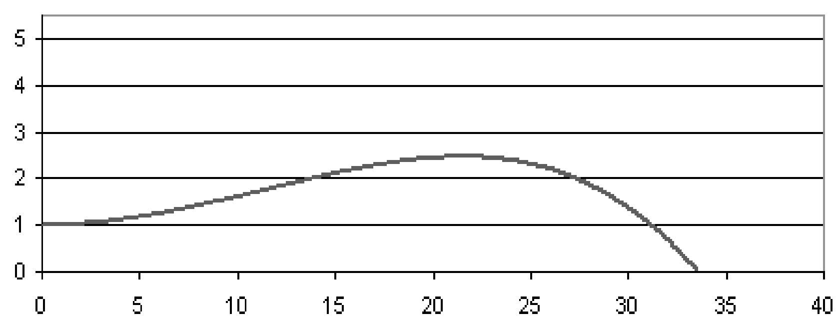

Figure 3. Plot of height(m) versus distance(m) for a Frisbee with initial velocity 14 m/s and angle of attack 7.5°.

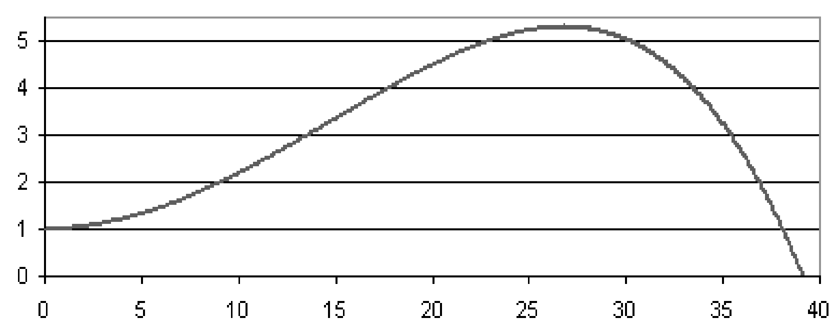

Figure 4. Plot of height(m) versus distance(m) for a Frisbee with initial velocity 14 m/s and angle of attack 10°.

5. Discussion

Although simplistic in nature, the results obtained from the program written provide a realistic simulation of the trajectory of an actual Frisbee. It was shown (using information from Hummel (2003) and Motoyama (2002)) what the various forces that act on a Frisbee are, and what they depend on, as well as how different angles of attack can vary the distance and height a Frisbee reaches greatly. In the future, further research may include developing a three dimensional model that includes the precession and rolling of the frisbee, as well as looking into the various physical properties of the Frisbee. These may include the different thicknesses of the Frisbee edges which varies the moment of inertia, flying rings which travel great distances and ridges placed on the Frisbee to reduce drag. By incorporating these properties it may be possible to design better Frisbees.

Appendix

A. Java code for plotting the trajectory of a Frisbee

The following code has been slightly modified so as to make it fit on the page.

```
import java.lang.Math;
import java.io.*;
/**
 *   The class Frisbee contains the method simulate which uses Euler's
 *   method to calculate the position and the velocity of a frisbee in
 *   two dimensions.
 *
 * @author Vance Morrison
 * @version March 4, 2005
 */
public class Frisbee {
    private static double x;
    //The x position of the frisbee.
    private static double y;
    //The y position of the frisbee.
    private static double vx;
    //The x velocity of the frisbee.
    private static double vy;
    //The y velocity of the frisbee.
    private static final double g = -9.81;
```

```
//The acceleration of gravity (m/s^2).
private static final double m = 0.175;
//The mass of a standard frisbee in kilograms.
private static final double RHO = 1.23;
//The density of air in kg/m^3.
private static final double AREA = 0.0568;
//The area of a standard frisbee.
private static final double CL0 = 0.1;
//The lift coefficient at alpha = 0.
private static final double CLA = 1.4;
//The lift coefficient dependent on alpha.
private static final double CD0 = 0.08;
//The drag coefficent at alpha = 0.
private static final double CDA = 2.72;
//The drag coefficient dependent on alpha.
private static final double ALPHA0 = -4;

/**
 * A method that uses Euler's method to simulate the flight of a frisbee in
 * two dimensions, distance and height (x and y, respectively).
 *
 */
public static void simulate(double y0, double vx0, double vy0,
                            double alpha, double deltaT)
{
    //Calculation of the lift coefficient using the relationship given
    //by S. A. Hummel.
    double cl = CL0 + CLA*alpha*Math.PI/180;
    //Calculation of the drag coefficient (for Prantl's relationship)
    //using the relationship given by S. A. Hummel.
    double cd = CD0 + CDA*Math.pow((alpha-ALPHA0)*Math.PI/180,2);

    //Initial position x = 0.
    x = 0;
    //Initial position y = y0.
    y = y0;
    //Initial x velocity vx = vx0.
    vx = vx0;
    //Initial y velocity vy = vy0.
    vy = vy0;


    try{
```

```
            //A PrintWriter object to write the output to a spreadsheet.
            PrintWriter pw = new PrintWriter(new BufferedWriter
                                       (new FileWriter("frisbee.csv")));

            //A loop index to monitor the simulation steps.
            int k = 0;

        //A while loop that performs iterations until the y position
        //reaches zero (i.e. the frisbee hits the ground).
        while(y>0){

            //The change in velocity in the y direction obtained setting the
            //net force equal to the sum of the gravitational force and the
            //lift force and solving for delta v.
            double deltavy = (RHO*Math.pow(vx,2)*AREA*cl/2/m+g)*deltaT;

            //The change in velocity in the x direction, obtained by
            //solving the force equation for delta v.  (The only force
            //present is the drag force).
            double deltavx = -RHO*Math.pow(vx,2)*AREA*cd*deltaT;

            //The new positions and velocities are calculated using
            //simple introductory mechanics.
            vx = vx + deltavx;
            vy = vy + deltavy;
            x = x + vx*deltaT;
            y = y + vy*deltaT;

            //Only the output from every tenth iteration will be sent
            //to the spreadsheet so as to decrease the number of data points.
            if(k%10 == 0){
            pw.print(x + "," + y + "," + vx);
            pw.println();
            pw.flush();
        }
            k++;
        }
        pw.close();
    }
     catch(Exception e){
         System.out.println("Error, file frisbee.csv is in use.");}
    }
}
```

References

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