Art of Insight in Science and Engineering

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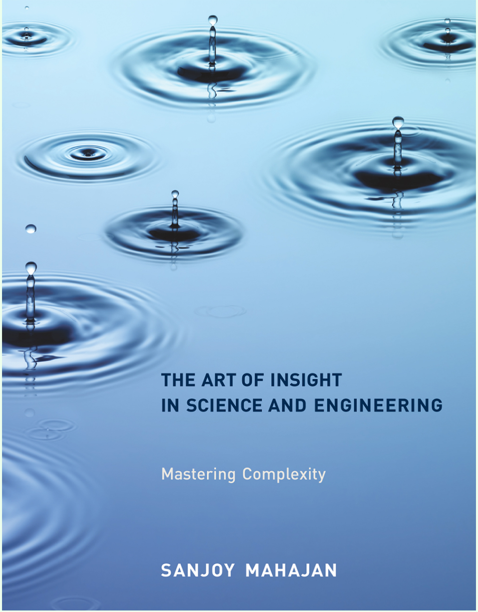
I hope to foster insight and contribute to the commons



Insight is hard to define but easy to recognize

You wonder whether your child is sick, and take her temperature. Raise your hand if the following temperature worries you:

Insight is hard to define but easy to recognize

You wonder whether your child is sick, and take her temperature. Raise your hand if the following temperature worries you:

40 °C

Insight is hard to define but easy to recognize

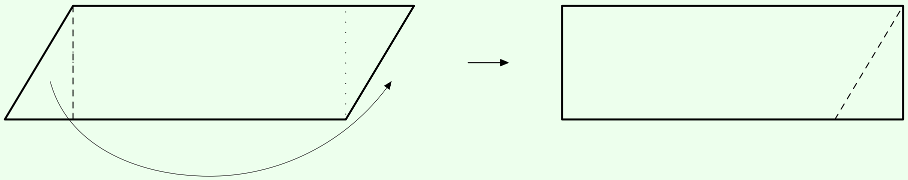
You wonder whether your child is sick, and take her temperature. Raise your hand if the following temperature worries you:

104 °F

Insight is hard to define but easy to recognize

$$\frac{243 + 243 + 243}{3} = ?$$

Insight is hard to define but easy to recognize



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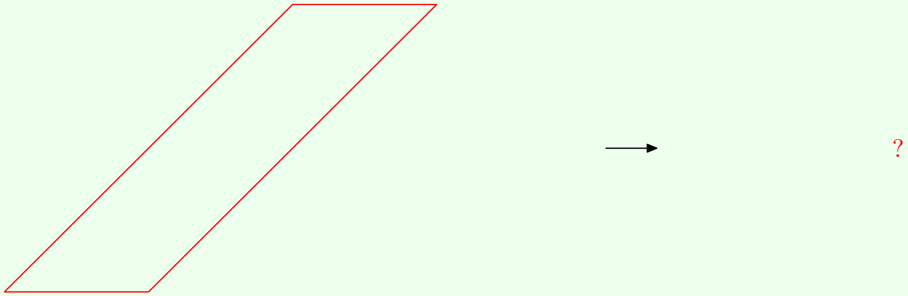
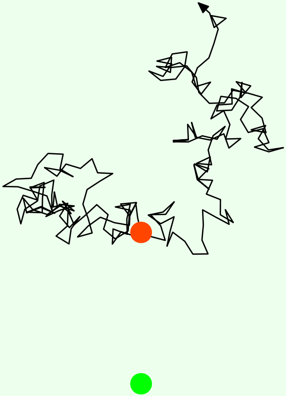


Without insight, problem solving turns into a random walk



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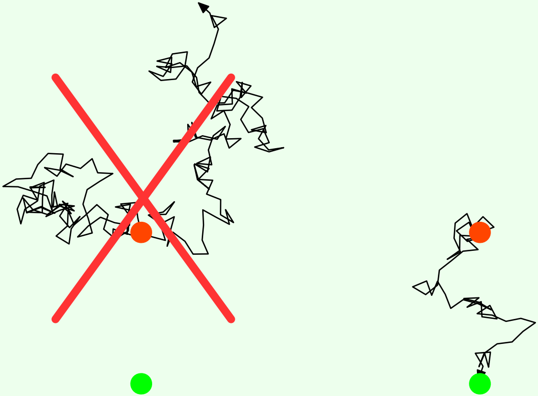
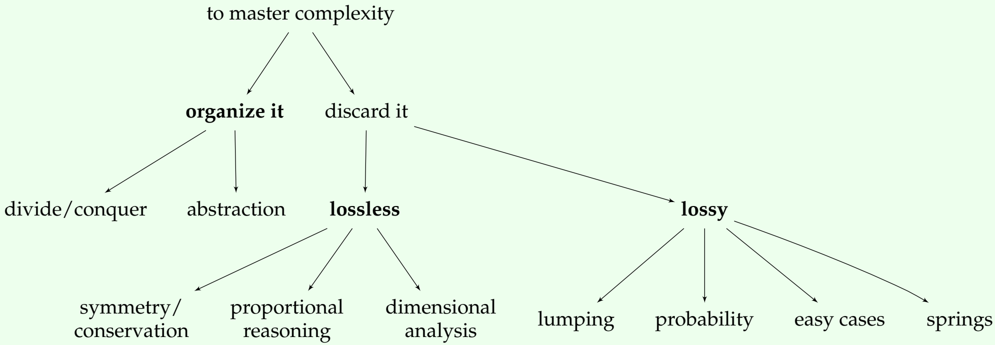
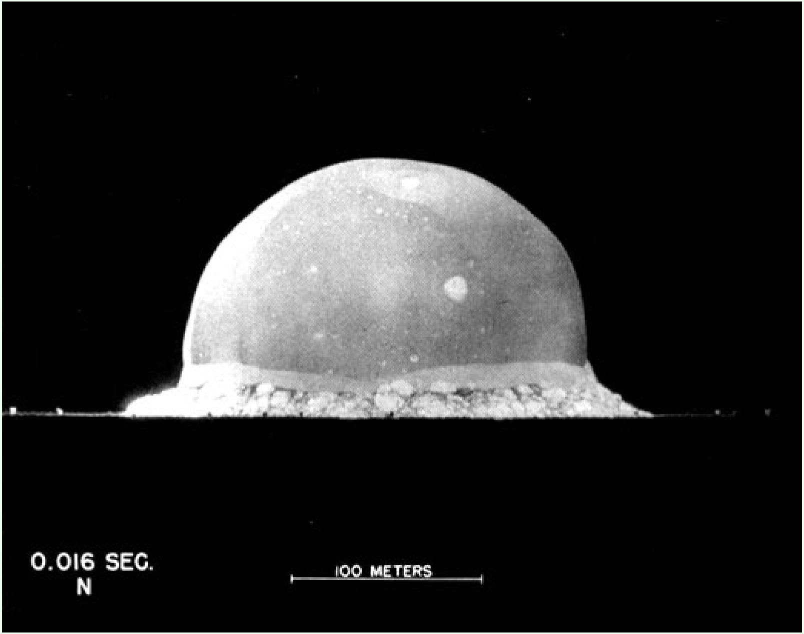


The book offers readers a toolchest to foster insight



Here is an insight-based approach to a famous problem

How much energy is released in this bomb blast?



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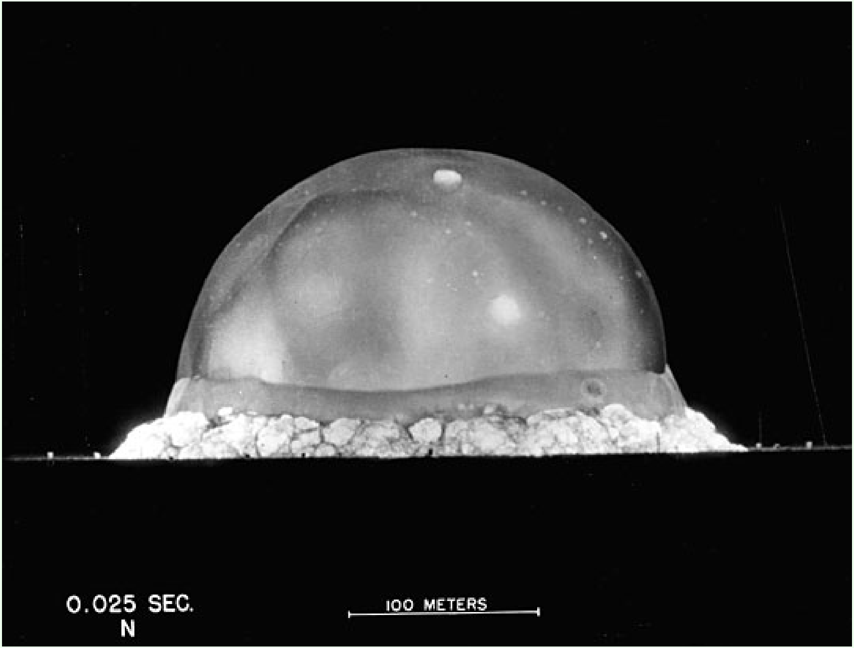

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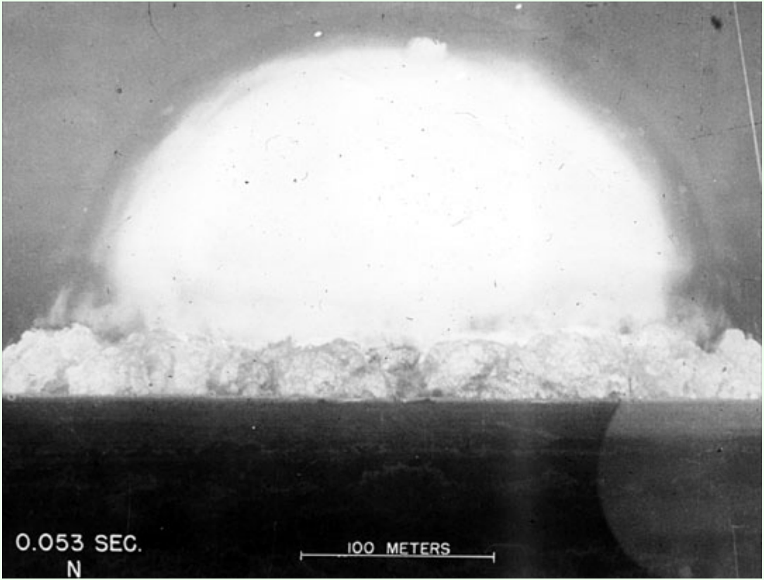


Here is a selection of the fireball data

t (ms)	R (m)
3.26	59.0
4.61	67.3
15.0	106.5
62.0	185.0

There is a famous, very complicated analysis

The equation of motion is $$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}=-\frac{p_0}{\rho}\frac{\partial y}{\partial r}. \tag{4}$$

Substituting from (1), (2) and (3) in (4) and writing f_1', ϕ_1' for $\frac{\partial}{\partial\eta}f_1$, $\frac{\partial}{\partial\eta}\phi_1$,

$$-(\tfrac{3}{2}\phi_1+\eta\phi_1')\,R^{-\frac{5}{2}}\frac{dR}{dt}+R^{-4}\left(\phi_1\phi_1'+\frac{p_0}{\rho_0}\frac{f_1'}{\psi}\right)=0. \tag{5}$$

This can be satisfied if $$\frac{dR}{dt}=AR^{-\frac{3}{2}}, \tag{6}$$

where A is a constant, and

$$-A(\tfrac{3}{2}\phi_1+\eta\phi_1')+\phi_1\phi_1'+\frac{p_0}{\rho_0}\frac{f_1'}{\psi}=0. \tag{7}$$

The equation of continuity is

$$\frac{\partial\rho}{\partial t}+u\frac{\partial\rho}{\partial r}+\rho\left(\frac{\partial u}{\partial r}+\frac{2u}{r}\right)=0. \tag{8}$$

Substituting from (1), (2), (3) and (6), (8) becomes

$$-A\eta\psi'+\psi'\phi_1+\psi\left(\phi_1'+\frac{2}{\eta}\phi_1\right)=0. \tag{9}$$

The equation of state for a perfect gas is

$$\left(\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}\right)(p\rho^{-\gamma})=0. \tag{10}$$

where γ is the ratio of specific heats.

Substituting from (1), (2), (3) and (6), (10) becomes

$$A(3f_1+\eta f_1')+\frac{\gamma f_1}{\psi}\psi'(-A\eta+\phi_1)-\phi_1 f_1'=0. \tag{11}$$

The equations (7), (9) and (11) may be reduced to a non-dimensional form by substituting

$$f=f_1a^2/A^2, \tag{12}$$

$$\phi=\phi_1/A, \tag{13}$$

where a is the velocity of sound in air so that $a^2=\gamma p_0/\rho_0$. The resulting equations which contain only one parameter, namely, γ, are

$$\phi'(\eta-\phi)=\frac{1}{\gamma}\frac{f'}{\psi}-\tfrac{3}{2}\phi, \tag{7a}$$

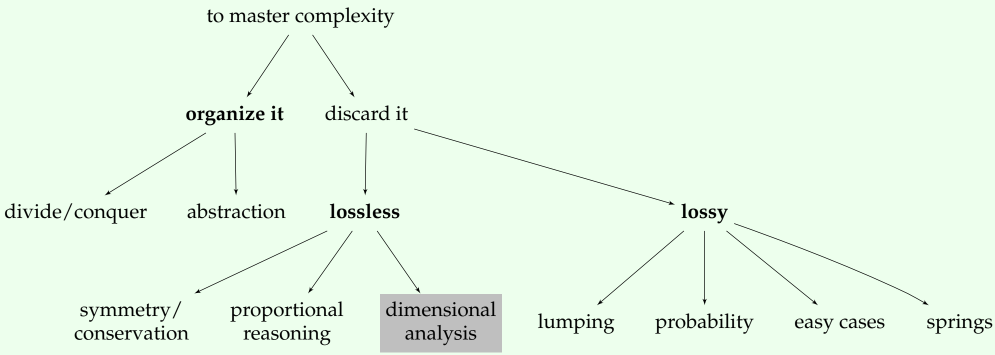
$$\frac{\psi'}{\psi}=\frac{\phi'+2\phi/\eta}{\eta-\phi}, \tag{9a}$$

$$3f+\eta f'+\frac{\gamma\psi'}{\psi}f(-\eta+\phi)-\phi f'=0. \tag{11a}$$

Eliminating ψ' from (11a) by means of (7a) and (9a) the equation for calculating f' when f, ϕ, ψ and η are given is

$$f'\{(\eta-\phi)^2-f/\psi\}=f\{-3\eta+\phi(3+\tfrac{1}{2}\gamma)-2\gamma\phi^2/\eta\}. \tag{14}$$

One route to insight is dimensional analysis



One route to insight is dimensional analysis

E	$\mathrm{ML}^2\mathrm{T}^{-2}$	blast energy
R	L	blast radius
t	T	time since blast
ρ_{air}	ML^{-3}	air density

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$\rightarrow \dfrac{E}{\rho_{\mathrm{air}}}$ has dimensions of $\mathrm{L^5T^{-2}}$.

$\rightarrow \dfrac{Et^2}{\rho_{\mathrm{air}}R^5}$ is dimensionless.

The dimensionless group makes a powerful prediction

$$\frac{Et^2}{\rho_{\text{air}} R^5} \sim 1$$

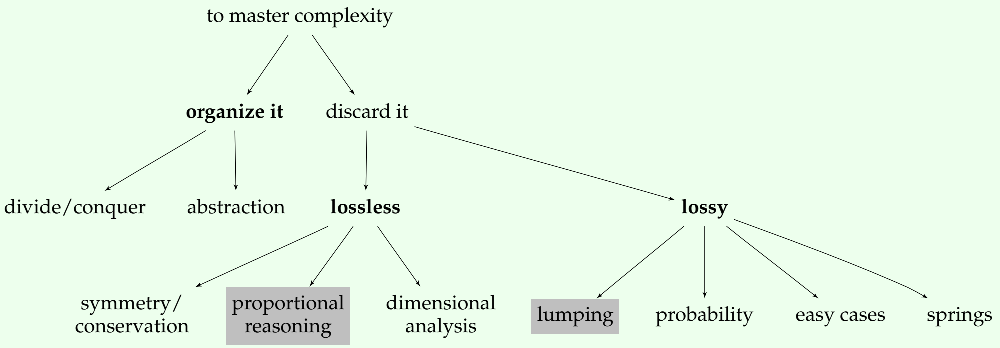
$$R \sim \left(\frac{E}{\rho_{\text{air}}} \right)^{1/5} t^{2/5}.$$

But the result still feels like magic

Dimensional analysis tells us what must be true, but not why.

We can get the “why” insight from a physical model

We can build the model using two of our tools



The model is based on the speed of the air molecules

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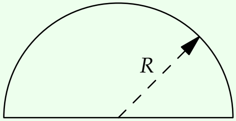
energy $\sim$ mass $\times$ speed^2.

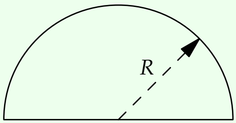
$$\rightarrow \text{speed} \sim \sqrt{\frac{\text{energy}}{\text{mass}}} \sim \sqrt{\frac{E}{\rho_{\text{air}} R^3}}.$$



The speed leads us to the fireball size

energy $\sim$ mass $\times$ speed^2.

$$\rightarrow \text{speed} \sim \sqrt{\frac{\text{energy}}{\text{mass}}} \sim \sqrt{\frac{E}{\rho_{\text{air}} R^3}}.$$



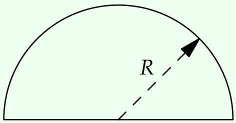
radius $R \sim$ speed $\times$ time t.

$$\text{radius } R \sim \sqrt{\frac{E}{\rho_{\text{air}} R^3}} \times t.$$

The two ways to represent the size connect the size and time to the blast energy

energy $\sim$ mass $\times$ speed^2.

$$\rightarrow \text{speed} \sim \sqrt{\frac{\text{energy}}{\text{mass}}} \sim \sqrt{\frac{E}{\rho_{\text{air}} R^3}}.$$



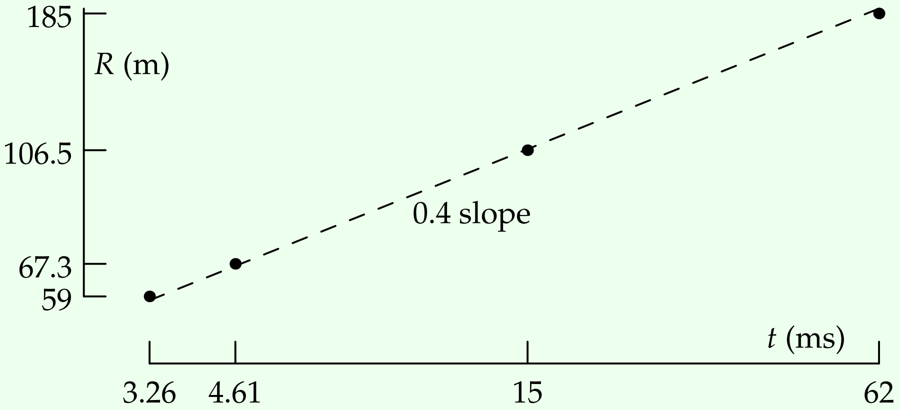
radius $R \sim$ speed $\times$ time t.

$$\text{radius } R \sim \sqrt{\frac{E}{\rho_{\text{air}} R^3}} \times t.$$

$$\rightarrow \frac{Et^2}{\rho_{\text{air}} R^5} \sim 1.$$

The scaling prediction fits the data on the fireball size

$$R \sim \left(\frac{E}{\rho_{\text{air}}}\right)^{1/5} t^{2/5}.$$



The scaling prediction gives an estimate for the blast energy

$E \sim 7 \times 10^{13}$ joules $\rightarrow E \sim 18$ kilotons of TNT.

The estimate is more accurate than we can expect

The classified value for the blast energy was 20 kilotons.

Insight is more important than accuracy

For almost 20 years, I wanted to publish under a free license

This book draws from the commons in software

compiling text to PDF	ConTeXt, LuaTeX, TexGyre Pagella
compiling figures to PDF	Asymptote, MetaPost, Python
editing source files	GNU Emacs
managing source files	Mercurial
managing compilations	GNU Make
underlying operating system	GNU/Linux (Debian)

Just this part of the commons is huge

Roughly 20 million lines of code.

A commons has three characteristics

1. resource that is easy to draw from but hard to exclude others from
2. people who want long-term access to the resource ("commoners")
3. rules for managing the resource

(George Caffentzis, "Russell Scholar Lecture IV," 2008)

For much of the software commons, the rules are the GNU General Public License (GPL)

For this book, the rules are the Creative Commons license

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The commons, a part of our infrastructure, is essential to public welfare

Charter of the Forest (September 11, 1217): protection of rights to the commons

⋮

Simon Patten (1852–1922): importance of reducing economic rent (difference between price and necessary cost of production)

⋮

free software, OpenCourseWare, MOOCs, ...

In 1815, Jefferson set us a riddle

[My] peculiar character, too, is that no one possesses [me] the less, because every other possesses the whole of [me].

Who am I?

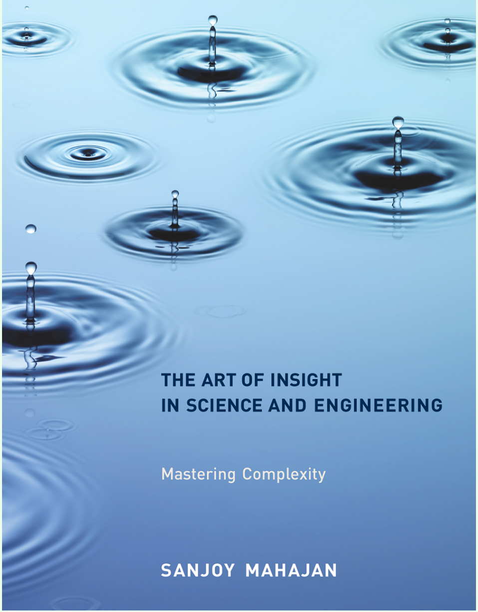
Solution to the riddle: I am an idea

Its peculiar character, too, is that no one possesses the less, because every other possesses the whole of it.

He who receives **an idea** from me, receives instruction himself without lessening mine; as he who lights his taper at mine, receives light without darkening me.

That ideas should freely spread from one to another over the globe, for the moral and mutual instruction of man, and improvement of his condition, seems to have been peculiarly and benevolently designed by nature[.]

I hope to have fostered insight and contributed to the commons



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Slides produced using free software:

GNU Emacs, GNU Make, LuaTEX, and ConTEXt
(on Debian GNU/Linux)