

Increased learning, student engagement & student satisfaction: **a research-based course transformation**

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Research/implementation on learning with different teaching methods.

These results likely apply to all sciences and many other subjects.

Course transformation and comparison of teaching methods:
identical courses (~200 each), Physical Sciences 2 (Mech) & 3 (E&M)

Fall 2013

Course head: Logan McCarty
(Various co-instructors)

Fall 2014

Course head: Logan McCarty
Co-instructor: Louis Deslauriers

traditional lecture

Active learning

Otherwise, same:

- student population
- syllabus/learning objectives
- content and coverage
- homeworks
- sections
- midterm designs
- labs

Identical 3 hour final exam (same grading rubric)

"Transformed" course?

Integration of multiple "best educational practices"...

1- Students are **actively engaged** with material

(e.g. they need to question, need to integrate with what they already know, etc.)

- *but engagement has to be productive...*

2- Need lots of components to show increased learning

(e.g. Peer interaction, Keeping track of own's progress, Making/testing predictions, etc)

Useful rule of thumb:

Approximate one-on-one tutoring with expert tutor as best we can

Practiced in all transformed courses.

- well articulated learning goals-- **guides everything**
- pre-class reading assignments with online quizzes
- clicker questions with student-student discussion
- in-class small group activities (5-10 min each)
targeted instructor feedback guided by observations
of student thinking, little pre-prepared lecture
- Two-stage exams

What does it look like?

Example: **5-10 min** in-class activity during lecture



<https://www.youtube.com/watch?v=Tw-ms15bsLs>

Physical Sciences 2: Lecture 5b October 14, 2014

Activity 3: Off-Axis Torques

- Now let's consider a bit more complicated example. We have an object (a top) that is spinning along an axis tilted at an angle ϕ from the vertical:

① $\vec{\tau} = \vec{R} \times \vec{F}$
 $= mgR \sin \phi$

$\vec{L} = I\omega$

Direction of precession

Direction of spin

Gravitational torque mgx produces precession

Origin

1. What is the (vector) angular momentum \vec{L} ? What is the (vector) torque arising from the gravitational force? (Take the origin to be the bottom point in the diagram, so that the contact force from the floor provides no torque.)

$\vec{\tau}$: magnitude $\tau = mgR \sin \phi$ points out of the page

2. Over a short time Δt , find the change in angular momentum, $\Delta \vec{L}$. (Use Newton's Second Law for rotation.) Will the top speed up? Slow down? Something else?

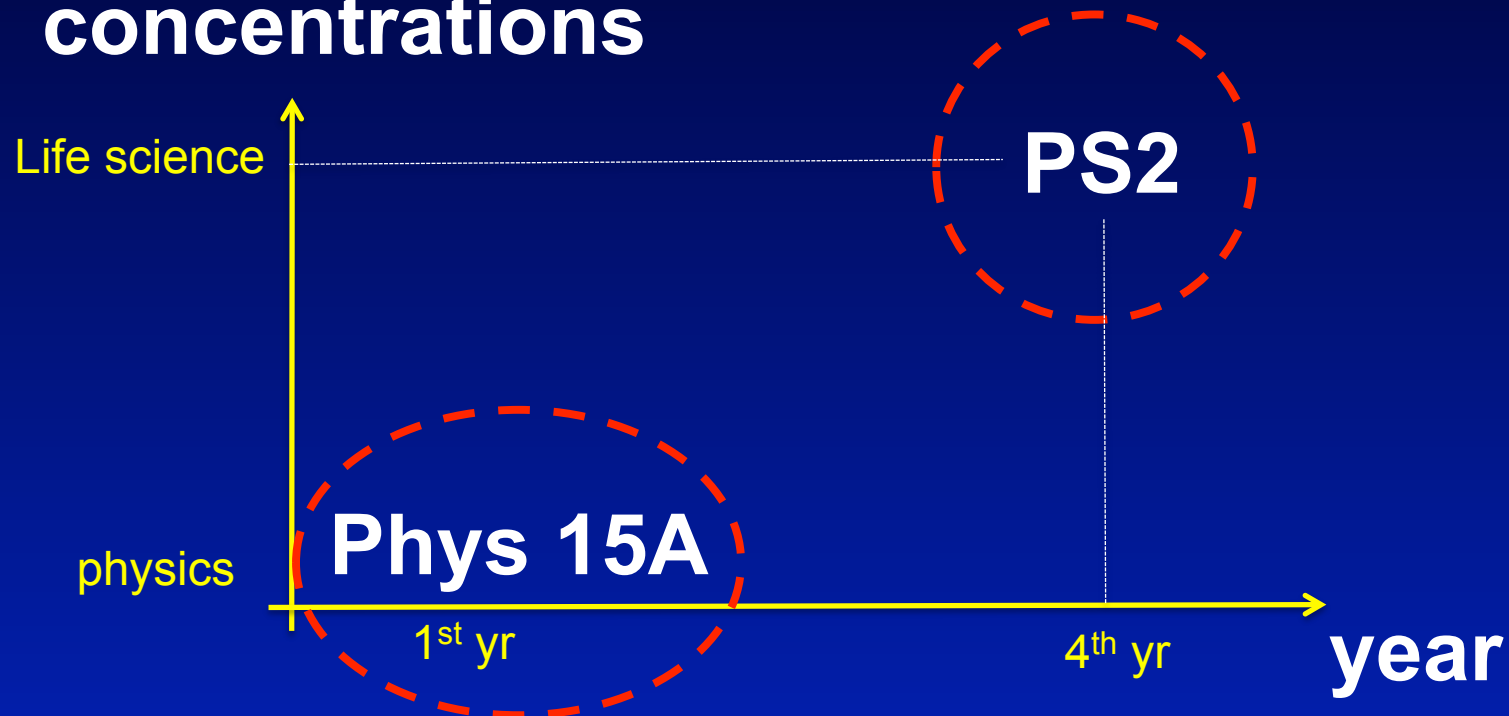
$\vec{\tau} \times \frac{\Delta \vec{L}}{\Delta t}$ so $\Delta \vec{L} = \vec{\tau} \cdot \Delta t$

BONUS! Precession arises because $\vec{\tau}$ is *perpendicular* to the angular momentum \vec{L} . What happens in linear motion when \vec{F} is perpendicular to the linear momentum \vec{p} ?

7

Student *year* and *concentration* matter

concentrations



- Advanced optics (Gaussian beams, Fourier optics, Optical processing, etc)
- Quantum Mechanics (1-D, 3-D, time dep./indep. perturbation, scattering etc)
- Modern Physics (intro to QM and Relativity)
- First year E&M for engineers, and 1st yr E&M for life science majors

Results:

- Student performance
- Course evaluations
- Expert vs. novice thinking

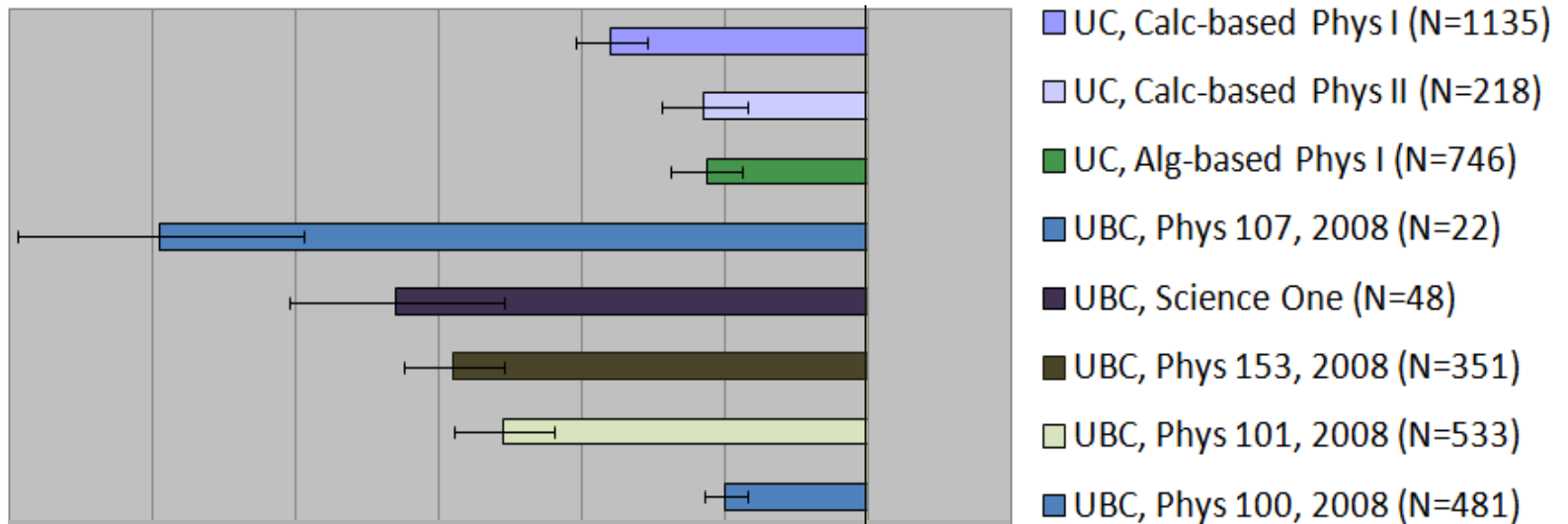
2014: mean 77.4 ± 0.8 (std. err.); N = 189

2013: mean 71.0 ± 1.0 (std. err.); N = 208

**Student
satisfaction (CUE)**

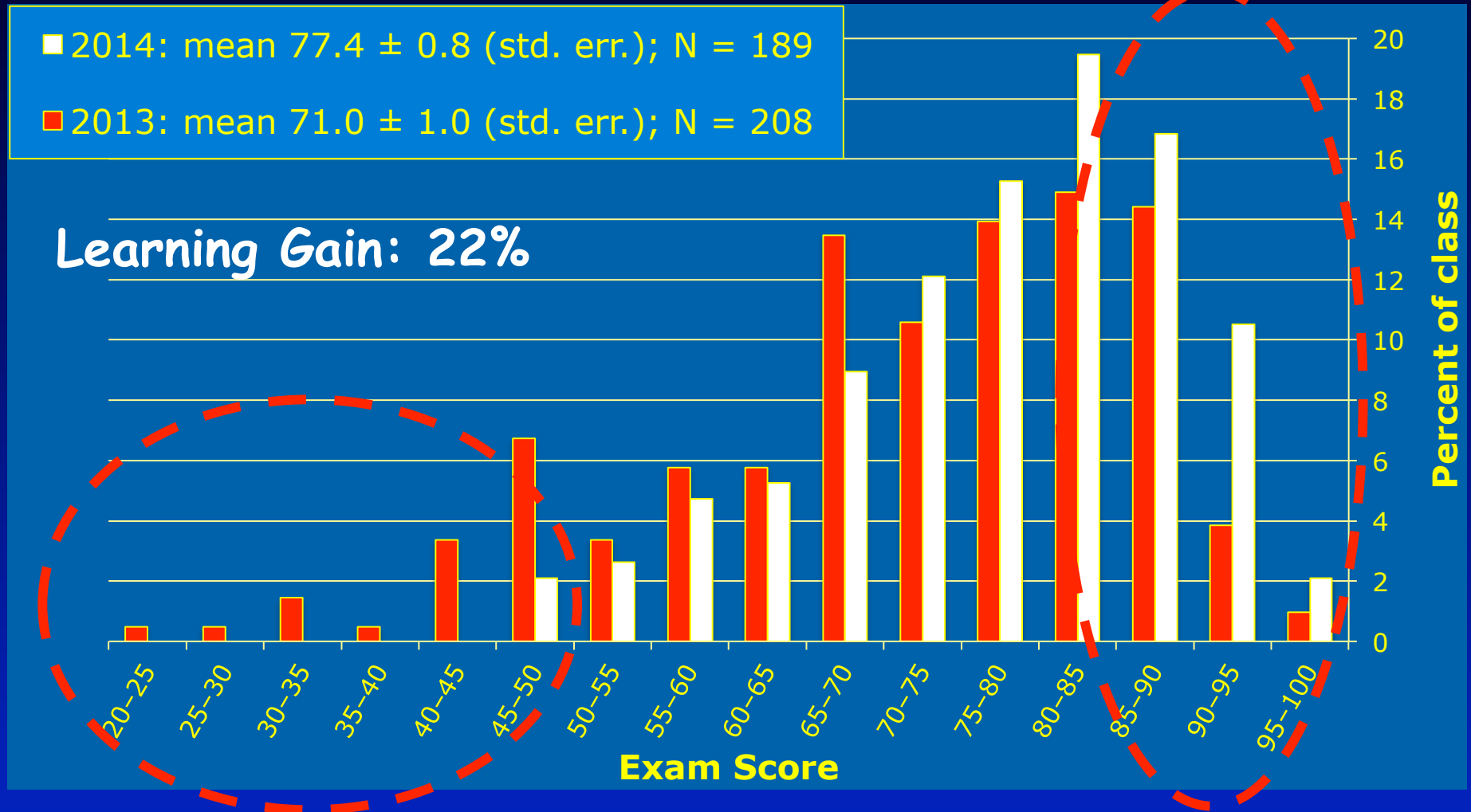
MORE NOVICE MORE EXPERT

-12 -10 -8 -6 -4 -2 0 2



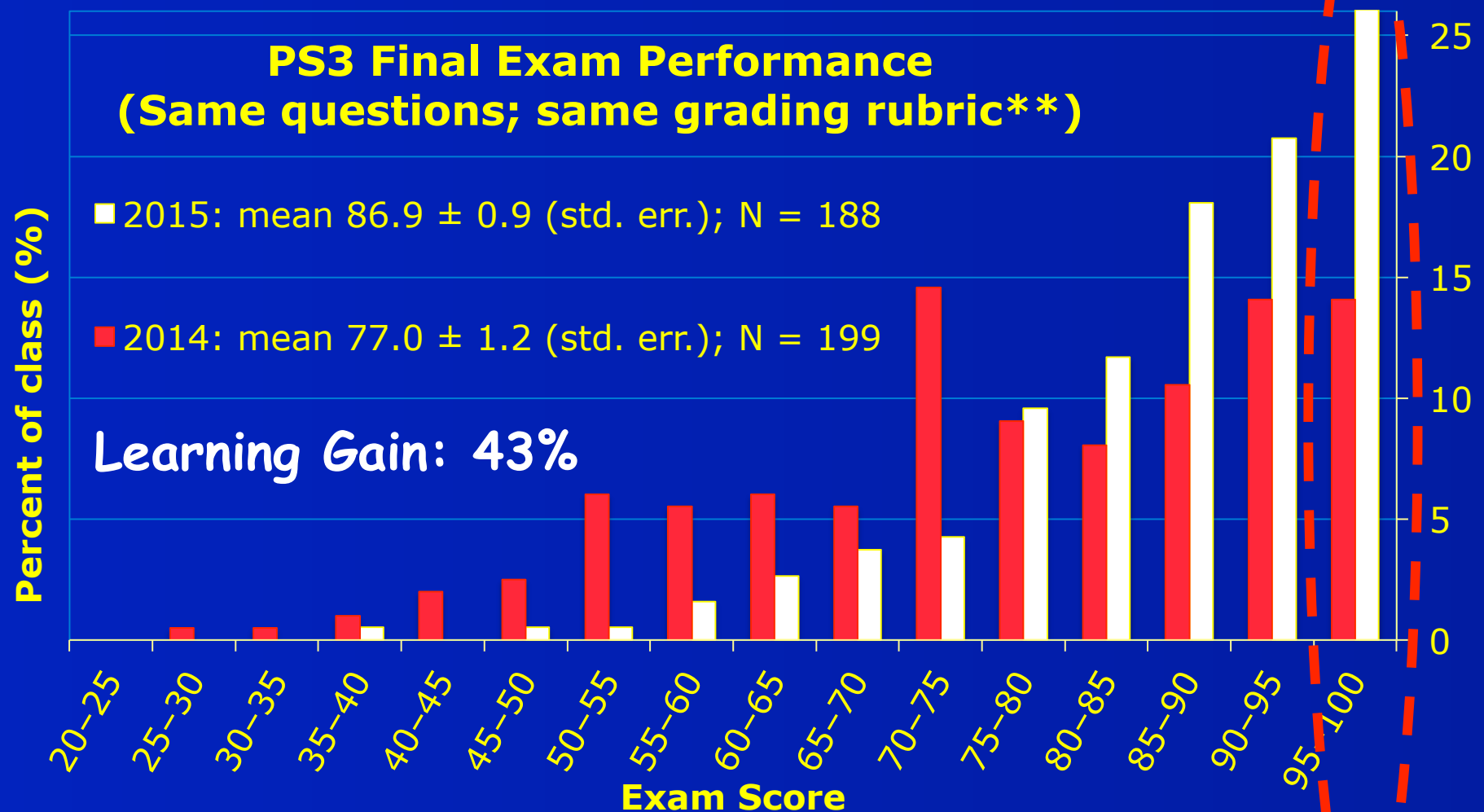
Shift in % Favorable (Post - Pre)

Results: PS2 Final Exam Performance (Same exam; same grading rubric)



****Similar increase in performance for midterm 1 & 2**
Active learning is good for everyone...

In PS3, we transformed: Lectures & HWs



****Same grading rubric?**

Salient fact: Number of perfect scores went from **3** to **20**!

What if exposure to material only occurred during lecture? (eg no HWs, no sections)

Comparing the learning in two identical sections of 1st year college physics. 270 students each.



Control--standard lecture class – highly experienced Prof with good student ratings.

Experiment – inexperienced teacher trained to use principles of effective teaching.

- Same learning objectives
- Same class time (3 hours, 1 week)
- Same exam (jointly prepared)

Results

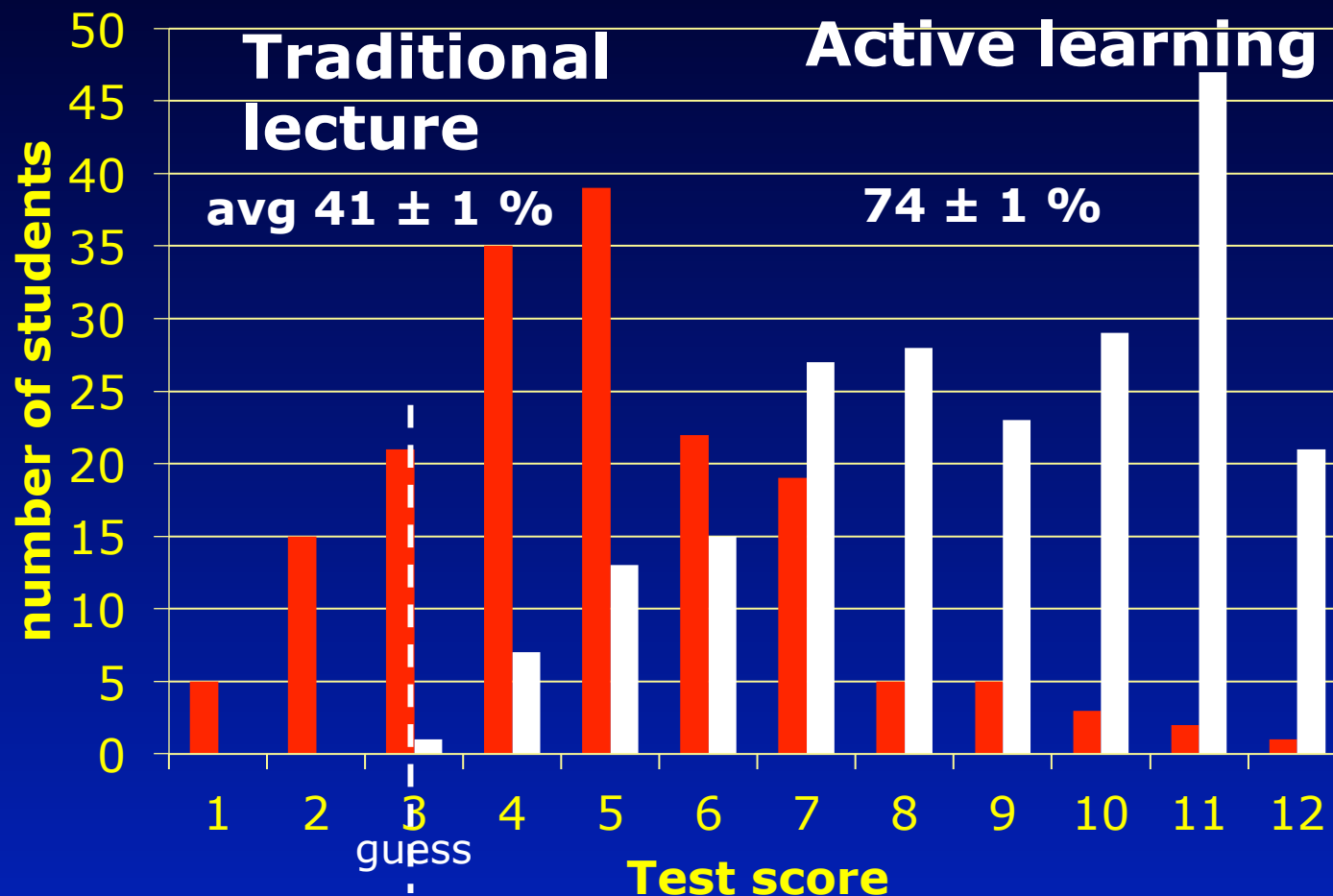
	<u>II. Trad</u>	<u>I. Transformed</u>
1. Attendance <i>N= 260 in both</i>	pre int 58% during 58 % →	58% (<i>wk 10 & 11</i>) 81%
2. Engagement (back ½ room)	pre int 50% during 50 % →	50% (<i>wk 10 & 11</i>) 85%
3. Learning (test) <i>above guessing</i> S. D. = 13%	41% 18%	74 % 51%

trad
18

trans
51

*other things practiced: scientific discourse,
critiquing scientific arguments, sense-making,
collaboration.*

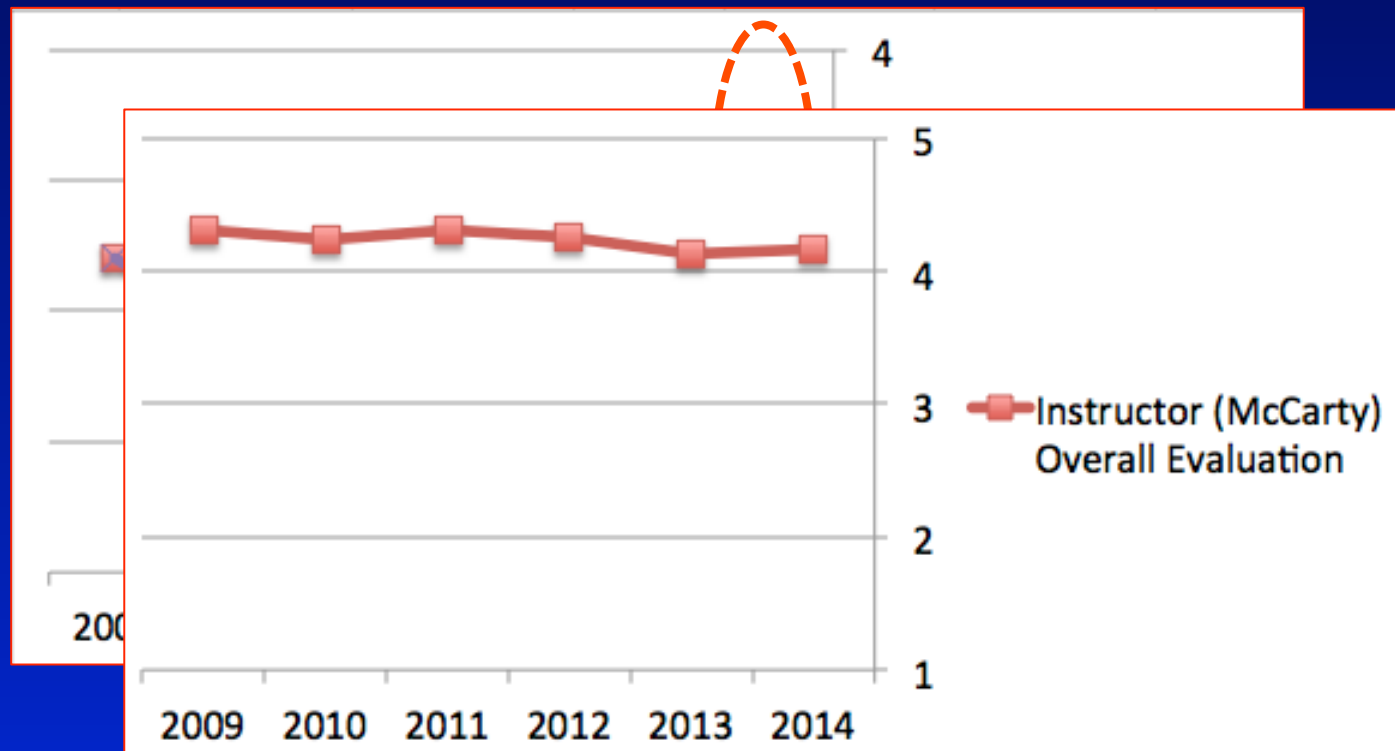
Learning from lectures **ONLY**



Clear improvement for entire student population.
Engagement 85% vs 50%.

Results: **Student satisfaction (CUE)**

- Most difficult transformation:
(1) Junior/Senior and (2) non-physics majors
- Significant number of students were initially skeptical
- ...But attitude (apparently) always changes after 1st exam



Results:

The CLASS Survey

(Colorado Learning Attitudes about Science Survey)

■ Main Goals:

- Focus on *perception about the discipline and learning the discipline*
- Valid/Reliable across university populations (non-sci to majors)
- Probe additional facets of perception (problem solving)

■ CLASS-Phys (42 statements)

Strongly Disagree 1 2 3 4 5 Strongly Agree

I think about the physics I experience in everyday life.

It is possible to explain physics ideas without mathematical formulas.

Builds on previous work in physics by (MPEX¹ & VASS²)

1. Redish, E., Saul, J. M. Steinberg, R. N., (1998). *Amer. Journal of Phys.* 2. Halloun, I. E., (1996). *Proceedings of the ICUPE.*

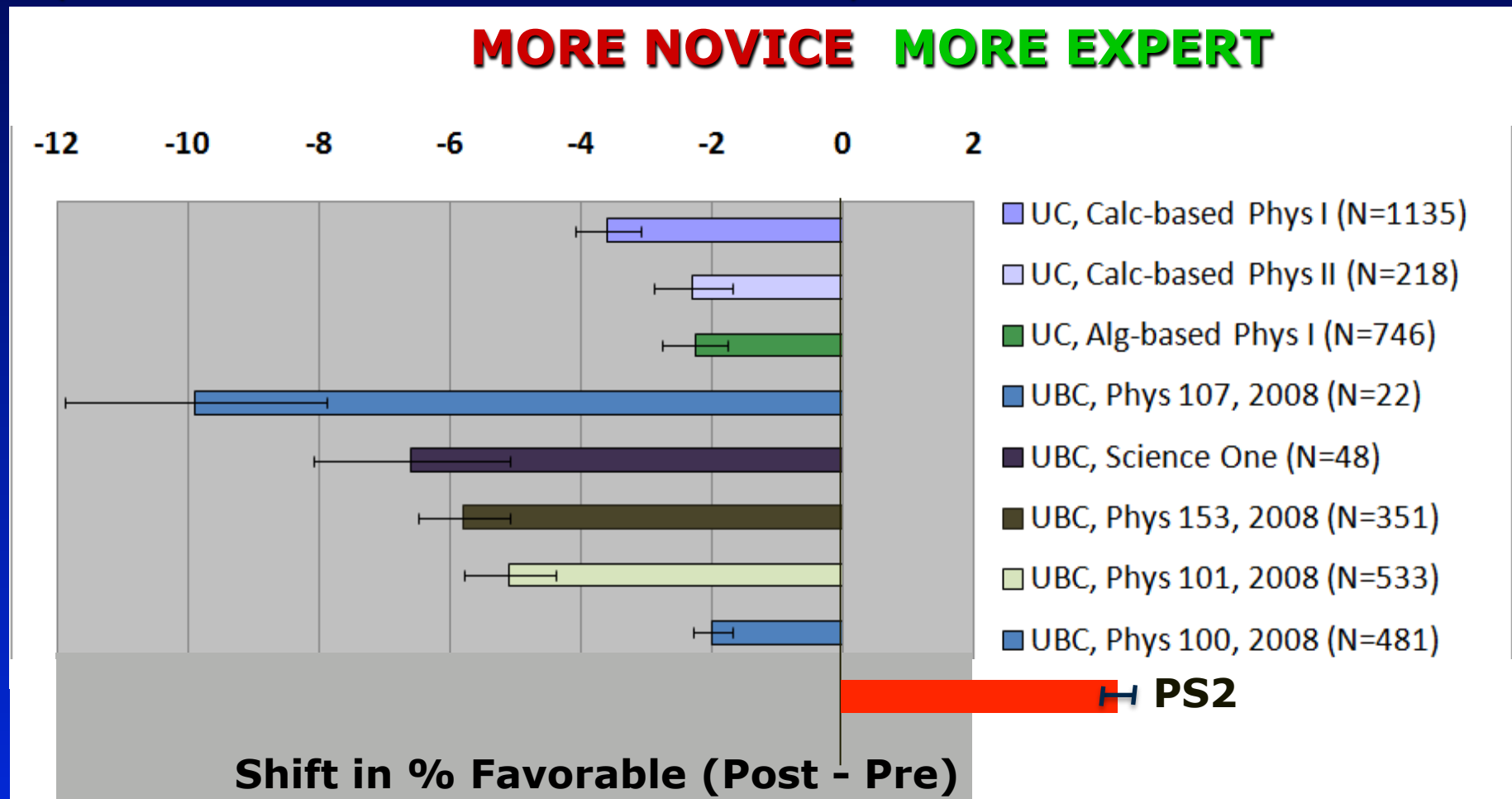
9. There is usually only one correct approach to solving a physics problem.
 10. If I get stuck on a physics problem on my first try, I usually try to figure out a different way that works.
 11. I am not satisfied until I understand why something works the way it does.
 12. I cannot learn physics if the teacher does not explain things well in class.
 13. I do not expect physics equations to help my understanding of the ideas; they are just for doing calculations.
-

Student perceptions of physics (almost) always **drops** from beginning to end of term.

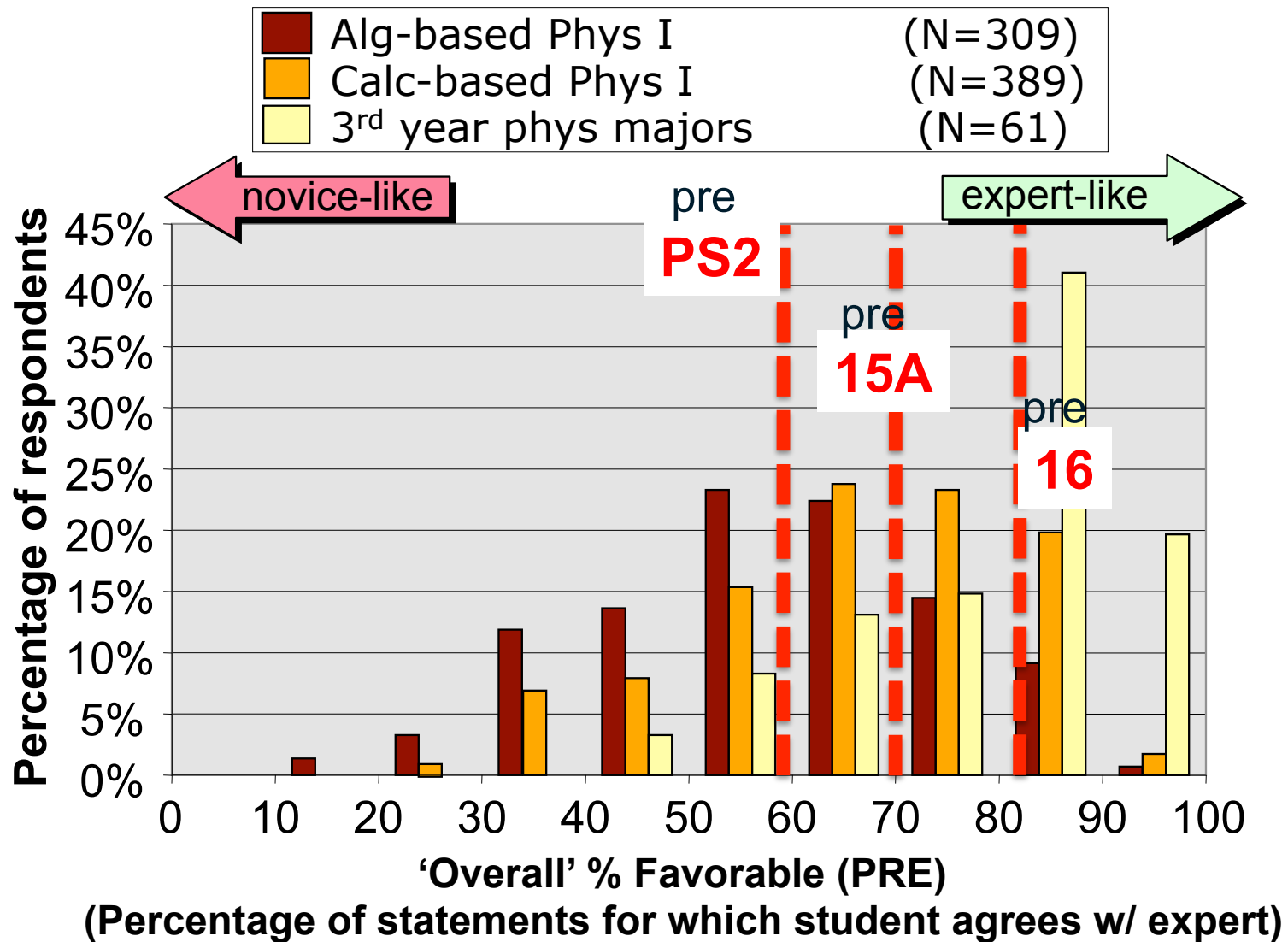
- Implies:
- 1- Student sees physics as less relevant in describing the world around them
- 2- They have less confidence they can use physics to solve problems they may encounter
- 3- They see physics as more about memorizing w/o any a need to understand
- 4- etc...

Results: Impact of teaching on students' beliefs

CLASS-Phys results at CU-Boulder and UBC:
Students' expectations shift to be more novice
(decline of ~5-8% in 'Overall' %fav)



Distribution of Beliefs About Physics



Kathy Perkins & Wendy Adams, CU

Who chooses to major in physics? And then stay on? ———> CLASS !

Transforming a course:

Why? How?

PS2 was going great...so why change it?

- Strengths:**
- Good course evaluations
 - Connections with biology/medicine
 - Solid conceptual understanding (40% norm. gain on FCI)
 - Problem solving skills (?)
- Concerns:**
- Poor student engagement
 - PER → lecturing is not optimal
-

Reluctant to change it...

- Concerns:**
- Course/Instructor evaluations might go down
 - Help weaker students at expense of top students
 - Cover less content
 - Conceptual learning at expense of problem solving
-

New:

Learning objectives
Interactive lectures
Two-stage exams

Unchanged:

Syllabus and content
Sections and labs
HW and exams

Anatomy of a course transformation:

- (1)** how do you actually transform lectures...
- (2)** how much work is it?
- (3)** What does it look like?
- (4)** transforming your own teaching

1- How do you transform a lecture?

- Write detailed list of (operational) learning goals for lecture
- Typically 4-7 major learning goals per lecture
- Design in-class activities for each learning objective
- Think carefully about expert feedback to students

2- How much work is it?

Process initially takes ~5-10 hours for each 1.5 hour lecture

With experience, typically ~3-4 hours (comparable to re-writing lecture from scratch)

Additional effort for pre-reading

Rotational Dynamics

- Highlights from the pre-reading:

The basic vector rotational quantity is the **angular velocity vector** $\vec{\omega}$. Its direction tells you the axis of rotation, and its magnitude tells the speed of rotation. The direction of rotation (clockwise or counterclockwise) is determined using the “right-hand rule.”

Once we’ve decided to make angular velocity a vector, lots of other rotational quantities must also become vectors. We can summarize the vector rotational quantities, along with their linear analogues:

Velocity \vec{v}	\Leftrightarrow	Angular velocity $\vec{\omega}$
Acceleration $\vec{a} = \frac{d\vec{v}}{dt}$	\Leftrightarrow	Angular acceleration $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$
Force \vec{F}	\Leftrightarrow	Torque $\vec{\tau}$
Mass m	\Leftrightarrow	Moment of inertia I
(linear) Momentum $\vec{p} = m\vec{v}$	\Leftrightarrow	Angular momentum $\vec{L} = I\vec{\omega}$
Newton’s 2 nd : $\sum \vec{F} = \frac{d\vec{p}}{dt}$	\Leftrightarrow	Newton’s 2 nd (rotation): $\sum \vec{\tau} = \frac{d\vec{L}}{dt}$

In an isolated system (with no net torque), angular momentum must be conserved—but that does not mean that the angular velocity will remain constant, because the moment of inertia could change as well. Only the *product* $I\vec{\omega}$ is constant for an isolated system.

When a rotating object does not have a fixed axis, you can apply a torque that is *perpendicular* to the angular momentum vector. In this case, the torque does not change the speed of rotation, but instead changes the *direction* of the rotation axis, in a phenomenon known as *precession*.

- Learning objectives: After this lecture, you will be able to...

1. Identify the angular velocity vector $\vec{\omega}$ for an object rotating around some arbitrary axis, using the right-hand rule to assign the correct direction of the vector.
2. Define and use the *cross product*, which multiplies two vectors to yield a third vector.
3. Determine the *vector torque* resulting from a force applied at some point on an object using the definition of the cross product and the right-hand rule.
4. Write Newton’s Second Law for rotational motion, and compare this law with the analogous law for linear motion.
5. Use the conservation of angular momentum to predict the behavior of an isolated system if its moment of inertia changes or if some aspect of its angular velocity changes.
6. Predict the direction of precession if a torque is applied to a rotating system in such a way that the torque is perpendicular to the angular momentum vector.
7. Calculate the rate of precession for a rotating system with a perpendicular torque.

3- What does it look like?

Example: 5-10 min in-class activity during lecture



<https://www.youtube.com/watch?v=Tw-ms15bsLs>

Physical Sciences 2: Lecture 5b October 14, 2014

Activity 3: Off-Axis Torques

Now let's consider a bit more complicated example. We have an object (a top) that is spinning along an axis tilted at an angle ϕ from the vertical:

1. What is the (vector) angular momentum L ? What is the (vector) torque arising from the gravitational force? (Take the origin to be the bottom point in the diagram, so that the contact force from the floor provides no torque.)

$\vec{\tau}$: magnitude $\tau = mgR \sin \phi$ points out of the page

2. Over a short time Δt , find the change in angular momentum, $\Delta \vec{L}$. (Use Newton's Second Law for rotation.) Will the top speed up? Slow down? Something else?

$\vec{\tau} \times \frac{\Delta \vec{L}}{\Delta t}$ so $\Delta \vec{L} = \vec{\tau} \cdot \Delta t$

BONUS! Precession arises because $\vec{\tau}$ is perpendicular to the angular momentum \vec{L} . What happens in linear motion when \vec{F} is perpendicular to the linear momentum \vec{p} ?

7

Instructor gets detailed, real-time feedback

Activity 1: Angular vs. linear motion

1. A point on the rim of a wheel of radius R starts out at angle θ_i and ends at angle θ_f . How far has it traveled (along the circumference)?



Turn a derivation into an activity

1. Show (explicitly!) that the change in K can be written as an integral from some initial state to some final state:

$$\Delta K = \int_{\text{initial}}^{\text{final}} dK = [K]_{\text{initial}}^{\text{final}} = K_f - K_i = \Delta K$$

2. Now let's look at " dK ." We can write the kinetic energy using the **dot product** as:

$$K = \frac{1}{2} m \vec{v} \cdot \vec{v}$$

Using this expression for the kinetic energy, the definitions of \vec{v} and \vec{a} , and the rules for "d," show that we can write dK in terms of the **infinitesimal displacement** $d\vec{r}$:

$$dK = m \vec{a} \cdot d\vec{r}$$

$$\begin{aligned} dK &= d\left[\frac{1}{2} m \vec{v} \cdot \vec{v}\right] = \frac{1}{2} m d[\vec{v} \cdot \vec{v}] \\ &= \frac{1}{2} m [d\vec{v} \cdot \vec{v} + \vec{v} \cdot d\vec{v}] = \frac{1}{2} m [2 d\vec{v} \cdot \vec{v}] \\ &= m [d\vec{v} \cdot \vec{v}] = \frac{m d\vec{v} \cdot \frac{d\vec{r}}{dt}}{dt} = m \left(\frac{d\vec{v}}{dt} \cdot d\vec{r}\right) \end{aligned}$$

(dot)
product
rule

3. Use parts (1) and (2) along with Newton's laws to find an expression for ΔK in terms of the **net force** \vec{F}_{net} :

$$\Delta K = \int_i^f dK = \int_i^f m \vec{a} \cdot d\vec{r} = \int_i^f m \vec{a} \cdot d\vec{r}$$

final

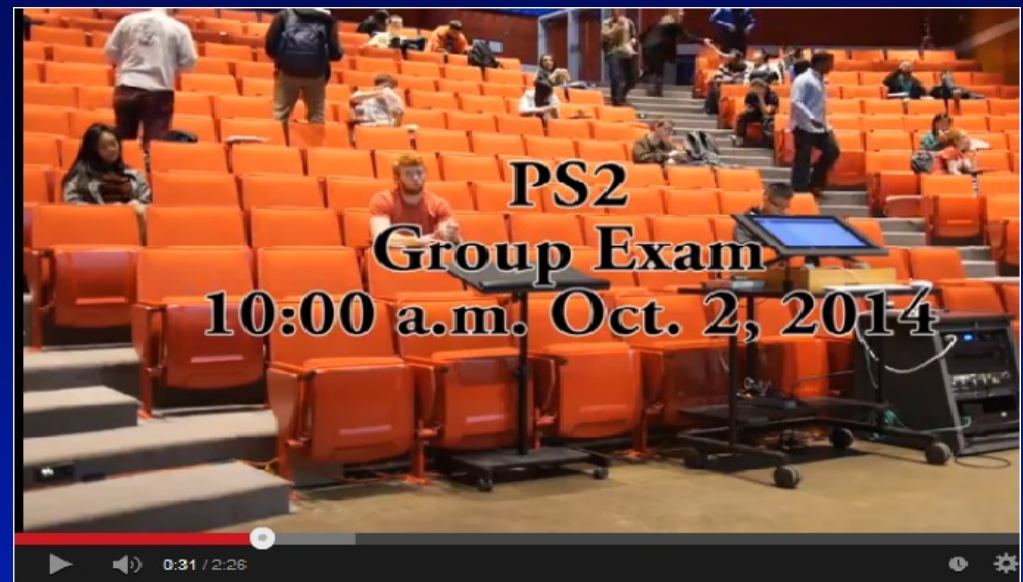
3- What does it look like? Cont'd

Two-stage Exams

Simple way to Introduces collaborative learning and formative assessment into an exam

Logistics:

- Individual exam (90 mins)
- Group exam (45mins)
- Students re-take the same ind. Exam in groups of 3 or 4
Each group hands-in 1 exam (requires consensus)
- 80% of midterm = individual score
- 20% of midterm = group score
(group score can't be lower than individual score)



3- What does it look like? Cont'd

Two-stage Exams

Benefits:

- immediate feedback on exam
- effective feedback with peer interaction
- Students enjoyed it
 - They reported benefits from the experience
 - intensely engaging experience for students
 - In 1 year post-course interviews, group exams is most remembered aspect of course



<https://www.youtube.com/watch?v=o1DN8z5DZWQ&feature=youtu.be>

4- Transforming the instructor...how did it feel?

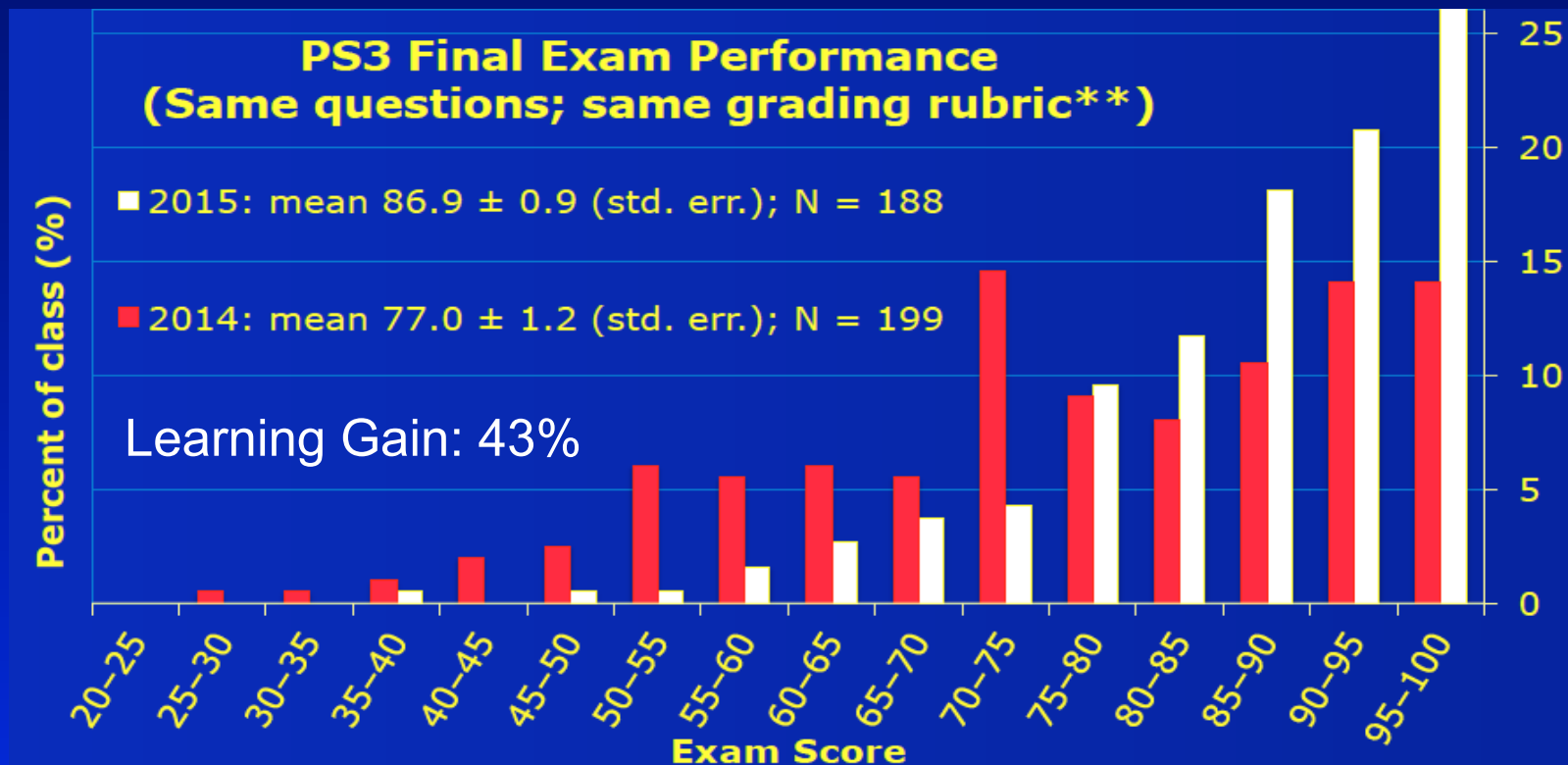
- **Initially**, felt challenging and uncomfortable
 - Give up control → frightening
 - Students seem skeptical
 - Need to adapt in real time:
 - how long per activity?
 - how much feedback?
 - classroom management
 - But still saw:
 - more sophisticated questions
 - sections were different
 - help room was different
 - 1st midterm always changes student attitudes...
-

Bottom line: *I love to lecture, but I could never go back*

This term in PS3, we...

- Transformed lectures in same way as in PS2
 - ...and also transformed weekly HWs
- With TF Kristina Callaghan

Lectures



Transforming weekly HWs

But weekly HWs are **already** active learning...

Used principles of deliberate practice:

- 1- Identified complex problem solving tasks
- 2- Broke them down into sub-tasks
Exercise targeted at sub-tasks, with feedback.
- 3- Synthesis portion of HW – exercises to integrate sub-tasks
- 4- Practice complex tasks with feedback

Made each step explicit to students

Upside: Transformed HWs are naturally adaptive to student' skill level

Thank you!

Greg Kestin (College Fellow and Extension School)

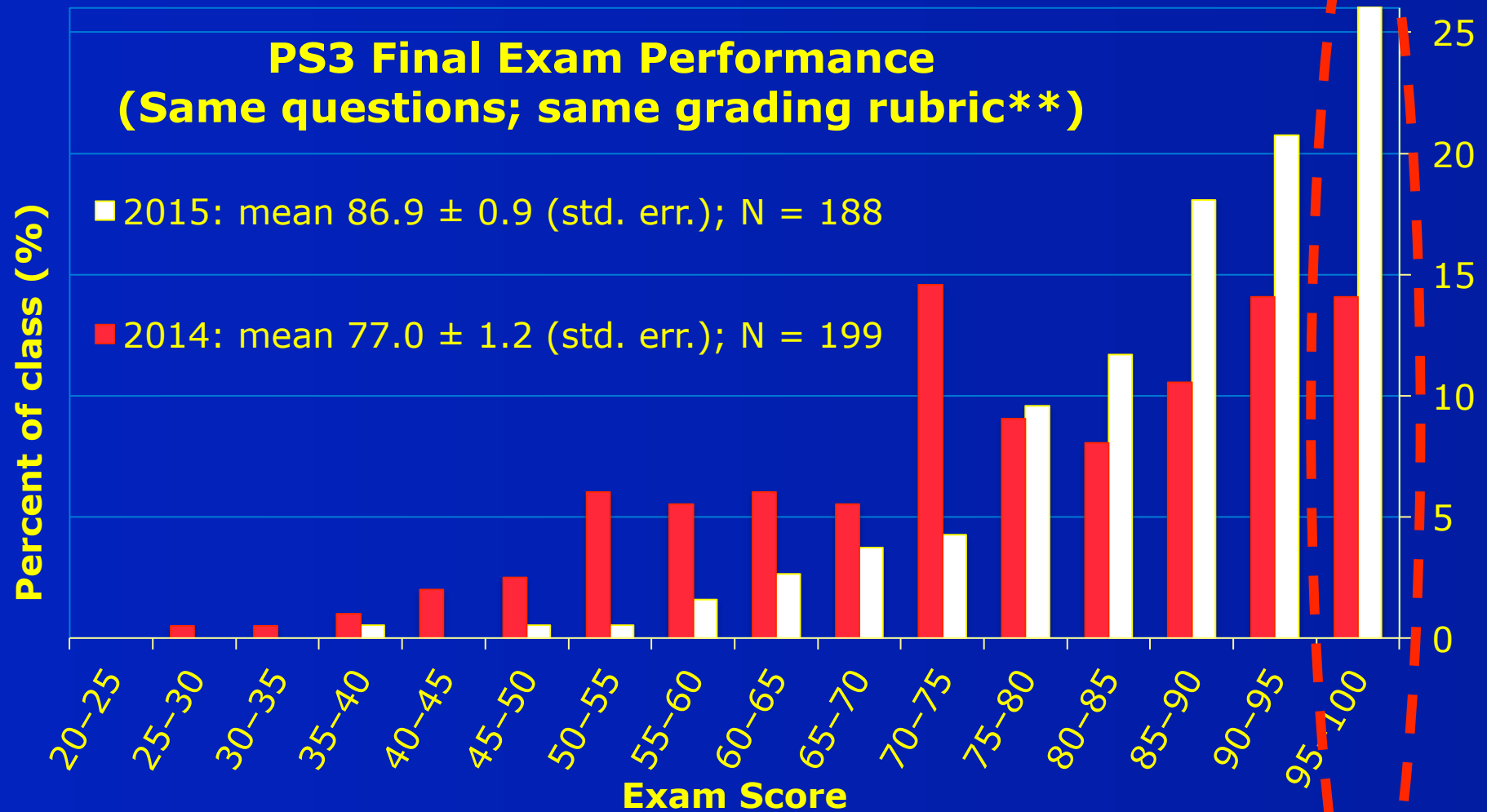
Carl Wieman Science Education Initiative, UBC
Harvard Physics colleagues
PS2 Teaching Fellows
PS2/PS3 students







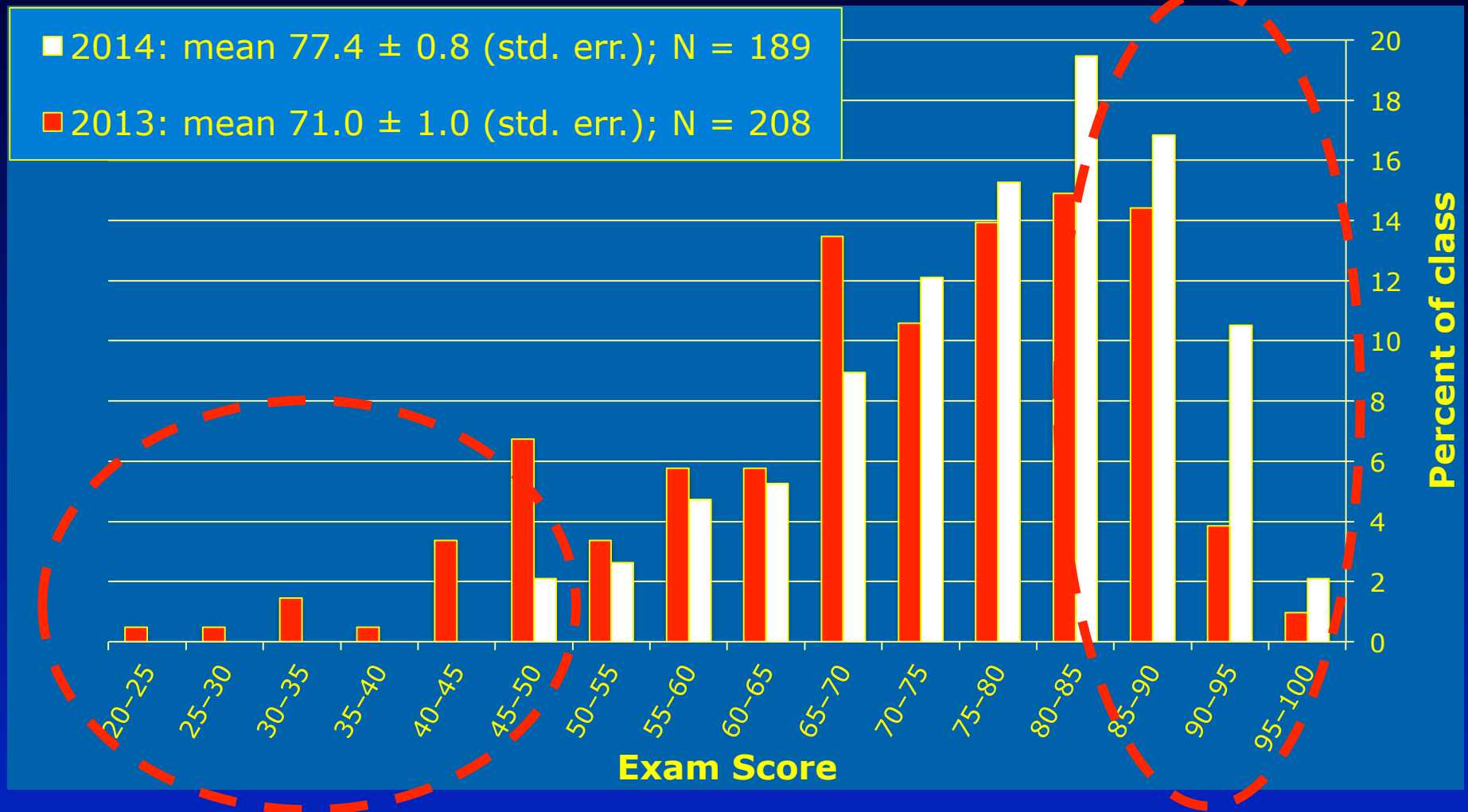
In PS3, we transformed: Lecture & HWs



****Same grading rubric?**

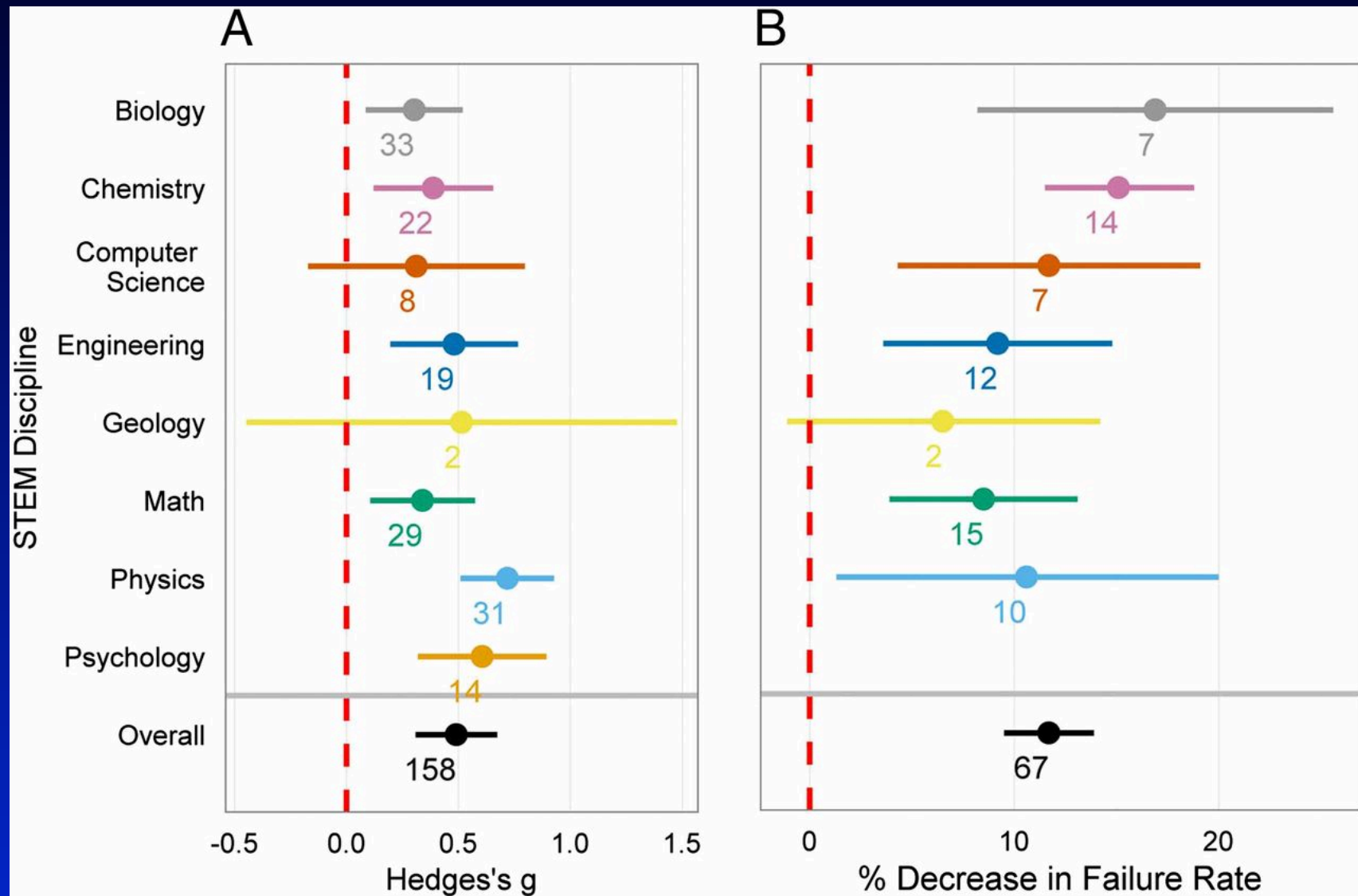
Salient fact: Number of perfect scores went from **3** to **20**!

Results: PS2 Final Exam Performance (Same exam; same grading rubric)



****Similar increase in performance for midterm 1 & 2**
Active learning is good for everyone...

Meta analysis of active learning interventions: effect sizes by discipline



Effective Science teaching for all students--transforming brains

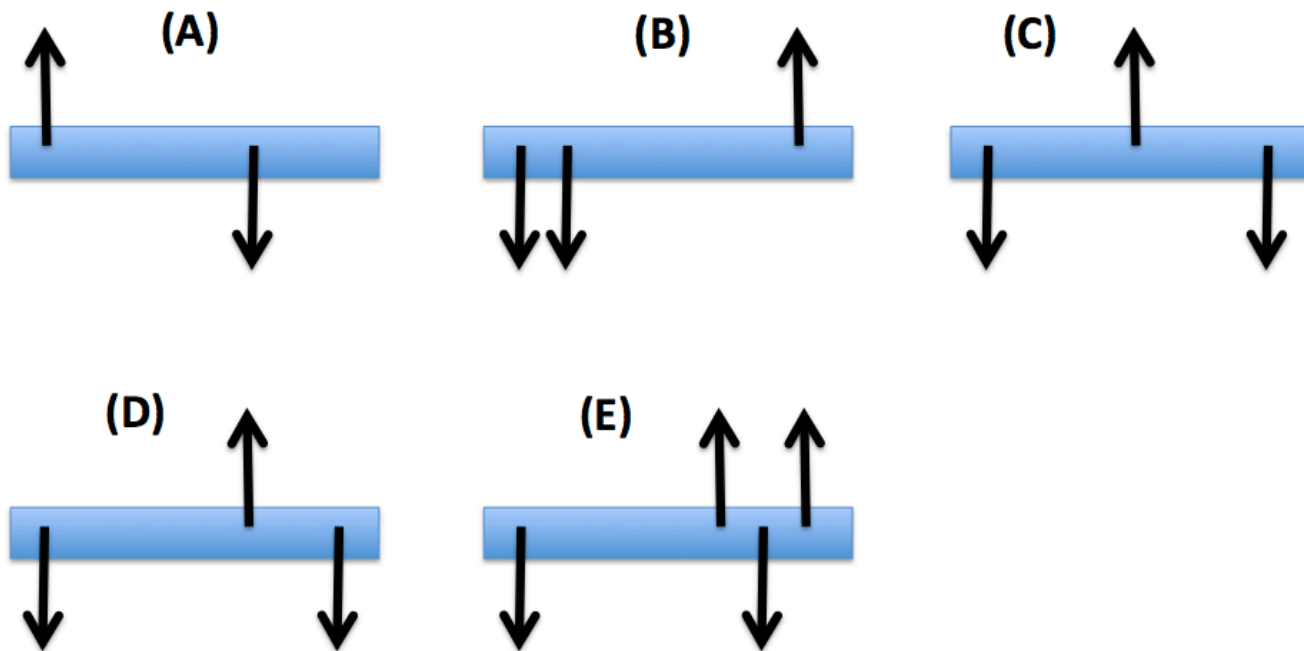
1. Motivation
2. Delineate expert thinking about topic → learning objectives
3. Create suitable tasks to explicitly practice
 - a. motivating- many hours of intense practice (thinking)
 - b. connect with prior knowledge
 - c. fit with how short & long term memory work
 - d. practice expert thinking
4. Provide timely and specific feedback/guidance

"The Influence of Experience and Deliberate Practice on the Development of Superior Expert Performance", K. Anders Ericsson; 2006

Instructor gets detailed, real-time feedback

Activity 1B: Balancing forces and torques

The figure shows six overhead views of a uniform rod on which two or more forces act perpendicularly to the rod? If the magnitudes of the forces are adjusted properly (*but kept nonzero*), in which situations can the rod be in static equilibrium? Make sure to provide explanations







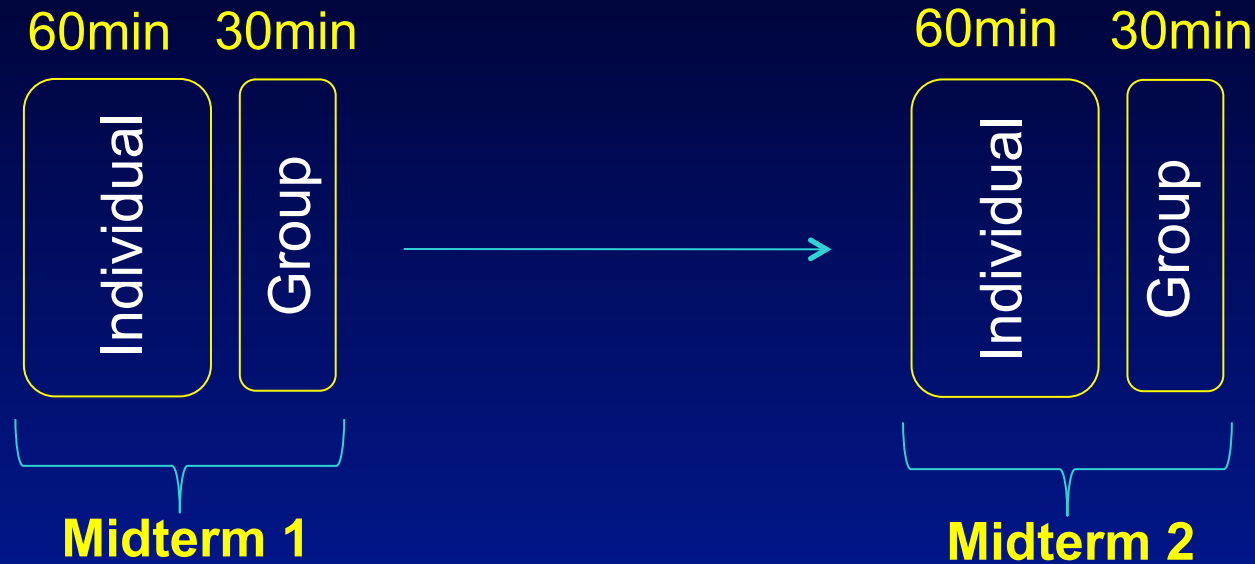
Effectiveness of group exams: UBC Quantum Mechanics (70 students)



First off, why do this?

- students get immediate feedback from peers
- students report benefits from the experience (they like it a lot!)
- its surprisingly easy to implement
- increased recall/learning (??)
- Improves group dynamics (??)

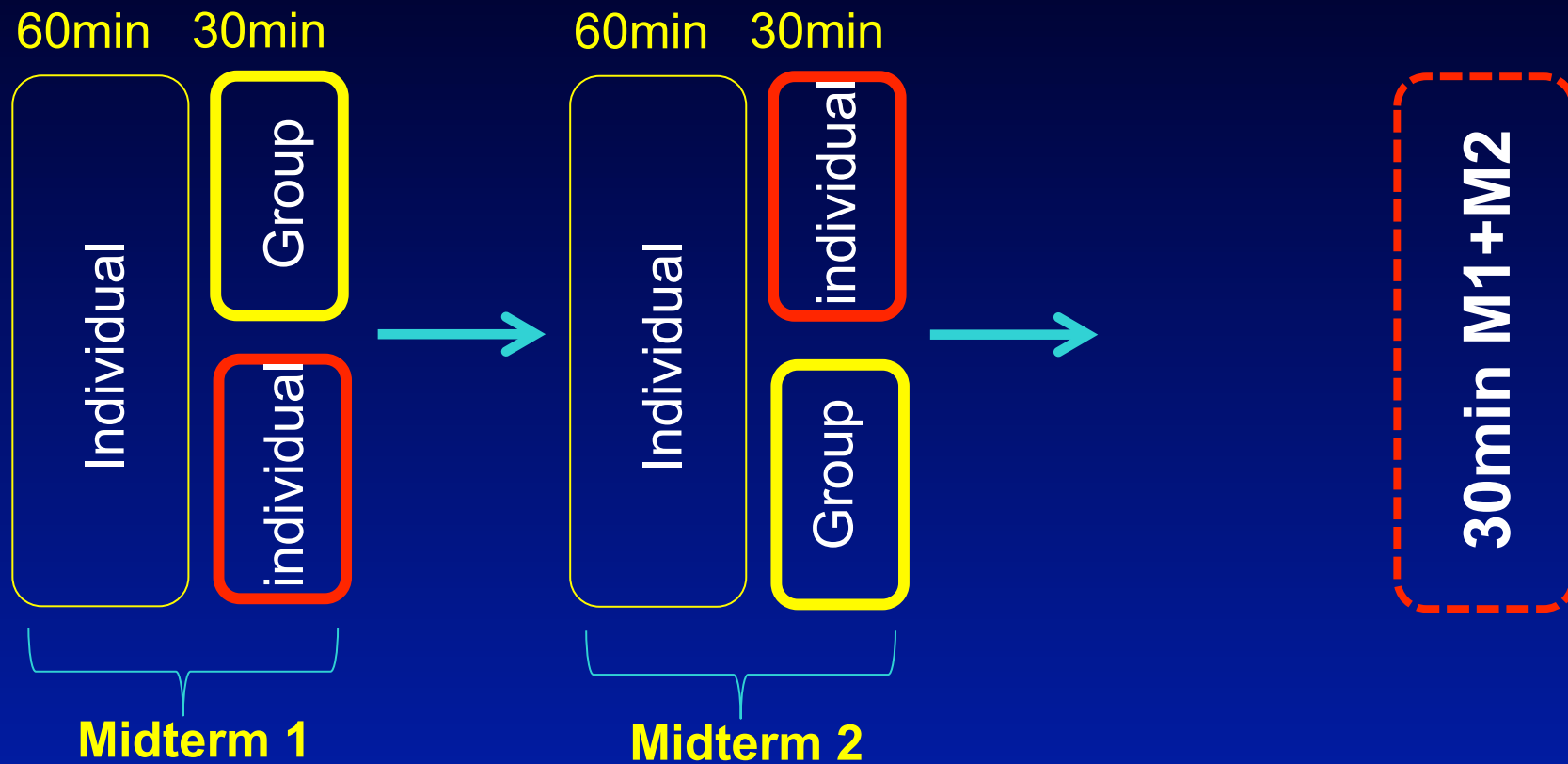
Effectiveness of group exams: UBC Quantum Mechanics (70 students)



The **individual** portion counts for 75% of midterm mark
The **group** portion counts for 25% of midterm mark

- Groups of 4
- Each group hand-in 1 exam (requires consensus)
- Group score can't be lower than individual score

Effectiveness of group exams: UBC Quantum Mechanics (70 students)



Results from last year:

30min midterm 1 Q's

Group = 81% +/-5%

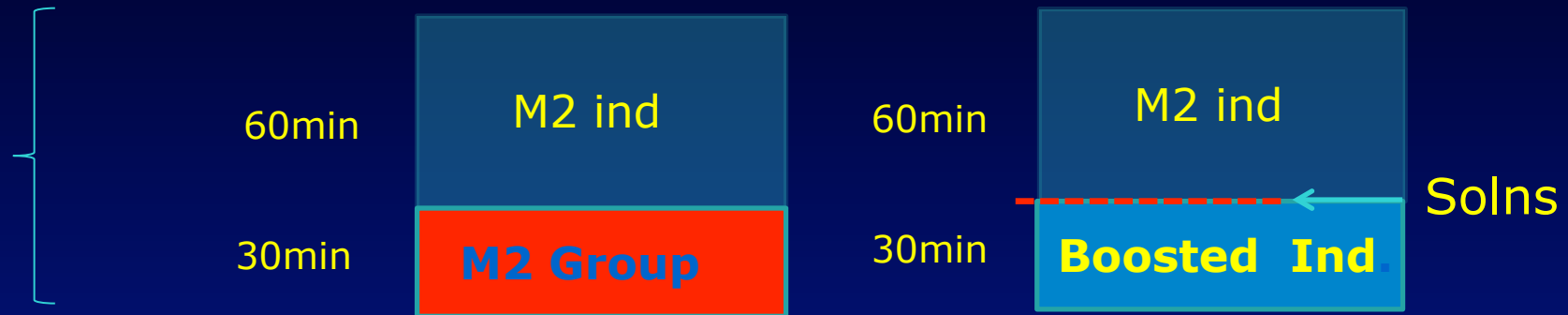
Ind = 78% +/-6%

30min midterm 2 Q's

Group = 80% +/-4%

Ind = 79% +/-5%

Group Vs Boosted Individual Exam



- What is this “Boosted Individual” 30min exam that replaces the 30min Group exam?

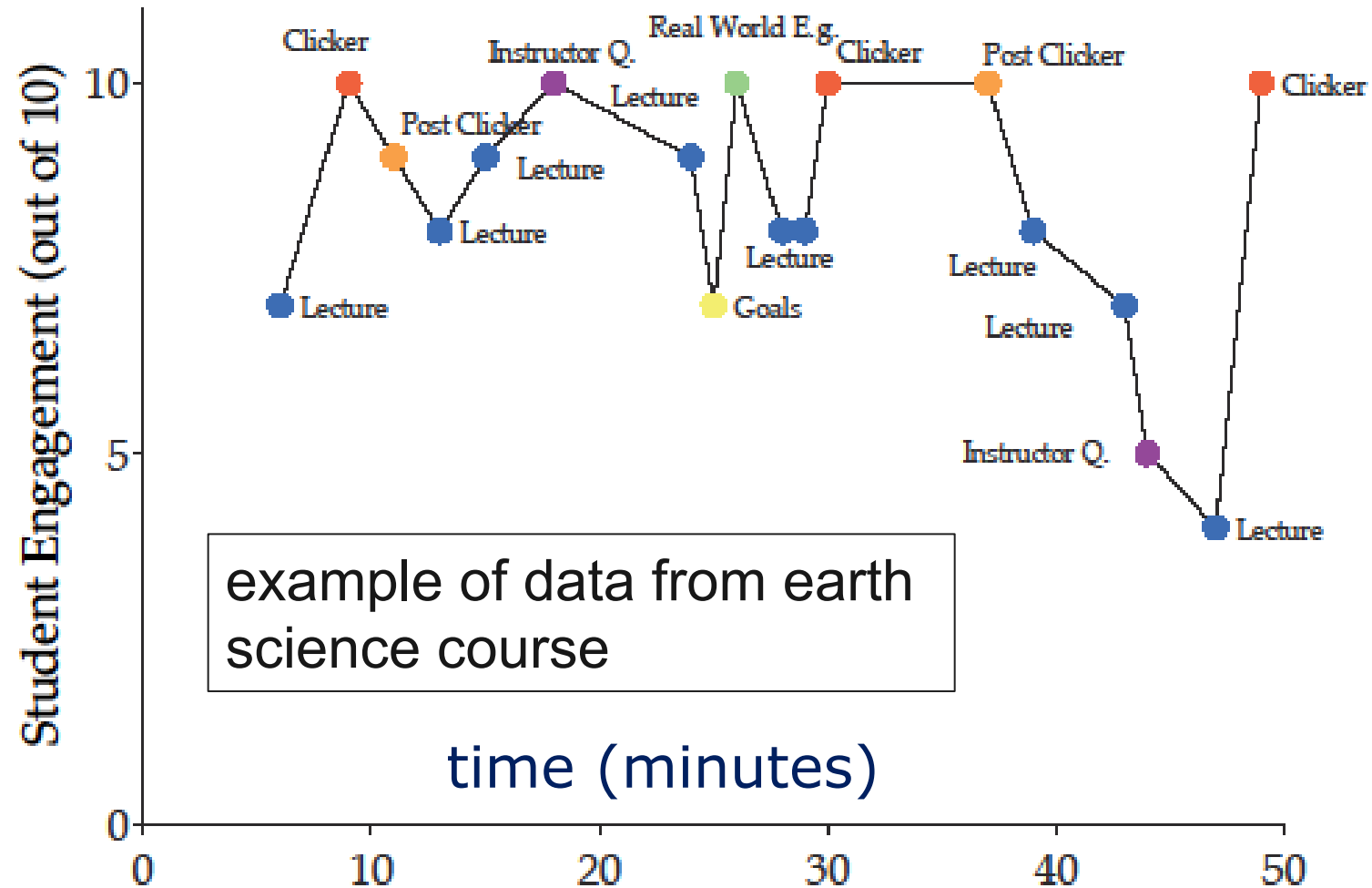
Immediately get detailed solutions to midterm 2

Then for each midterm question:

- Step 1: Follow the guide to identify type (and *flavor*) of error(s)**
- Step 2: Identify the core concept(s) of the midterm question AND irrelevant information(s)**
- Step 3: Using info in step (2), produce ‘near’ and ‘far’ transfer analogies**

Boosted Individual tasks are guided...so can't be stuck even if by yourself

Measuring student (dis)engagement. *Erin Lane*
Watch random sample group (10-15 students). Check against list of disengagement behaviors each 2 min.



6. Bringing up the bottom of the distribution

“What do I do with the weakest students? Are they just hopeless, or is there anything I can do to make a difference?”
*many papers showing things that **do not** work*

Demonstration of how to transform lowest performing students into medium and high.

Intervened with bottom 20-25% of students after midterm 1.

- a. very selective physics program 2nd yr course
- b. general interest intro climate science course

What does an intervention look like?

Email after M1-- concerned about performance. 1) Want to meet and discuss; or 2) 4 specific pieces of advice on studying.**[old]**

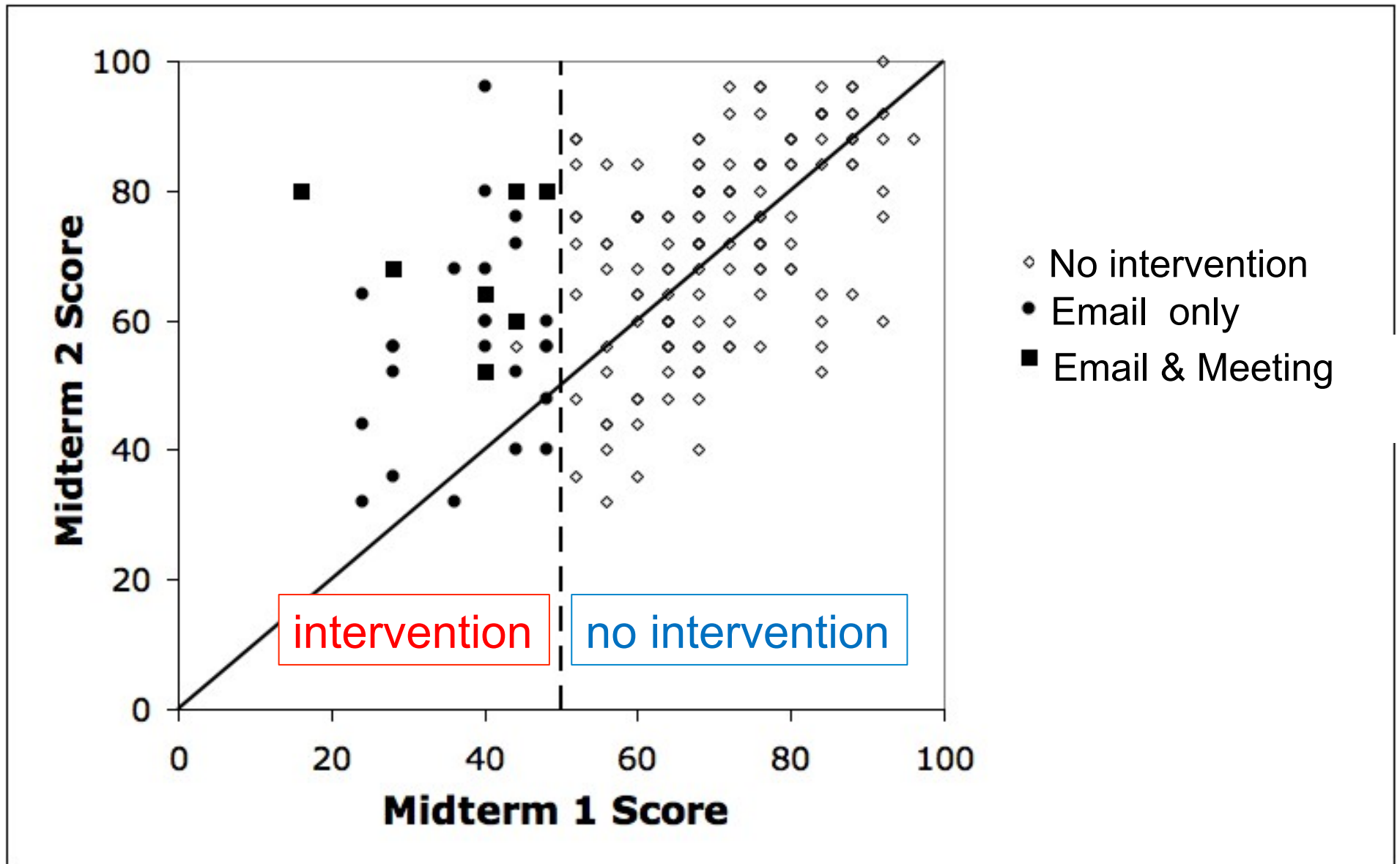
Meetings-- *How did you study?*

mostly just looked over stuff, tried to memorize book & notes

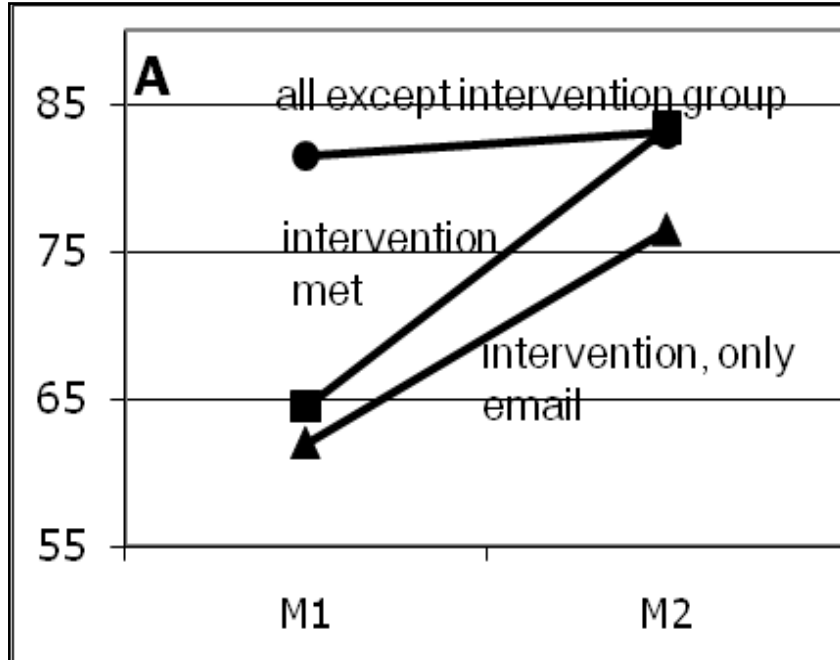
Given **small number** of **specific** things to do:

1. test yourself as review the homework problems and solutions.
2. test yourself as study the learning goals for the course given with the syllabus.
3. actively (explain to other) the assigned reading for the course.
4. Phys only. Go to weekly (optional) problem solving sessions.

Intro climate Science course (S. Harris and E. Lane)

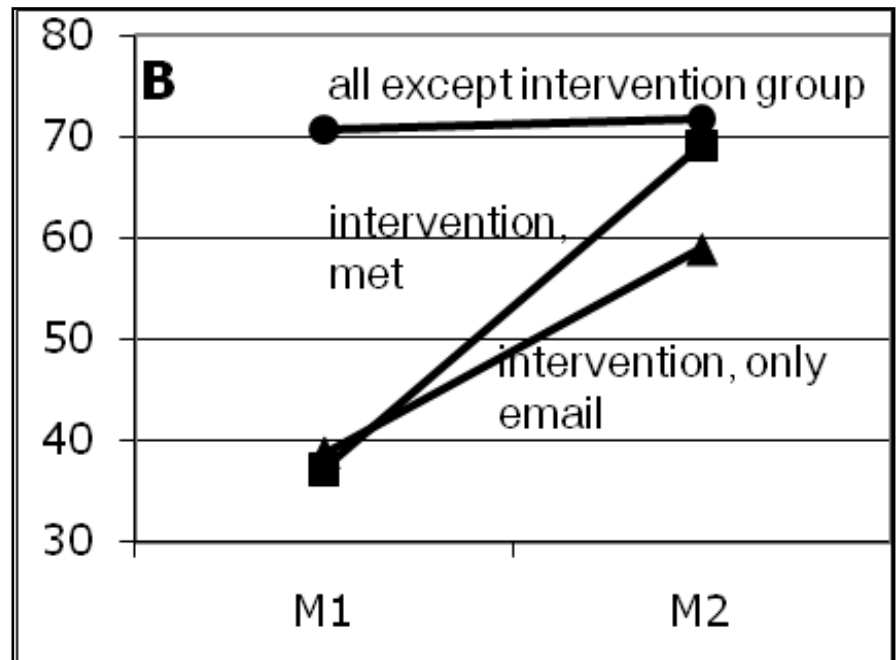


- End of 2nd yr Modern physics course (very selective and demanding, N=67)



bottom 25% averaged +19% improvement on midterm 2 !

- Intro climate science course. Very broad range of students. (N=185)



Averaged +30% improvement on midterm 2 !

Met -Reported less time studying ⇒ mean
Email only- more time ⇒ mean

Bunch of survey and interview analysis end of term.

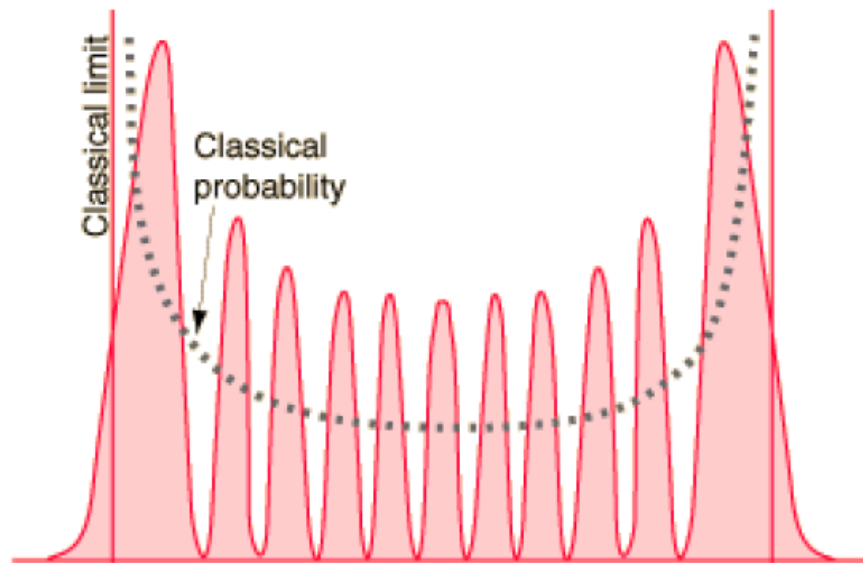
⇒ students changed how they studied

*(but did not think this would work in most courses,
⇒ doing well on exams more about figuring out instructor
than understanding the material)*

Instructor can make a dramatic difference in the performance of low performing students with small but appropriately targeted intervention to improve study habits.

Plotted below is the **probability distribution** corresponding to an eigenstate of the harmonic oscillator. Which eigenstate is this?

- A. $|\psi_1\rangle$
- B. $|\psi_{100}\rangle$
- C. $|\psi_5\rangle$
- D. $|\psi_{10}\rangle$
- E. $|\psi_{20}\rangle$



clicker

Bonus: ● if you computed the expectation value of position, would it be time dependent for this state?

◆ Why is the extension into the classically forbidden region less than for the ground state?

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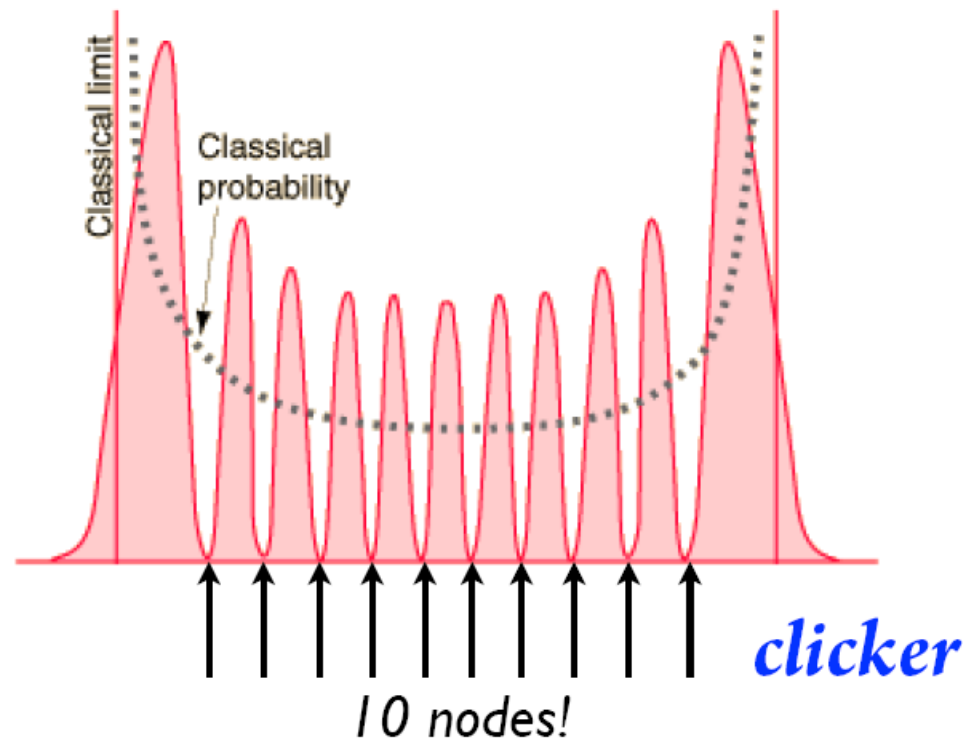
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B. $|\psi_{100}\rangle$

C. $|\psi_5\rangle$

D. $|\psi_{10}\rangle$

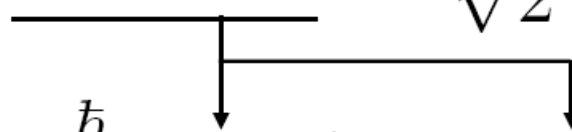
E. $|\psi_{20}\rangle$



Bonus: ● if you computed the expectation value of position, would it be time dependent for this state? **No, this is a stationary state!**

◆ Why is the extension into the classically forbidden region less than for the ground state? **The potential is steeper at higher energies.**

What about the expectation value of position squared?

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}} (|\psi_n\rangle + |\psi_{n+2}\rangle)$$
$$\langle \hat{x}^2 \rangle = \frac{\hbar}{2m\omega} \langle \psi | (\hat{a}^\dagger + \hat{a})^2 | \psi \rangle$$


What is the oscillation frequency of the expectation value of the square of the position position given this state?

A. It doesn't oscillate.

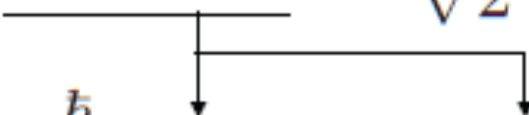
B. 2 x omega

C. omega

clicker

Bonus: ● what does this imply for the width of this state? σ_x

What about the expectation value of position squared?

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}} (|\psi_n\rangle + |\psi_{n+2}\rangle)$$
$$\langle \hat{x}^2 \rangle = \frac{\hbar}{2m\omega} \langle \psi | (\hat{a}^\dagger + \hat{a})^2 | \psi \rangle$$
A diagram consisting of a horizontal line with a vertical arrow pointing down from its center to the state $|\psi\rangle$ in the expectation value formula below. Two other vertical arrows point down from the ends of this horizontal line to the \hat{a}^\dagger and \hat{a} terms in the same formula.

What is the oscillation frequency of the expectation value of the square of the position position given this state?

A. It doesn't oscillate.

B. 2 x omega

C. omega

clicker

Bonus: ● what does this imply for the width of this state? σ_x

Suppose a particle in a harmonic oscillator is in the following superposition state of energy eigenfunctions at $t=0$. If we measure the energy, what is the probability of finding $E = \frac{9}{2}\hbar\omega$

$$|\Psi(t=0)\rangle = \frac{1}{4}(\sqrt{5}|n=1\rangle + \sqrt{6}|n=2\rangle - e^{i\phi}\sqrt{3}|n=4\rangle + e^{i\phi}\sqrt{2}|n=9\rangle)$$

- A. $\sqrt{5}/4$
- B. $\sqrt{2}$
- C. $2/16$
- D. $6/16$
- E. That is not an allowed energy.

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Bonus: ● if you computed the expectation value of position, would it be time dependent for this state?

◆◆◆ If so, what frequencies would you expect?

Given that $\hat{a} = \frac{1}{\sqrt{2\hbar m\omega}} (+i\hat{p} + m\omega\hat{x})$, what is \hat{a}^\dagger (\hat{a}^\dagger is the hermitian conjugate of \hat{a})?

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A. $\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar m\omega}} (-i\hat{p} + m\omega\hat{x})$

B. $\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar m\omega}} (-i\hat{p}^\dagger + m\omega\hat{x}^\dagger)$

C. $\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar m\omega}} (+i\hat{p}^\dagger + m\omega\hat{x})$

D. $\hat{a}^\dagger = \frac{1}{\sqrt{2\hbar m\omega}} (+i\hat{p} + m\omega\hat{x})$

E. More than one of the above are correct!

Bonus: ● Are x and p hermitian?

◆◆ How could you figure this out?

Consider a two state superposition of energy eigenstates

$$|\psi(t=0)\rangle = \frac{1}{\sqrt{2}} (|\psi_i\rangle + |\psi_j\rangle) \quad \mathbf{A}$$

compute

$$\langle \hat{A} \rangle = \langle \psi | \hat{A} | \psi \rangle$$

*individual papers
discuss in group*

1) rewrite this state at time t .

2) write down the expression for the expectation value grouping the terms into those that are time dependent and those that are not.

3) explain under what conditions is this expectation value not constant

Bonus: ● For the “apple” operator to be an observable, it must be Hermitian. Does this property simplify the expression in 2? Yes/No?

$$\hat{A}^\dagger = \hat{A}$$

Bonus: ♦ If yes, then write down the simplified expression.

Bonus: ♦♦ do the same problem for a superposition of 3 eigenstates

