Doping $\alpha$-Al$_2$O$_3$ to reduce its hydrogen permeability: Thermodynamic assessment of hydrogen defects and solubility from first principles

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Abstract

This paper assesses the role of doping on the hydrogen permeability and electronic properties of $\alpha$-Al$_2$O$_3$. Formation energies of intrinsic and extrinsic defects in $\alpha$-Al$_2$O$_3$ were calculated using density functional theory. Using these energies as input, a thermodynamic model was utilized to identify the equilibrium defect concentrations (barring hydrogen defects) in undoped and doped $\alpha$-Al$_2$O$_3$ under aluminization conditions of 1100 K, over a range of $p_{O_2}$ at a fixed doping level of 1 ppm. Defect concentrations calculated at 1100 K under $p_{O_2}$-rich conditions were used as input to establish hydrogen and electronic defect concentrations under functional conditions of 300 K, over a range of $p_{O_2}$. The effect of dopants on the fraction of free hydrogen interstitials, which has implications on diffusivity, and the overall hydrogen solubility, was found to be substantial and distinct. Relative to the undoped case, Mg-doping increased the concentration of free hydrogen interstitials, the primary diffusing species, by 10$^7$ times, whereas Ti-, Si-, Fe-, Cr-doping eliminated it to negligible amounts. Comparing the impact on total hydrogen solubility, Mg-doping increased it by 10$^4$ times; Fe- and Cr-doping increased it negligibly by ~1.5 times. In contrast, Ti- and Si-doping decreased it to nearly 1/3 that of the undoped case. Analyzing the role of isolated defect concentrations and binding energies of defect complexes helps elucidate these effects. Effect of dopant concentrations of 10 and 100 ppm was also investigated, with the conclusion that doping with Si and Ti at 1 ppm is the best strategy to reduce hydrogen diffusivity and solubility by the greatest amount. The findings aid in the design of effective hydrogen permeation barrier layers for use in hydrogen storage and transport infrastructure as well as in the understanding of defect states in Al$_2$O$_3$ used in electronic devices, such as resistive switching.

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1. Introduction

The past few years have witnessed a growing interest in the study of hydrogen in oxides. One area of key focus has been on understanding the factors influencing hydrogen permeability in barrier oxides like Al$_2$O$_3$ [1–5]. Hydrogen permeation barriers (HPBs) are needed for high temperature applications such as that in nuclear reactors to prevent tritium escape into the environment [6], as well as for room temperature applications such as preventing embrittlement of steels used in hydrogen storage and transport infrastructure [7]. This will play a key role in enabling the hydrogen economy.

From the various material classes considered for use as HPBs on steels [8], it appears that $\alpha$-Al$_2$O$_3$ performs as one of the best [6,9]. Its experimentally determined permeation reduction factor is the highest among candidate HPB materials [9]. The activation energy for hydrogen diffusion in the steel lattice is only 0.08 eV [10], whereas for diffusion in sintered alumina it is 1.89 eV [11]. This results in hydrogen diffusivity of alumina being more than 10$^{20}$ times lower than that of steel. Moreover, the low solubility of hydrogen in $\alpha$-Al$_2$O$_3$ [9], its good adherence [12], and its high chemical stability under reducing conditions [13], make it an ideal choice for use as a HPB.

Computational studies have helped better understand the behavior of hydrogen in $\alpha$-Al$_2$O$_3$. Belonoshko et al. [14] computed the diffusivity of neutral H in bulk $\alpha$-Al$_2$O$_3$ to be $\sim 10^{-29}$ m$^2$/s.
2. Methodology

2.1. DFT calculations

The energetics of the $2 \times 2 \times 1$ perfect supercell (a = 9.62 Å, c = 13.13 Å) and of all the defects were calculated using density functional theory (DFT) using a plane-wave basis set, projector-augmented wave pseudopotentials [27] and the Perdew-Burke-Ernzerhof (PBE) parameterization of the generalized gradient approximation (GGA) [28] as the exchange-correlation functional, as implemented in the Vienna ab initio Simulation Package (VASP) [29] v.5.4.1. A kinetic energy cutoff of 520 eV and a gamma-centered $2 \times 2 \times 2$ k-point mesh were used, resulting in a convergence accuracy of <1 meV/atom. All calculations were performed with a Gaussian smearing width of 0.05 eV and spin-polarized setting. Atomic positions were relaxed until the force on each atom was below 0.05 eV/Å. The first order Makov-Payne (MP1) correction [30] was adopted to account for the finite-size errors arising from periodic boundary conditions as imposed by DFT. We used the dielectric constant of $\alpha$-Al$_2$O$_3$ as 9 [31]. Comparison of the results of these settings with experiments and prior GGA calculations, and relaxed structures of the native defects (vacancies and interstitials of Al and O) can be found in Table S1 and Fig. S1 respectively of the Supplementary Information (SI). Details of the hydrogen defects studied (interstitials, complexes with Al-vacancies, O-vacancies and dopants) are elaborated upon in Section 3.3. The substitutional dopants considered were Mg, Si, Ti, Fe and Cr. These are common acceptor and donor substitutional impurities in $\alpha$-Al$_2$O$_3$ [20]. Systematically studying this range of dopants helps compare and contrast the effect on electrical properties and hydrogen solubility of $\alpha$-Al$_2$O$_3$ doped with a prototypical acceptor ($M_{\text{GaN}}$), a prototypical donor ($Si_{\text{Al}}$), and dopants that show multiple charge states ($Ti_{\text{Al}}^\text{+}$, $Fe_{\text{Al}}^{3+}$, $Cr_{\text{Al}}^{4+}$), thus covering a broad spectrum of possibilities.

2.2. Thermodynamic model to evaluate defect concentrations in native and doped $\alpha$-Al$_2$O$_3$ at 1100 K in oxygen atmosphere

The concentration $N_i$ (in units of per f.u. of $\alpha$-Al$_2$O$_3$) of defect $i$ is given by the equation [32].

$$N_i = \frac{N_i^{\text{conf}} N_i^{\text{sites}}}{N_i^{\text{conf}} + \exp \left( \frac{E_i^{\text{tot}} - E_i^{\text{conf}}}{kT} \right)} = \frac{N_i^{\text{conf}} N_i^{\text{sites}} \exp \left( -\frac{E_i^{\text{tot}}}{kT} \right)}{1 + \exp \left( -\frac{E_i^{\text{tot}}}{kT} \right)}$$

(1)

where $E_i^{\text{tot}} = E_i^{\text{def}} + E_i^{\text{conf}} - \sum_j n_j \mu_j + q_i \mu_e$, the formation energy of defect $i$ with charge $q_i$. The notations are as follows: $N_i^{\text{sites}}$ = total number of sites over which $N_i$ defects of type $i$ can be distributed; $N_i^{\text{conf}}$ = number of equivalent configurations in which the defect $i$ can be incorporated on a single site; $E_i^{\text{tot}}$ = DFT total energies of the defect and perfect supercells; $\Delta E_i = \sum_j n_j \mu_j + q_i \mu_e$, the chemical potential of $j(Al, O)$ atoms in the defect and perfect supercell; $\mu_e$ chemical potential of $j(Al, O)$ in the system; $\mu_e = \mu_e^\text{valence} + \mu_f$. Where $\mu_e^\text{valence}$ is the valence band maximum of the perfect supercell and $\mu_f$ is the Fermi level relative to the valence band maximum; $k = \text{Boltzmann constant}; T = \text{temperature}$.

The chemical potential of oxygen $\mu_O$ is given as
\[
\mu_O(T, P_{O_2}) = \frac{1}{2} \left[ E^{\text{DFT}}_{O_2} + E_{\text{over}} + \mu_O^G(T, P^0) + kT \ln \left( \frac{P_{O_2}}{P^0} \right) \right]
\]

where \( E^{\text{DFT}}_{O_2} \) = DFT energy of the \( O_2 \) molecule; \( E_{\text{over}} \) = correction for the \( O_2 \) overbinding error caused by GGA, taken as 1.36 eV as identified by Wang et al. [33]; \( \mu_O^G(T, P^0) \) is the difference in chemical potential of \( O_2 \) gas between \( T = 0 \)K and the temperature of interest, at a reference pressure of \( P^0 = 1 \text{ atm} \), obtained from thermochemical tables [34]; \( P_{O_2} \) is the partial pressure of oxygen gas.

The chemical potential of Al can be directly obtained from \( \mu_O \), and is given as

\[
\mu_{Al}(T, P_{O_2}) = \frac{1}{2} \left[ E^{\text{DFT}}_{Al_{2}O_3} - 3\mu_O(T, P_{O_2}) \right]
\]

where \( E^{\text{DFT}}_{Al_{2}O_3} \) is the DFT energy of one formula unit of the perfect crystal of \( \alpha-\text{Al}_2\text{O}_3 \).

Equation (1) does not include the chemical potential of the dopant, \( \mu_M \), as we fix the total concentration of each dopant considered at 1 ppm. Thus, the system is no longer thermodynamically open to the dopant \( M \), therefore \( \mu_M \) does not appear in the equation. This reflects the experimental conditions of addition of dopants in fixed concentrations during the formation of \( \alpha-\text{Al}_2\text{O}_3 \) film. 1 ppm is simply a representative doping concentration, chosen to be within the solubility limit of the various dopants in \( \alpha-\text{Al}_2\text{O}_3 \) and to ensure that the dilute defect approximation is still valid.

The derivation of Equation (1) and approximations used can be found in SI Section S3. The comparison of our DFT calculations of \( O_2 \) and \( H_2 \) molecules with experiments and previous theoretical work can be found in SI Table S2.

2.3. Thermodynamic model to evaluate defect concentrations in native and doped \( \alpha-\text{Al}_2\text{O}_3 \) at 300 K in hydrogen atmosphere

The concentrations of various hydrogen defects considered at 300 K is given by

\[
N_{H^\#} = N^\text{conf}_{\text{sites}} N^\text{conf}_{\text{con}} \exp \left( -\frac{E^H}{kT} \right)
\]

\[
N_{X-nH^\#} = \left( \frac{N^\text{conf}_{\text{sites}}}{N^\text{conf}_{\text{con}}} \right)^n \left( \frac{N^\text{conf}_{\text{sites}}}{N^\text{conf}_{\text{con}}} \right)^n \exp \left( \frac{E^H}{kT} \right)
\]

where \( E^H \) signifies hydrogen interstitials and \( [X-nH]^\# \) signifies hydrogen complexes considered, with \( X = \text{V}_{Al} \) or \( \text{V}_{O} \) or \( M_{Al} \) (aluminum vacancy, oxygen vacancy, substitutional dopant on Al site, respectively) and \( n \) = number of hydrogen interstitials forming the complex, \( q \) is the charge state of the defect. \( E^H \) is the binding energy of the complex, given as \( E^H(AB) = E^H_f + E^H_i - E^{AB} \). Only those complexes with positive binding energy and which form realizable charges \( (q_1, q_2) \) upon dissociation are considered (see SI Table S3 for list).

These equations result from minimization of the constrained grand canonical potential (derivation in SI Section S5), in which we consider the system to be open to hydrogen and electrons, and closed to aluminum and oxygen. The experimentally determined activation energies for O, Al, and H diffusion in polycrystalline \( \alpha-\text{Al}_2\text{O}_3 \) are ~6 eV [35], ~4 eV [35], and ~1.89 eV [11] respectively. This wide difference in activation energies results in H being comparatively more mobile than Al and O. Thus, H can diffuse through the system and equilibrate with the environment even at low temperatures of 300 K, whereas Al and O (and other ionic defects in general), due to their larger sizes, are immobile at such low temperatures and do not equilibrate with the environment. Native and dopant defect concentrations are thus fixed to the values obtained for each charge state at 1100 K, O-rich conditions.

The chemical potential of hydrogen is computed as

\[
\mu_H(T, P_{H_2}) = \frac{1}{2} \left[ E^{\text{DFT}}_{H_2} + \mu_H^G(T, P^0) + kT \ln \left( \frac{P_{H_2}}{P^0} \right) \right],
\]

where \( E^{\text{DFT}}_{H_2} \) = DFT energy of a \( H_2 \) molecule; \( \mu_H^G(T, P^0) \) is calculated from thermochemical tables, analogous to \( \mu_O^G(T, P^0) \); \( P_{H_2} \) is the partial pressure of hydrogen gas.

The above equations are solved with the constraint

\[
\sum_i q_i N_i + p_e - n_c = 0
\]

(7)

here, \( N_i \) = concentration of point defect \( i \) of charge \( q_i \), \( p_e \) = concentration of holes in the valence band and \( n_c \) = concentration of electrons in the conduction band, which are calculated by integrating the electronic density of states \( g(E) \)

\[
p_e = \int_{-\infty}^{E_{\text{CBM}}} g_e(E) \frac{dE}{1 + \exp\left( \frac{E - E_{\text{Fermi}}}{kT} \right)} \tag{8a}
\]

\[
n_c = \int_{E_{\text{CBM}}}^{\infty} g_e(E) \frac{dE}{1 + \exp\left( \frac{E - E_{\text{Fermi}}}{kT} \right)} \tag{8b}
\]

\( E_{\text{CBM}} \) is the energy of the conduction band minimum.

Each of the terms in Equation (7) is a function of the Fermi level \( E_{\text{Fermi}} \). Solving for this \( E_{\text{Fermi}} \) that achieves charge neutrality at each \( P_{O_2} \) and \( P_{H_2} \) using a standard root-finding algorithm, we can then obtain the concentration \( N_i \) of point defect \( i \) and the concentration of valence band holes \( p_e \) and conduction band electrons \( n_c \) at a particular temperature over a range of \( P_{O_2} \) and \( P_{H_2} \).

3. Results and discussion

3.1. Defect equilibria in undoped \( \alpha-\text{Al}_2\text{O}_3 \) at 1100 K in oxygen atmosphere

Fig. 1 presents the calculated defect equilibria of undoped \( \alpha-\text{Al}_2\text{O}_3 \) at 1100 K. The formation energies used as input to generate such Kröger-Vink diagrams and their implications are discussed in SI Fig. S2.

GGA-PBE [28] is known to underestimate the band gap of semiconductors and insulators, as it does for \( \alpha-\text{Al}_2\text{O}_3 \) as well (SI Table S1). In many cases, it also fails to predict charge localization originating from narrow bands (as in transition metal oxides) or due to local lattice distortions around defects. This can affect the computed formation energies and defect transition levels, which can affect the trends in calculated defect concentrations. Therefore,
it is important to compare the GGA-PBE computed formation energies and defect levels with more accurate (but more computationally expensive) functionals like the Heyd-Scuseria-Ernzerhof (HSE) [36] hybrid functional. Comparison with calculations by Choi et al. [37], who used HSE, show that GGA-PBE here successfully captures the trends in formation energies and defect levels in α-Al2O3. This fortuitous correspondence is likely because α-Al2O3 does not exhibit any charge localization arising from narrow bands or lattice distortions. Thus, from a semi-quantitative standpoint, the trends in predicted defect concentrations will remain the same, with no significant impact on the results of this study. This comparison and discussion is further elaborated upon in SI Section S8.

At high pO2, p-type conductivity is exhibited, where holes p are compensated by Va. At strongly reducing atmospheres, V0 is the predominant defect, and n-type conductivity is exhibited, where electrons n are compensated by V0. In the high (low) pO2 cases, compensation of Va (V0) does not occur via Ionic defects like V0 (Va) to form a Schottky quintet or Al+ (O2-) to form a Frenkel pair, as such defects have prohibitively high formation energies (>4 eV, see SI Table S5). Compensation by partially charged Va+, Al+ (Va, Va0, O2-) would similarly require high formation energies. The equilibrium Fermi level established in both the cases is such that compensation by electronic defects is energetically more favorable. Therefore, in order to enhance the electrical conductivity of undoped α-Al2O3, one could either process the sample under po2-rich conditions to increase the concentration of Va and holes, or alternatively, process the sample under po2-poor conditions to increase the concentration of V0 and electrons.

Our calculated plot captures the experimental defect compensation mechanism at high po2 successfully, with the computed slope of -0.19 in good accordance with the experimental po2 dependence [38]. At intermediate po2, po2-independent extrinsic ionic conductivity has been observed experimentally [38], given the high levels of impurities found in nominally ‘pure’ single crystal α-Al2O3 [39], and the high energies of formation of intrinsic Schottky/Frenkel defects mentioned in the previous paragraph. We therefore see no such flat region in our calculated plot, as our calculations consider only pure α-Al2O3. At low po2, although our computed plot captures the experimentally observed n-type conductivity [38], the computed slope is 0.25, in contrast to the range experimentally observed -0.15–0.2 [38]. We attribute the difference to the presence of V0 and V0 defects (known as F and F- color centers respectively). The samples used in experiments were either heated to very high temperatures (>1700 °C) to obtain steady state conductivity measurements or were hot-pressed [38]. This heating and holding at high temperature (for example, for 5 h at 1800 °C [39]) is done to remove the F and F- color centers in α-Al2O3, as exemplified by Levy [39]. Thus, in the experiments conducted, the defect species at low po2 are most likely electrons compensated by V0. In our computed Kroger-Vink diagram, at low po2, we see that the predominant charged defect species are V0 compensated by electrons. We see V0 to be the predominant defect overall. These electron-vacancy associated defects and cannot be ‘annealed’ out and dissociated at the temperature (1100K) in these calculations.

Another difference between the experimental conductivity plots and our calculated Kroger-Vink diagram is the po2 at which transition between p-type to n-type conductivity occurs. The calculated transition po2 is ~10−12 atm, as compared to 10−12 atm in experiments at 1500 K [38]. This difference can be attributed to the higher temperatures implemented in conductivity experiments and to the impurities present in the experimental samples, which alter the transition po2. These effects can be seen in SI Fig. S4.

Despite the good agreement, comparison of the computed slopes and defect compensation mechanisms with experimental studies at high po2 and the logical justification for the discrepancy at low po2, are encouraging and ratify the robustness of our model. Moreover, since we are concerned with mainly the high po2 results for establishing hydrogen defect concentrations, the validation of our computed trends with that of experiments in this regime gives us confidence in the results presented in the following sections.

3.2. Defect equilibria in doped α-Al2O3 at 1100 K in oxygen atmosphere

Fig. 2 shows the defect equilibria established in α-Al2O3 doped with Mg, Si, Ti, Fe and Cr at 1100 K. The total dopant concentrations are fixed at 1 ppm.

Fig. 2a shows the defect equilibria established in Mg-doped α-Al2O3. Mg exhibits predominantly a 2+ charge state on the Al000 site (MgA) and thus is an acceptor dopant. At high po2, MgA is compensated by holes. The hole concentration is entirely dictated by the concentration of MgA and is nearly 104 times higher than in the undoped case. Thus, the electronic conductivity of α-Al2O3 can be increased at high po2 by doping with Mg. In contrast, the concentration of VAI is lowered by nearly 104 times. An increase in other positively charged defects, namely V0 and Ali+ is observed at high po2, which was not seen in the undoped case. As the po2 is reduced, compensation of MgA is predominantly first by V0 and Al0+ and then at extreme reducing regimes, compensation is by V0. V0 also exhibits significant concentrations as po2 is reduced. The concentration of holes is found to decrease with decrease in po2. Conductivity experiments on 10 ppm Mg-doped single crystal α-Al2O3 [40] revealed that the electronic conductivity decreased monotonically, while the ionic conductivity increased and approached a constant value with decrease in po2. The electronic conductivity was attributed to holes, but the study was unable to determine whether to attribute ionic conductivity to V0 or Al0+.

The same trend is seen in our calculated Kroger-Vink plot, with the concentration of holes decreasing and the concentration of V0 and Al0+ increasing and approaching a (near-equal) constant value with decreasing po2. Unlike our calculated plot, a slight increase in ionic conductivity due to V0 is not observed experimentally, possibly due to use of hot-pressed samples/high experimental temperatures which anneal out such electron-vacancy associated defects, as explained in Section 3.1.
At the other extreme, Fig. 2b shows the defect equilibria established in Si-doped α-Al₂O₃. Si exhibits predominantly a 4+ charge state on the Al³⁺ site ([Si⁴⁺ₐ], and thus acts as a donor. Unlike Mgₐ⁺, which is compensated by an electronic defect, Siₐ⁺ is compensated almost entirely by the ionic defect Vₐ⁻ (and not electrons) across the pO₂ range studied. The Fermi level established in Si-doped α-Al₂O₃ across the pO₂ range is too far from the computed conduction band minimum to result in compensation by electrons (see SIFig. S5 for calculated equilibrium Fermi levels at high and low pO₂ for all doped cases). Thus, the concentration of Vₐ⁻ can be entirely tuned by controlling the donor dopant concentration, and thus, it is increased by over 10³ times upon 1 ppm Si doping. Density measurements on Si-doped single-crystal α-Al₂O₃ also suggest compensation of Si⁴⁺ by Vₐ⁺ [41].

In Ti-doped α-Al₂O₃, we see that at high pO₂, Ti⁴⁺ ([Tiₐ⁺]) is the predominant charge state and it is compensated by Vₐ⁻. Similar to the Si-doped case, an increase in the concentration of Vₐ⁻ as compared to the undoped case is seen, by over 10³ times. Concentration of holes is 10 times lower than the undoped case at high pO₂. Thus, ionic conductivity of α-Al₂O₃ can be increased by addition of Ti under high pO₂ conditions. As pO₂ is reduced, Ti³⁺ becomes the predominant charge state ([Tiₐ⁻]), and the concentrations of Vₐ⁻ defects reduce as well, decreasing ionic conductivity. A steady increase in the concentration of Tiₐ⁻ and electrons is seen as well, along with Vₐ⁺ and Vₐ⁻. Experimental studies on Ti-doped single crystal α-Al₂O₃ [41,42] have also observed that Vₐ⁻ primarily compensates for Tiₐ⁺ at high pO₂. Optical absorption measurements [42] with varying pO₂ revealed that at high pO₂, Tiₐ⁻ is the predominant charge state, and as pO₂ decreases, concentration of

![Fig. 2. Defect equilibria in (a) Mg-doped (b) Si-doped (c) Ti-doped (d) Fe-doped (e) Cr-doped α-Al₂O₃ at 1100 K with dopant concentration fixed at 1 ppm. Only defects with concentration [D]>10⁻¹⁶ per f.u. are shown. Point defects are represented by Kröger-Vink notation, n_e denotes conduction band electrons and p_h denotes valence band holes.](image-url)
Ti\text{Al} decreases while that of T\text{Al}\text{H}\text{Al} increases. Electrical conductivity studies in the same paper revealed that as p\text{O2} decreases, ionic conductivity (attributed to V\text{Al}\text{H}\text{Al}) decreases, electronic conductivity increases (attributed to electrons). The same trend is captured by our calculations.

In Fe-doped \(\alpha\)-Al\text{2}O\text{3}, as seen in Fig. 2d, Fe\(^{3+}\) (Fe\(^{4+}\)) is the predominant charge state across the p\text{O2} range studied. At high p\text{O2}, the concentration of Fe\(^{4+}\) is only \(-10^{-9}\) per f.u., 10⁶ times lower than the total concentration of Fe in the lattice (1 ppm). The concentration of the compensating V\text{Al}\text{H}\text{Al} is \(4 \times 10^{-10}\) f.u., only 10 times higher than that in the undoped case. Experimental studies on Fe-doped single crystal [43] and polycrystalline [44] \(\alpha\)-Al\text{2}O\text{3} have also concluded that Fe\(^{4+}\) is the predominant defect in Fe-doped \(\alpha\)-Al\text{2}O\text{3}. The model based on conductivity tests [43] claimed that high p\text{O2} conductivity is due to holes, with a slope of +3/16. While we see holes having a significant concentration in our calculated plots, with a p\text{O2} dependency of +3/16, holes are not the predominant charged defect in our case. Fe\(^{4+}\) is the predominant charged defect, compensated by V\text{Al}\text{H}\text{Al}, with the same p\text{O2} dependency as that of holes and the experiments. The conductivity studies [43] did not consider calculating the ionic transference number by measuring EMF as a function of p\text{O2}, and analysis was done assuming Fe\(^{3+}\) and Fe\(^{4+}\) as the only charge states that Fe expresses, so it is possible that the presence of ionic conductivity due to V\text{Al}\text{H}\text{Al} was missed. At low p\text{O2}, our model reveals that the main defect compensation mechanism is V\text{Al}\text{H}\text{Al} compensating Fe\(^{4+}\). However, creep tests on polycrystalline \(\alpha\)-Al\text{2}O\text{3} concluded that Al\(^{1}\) is the predominant defect [44]. Although the creep rates gave good agreement with p\text{O2} dependency of both V\text{Al}\text{H}\text{Al} and Al\(^{1}\), the better match of the diffusion coefficient of Al\(^{1}\) with that calculated in creep tests resulted in the conclusion that Al\(^{1}\) is the main defect. The fact that we are considering single crystal \(\alpha\)-Al\text{2}O\text{3} in this study, and given that \(\alpha\)-Al\text{2}O\text{3} grain boundaries are fast diffusion pathways for oxygen (which precluded its consideration as a rate limiting step in creep) could be the reason behind the difference at low p\text{O2}.

In Cr-doped \(\alpha\)-Al\text{2}O\text{3}, as seen in Fig. 2e, similar to the case of Fe-doped \(\alpha\)-Al\text{2}O\text{3}, Cr\(^{3+}\) (Cr\(^{4+}\)) is the predominant charge state across the p\text{O2} range. At high p\text{O2}, the concentration of Cr\(^{4+}\) is only \(-10^{-11}\) per f.u. Cr\(^{4+}\) is compensated by V\text{Al}\text{H}\text{Al}; the concentration of which is \(8 \times 10^{-11}\) per f.u., nearly equal to that in the undoped case. Apart from Cr\(^{4+}\) and V\text{Al}\text{H}\text{Al}, holes are also seen to have predominant concentration at high p\text{O2}. Except for Cr\(^{4+}\) and Cr\(^{3+}\), no other charge states of Cr (+5, +6) are found to have significant concentrations in the p\text{O2} range considered. To our knowledge, no experimental conductivity studies have been conducted for Cr-doped \(\alpha\)-Al\text{2}O\text{3}, but Cr is known to be primarily an isovalent dopant in \(\alpha\)-Al\text{2}O\text{3} [41]. This is reflected in our plots.

Thus, from the above analysis, at high p\text{O2}, \(\alpha\)-Al\text{2}O\text{3} formation conditions, to increase the electronic conductivity of \(\alpha\)-Al\text{2}O\text{3}, acceptor doping such as that with Mg should be employed; to increase the cationic conductivity of \(\alpha\)-Al\text{2}O\text{3}, donor doping such as that with Si or Ti, which exhibit predominantly +4 charge states, should be employed. Fe and Cr doping will prove to be unsatisfactory dopants if the aim is to increase electronic/ionic conductivity, as these dopants are primarily isovalent in the \(\alpha\)-Al\text{2}O\text{3} lattice. The trends observed above can further be rationalized from the equilibrium Fermi levels established, which is elaborated upon in SI Fig. S5.

### 3.3. Hydrogen defects in \(\alpha\)-Al\text{2}O\text{3}

Before going into the defect equilibria of hydrogen and electronic defects at 300 K, H\text{2} gas environment conditions, we first define the relevant hydrogen defects in \(\alpha\)-Al\text{2}O\text{3}. The defects considered were: mono-hydrogen interstitials H\text{1}\text{Al}, Al vacancy-H complexes [V\text{Al} – nH\text{Al}\text{H}], substitution in the O site H\text{O} (denoted as [V\text{O} – H\text{O}]) in Section 2.3, and dopant-H complexes [M\text{Al} – H\text{Al}]. We do not consider H complexes with native interstitials as the latter have high energies of formation across most of the Fermi level range (see SI Fig. S2), rendering their concentrations and thereby their complexes with hydrogen, negligible. Fig. 3 shows the fully relaxed atomic configurations of some of the key hydrogen defects. In H\text{1}, hydrogen is oriented towards the empty octahedral interstitial site, as a proton attached to one of the 6 degenerate oxygen atoms. This orientation matches previous experimental [15,16] and theoretical [4] studies. Given that Al vacancies are known to be ‘hydrogen sinks’ in Al\text{2}O\text{3} [45], it is important to consider Al vacancy-H clusters. In all [V\text{Al} – nH\text{Al}\text{H}] complexes, hydrogen atoms exist as protons attached to oxygen atoms, pointing towards the cation vacancy center.

The tendency to add/remove a hydrogen atom to an existing [V\text{Al} – nH\text{Al}\text{H}] complex is determined by a positive/negative binding energy:

\[
E_b (\text{[V}_\text{Al} – n\text{H}\text{Al}\text{H}]) = E_f^{\text{Vac}} - (n-1)E_f^{\text{H}} + E_f^{\text{H}} - E_f^{\text{Vac} – n\text{H}\text{Al}\text{H}}
\]

(9)

From Table 1, it is clear that [V\text{Al} – nH\text{Al}\text{H}] up to n = 5 are stable.

While other charge states of these defect complexes are theoretically possible, the number of possibilities are intractable when n > 1. Intuitively, we believe that the charge states listed in Table 1 will be the predominant charge states, given that the V\text{Al}\text{H}\text{Al} is known to be a very attractive site for protons, as observed from experimental infrared absorption peaks in cooled γ-irradiated single crystal \(\alpha\)-Al\text{2}O\text{3} specimen [47] and from theoretical studies [45]. In H\text{O}, the hydrogen occupies the oxygen vacancy site. Charge density plots and local density of states (shown in SI Fig. S6) reveal that hydrogen multicanter bonds are formed between the reactive hydrogen and the nearest neighbor Al cations. This also explains the high binding energy of the complex (3.12 eV, see SI Table S3).

### 3.4. Defect concentration profiles at 300 K in hydrogen atmosphere

We examine next the concentrations of hydrogen and electronic defects established at 300 K over a range of p\text{O2}, while keeping the individual ionic defect concentrations fixed as found in undoped and doped \(\alpha\)-Al\text{2}O\text{3} at 1100 K high p\text{O2} conditions. At such low temperatures, binding energy effects become significant. As a result, while the total native and dopant concentrations are fixed, hydrogen may bind to these defects, thus the concentration of isolated native and dopant defects may decrease and the concentration of defect complexes increase.

Fig. 4a shows the defects in undoped \(\alpha\)-Al\text{2}O\text{3}. We see that the predominant defect is [V\text{Al} – 3H\text{Al}\text{H}], if one compares this with Fig. 1, it is readily noticed that the concentration of [V\text{Al} – 3H\text{Al}\text{H} + [V\text{Al} – 4H\text{Al}\text{H}], whose constituent defects are V\text{Al}\text{H}\text{Al}\text{H} and H\text{Al}, is nearly equal to that of V\text{Al}\text{H}\text{Al} established at 1100 K, p\text{O2} = 1 atm (5.3 × 10^{-11} per f.u.). The concentration of free hydrogen interstitials H\text{Al} is \(-10^{-16}\) per f.u. The concentrations of [V\text{Al} – H\text{Al}\text{H}, V\text{Al} – 2H\text{Al}\text{H} and [V\text{Al} – 5H\text{Al}\text{H}] are not significant, as all the [V\text{Al} – nH\text{Al}\text{H}] complexes compete for the fixed number of V\text{Al}\text{H}\text{Al} sites. The significantly high binding energies of the clusters up to n = 4 as seen in Section 3.3 results in the predominance of [V\text{Al} – 3H\text{Al}\text{H} and [V\text{Al} – 4H\text{Al}\text{H}]. Moreover, concentration of [V\text{Al} – 5H\text{Al}\text{H}] is lower due to the comparatively lower binding energy and due to the (N\text{Al})\text{Al}\text{H} term involved in determining its concentration, which rapidly reduces the right-
hand side term of Equation (5). The total H concentration is ~6.8 \times 10^{-11} per f.u., the contribution to which comes almost entirely from bound defect complexes seen in Fig. 4a. This explains the superior hydrogen permeation resistance behavior of $\alpha$-Al$_2$O$_3$; that is, most of the hydrogen is immobile, trapped at Al vacancy sites with high binding energy.

In Mg-doped $\alpha$-Al$_2$O$_3$, as seen in Fig. 4b, the predominant defect is the neutral defect complex $[\text{Mg}_\text{Al} - H]^+$, and its concentration is nearly equal to that of Mg (1 ppm). The concentration of free $H_i$, along with the significant effect of binding energy (0.68 eV for $[\text{Mg}_\text{Al} - H]^+$), leads to the formation of the complex and a decrease in the concentration of isolated $\text{Mg}_\text{Al}$. The isolated $H_i$ is compensated by the remaining isolated $\text{Mg}_\text{Al}$ (concentration $-3.24 \times 10^{-3}$ per f.u.). The total concentration of hydrogen is almost dictated entirely by the dopant concentration due to the complex formation and thus is $-10^{-5}$ per f.u. This $10^4$ times increase in the total concentration of hydrogen, as well as the $10^7$ times increase in the concentration of diffusible hydrogen ($H_i$), as compared to the undoped case, makes Mg an unfavorable dopant to be used for improving the barrier properties of $\alpha$-Al$_2$O$_3$.

In contrast to the above two cases, in Si-doped $\alpha$-Al$_2$O$_3$, hydrogen defects are not the predominant defects. Instead, the

<table>
<thead>
<tr>
<th>Defect cluster</th>
<th>Binding energy (eV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[\text{V}_\text{Al} - H]^+$</td>
<td>3.24</td>
</tr>
<tr>
<td>$[\text{V}_\text{Al} - 2H]^+$</td>
<td>1.90</td>
</tr>
<tr>
<td>$[\text{V}_\text{Al} - 3H]^+$</td>
<td>1.99</td>
</tr>
<tr>
<td>$[\text{V}_\text{Al} - 4H]^+$</td>
<td>0.912</td>
</tr>
<tr>
<td>$[\text{V}_\text{Al} - 5H]^+$</td>
<td>0.248</td>
</tr>
<tr>
<td>$[\text{V}_\text{Al} - 6H]^+$</td>
<td>-0.716</td>
</tr>
</tbody>
</table>

**Table 1**

Binding energy of various $[\text{V}_\text{Al} - nH]^+$ clusters.
predominant defects are $\text{Si}_{\text{Al}}$ compensated by $\text{V}^{0}_{\text{Al}}$. The concentration of free hydrogen interstitial $H_i$ is negligible ($\sim 10^{-10}$ per f.u.), due to its high formation energy at the equilibrium Fermi level established (see SI Fig. S7 for further discussion on equilibrium Fermi level effects on $H_i$ concentration in the undoped and doped $\alpha$-$\text{Al}_2\text{O}_3$ cases). This extremely low concentration of $H_i$ offsets the effect of high binding energy and of the $10^3$'x increase in $V^{+}_{\text{Al}}$ concentration in the Si-doped system seen in Section 3.2 (which would have served to increase the concentration of $[V^{+}_{\text{Al}} - nH_i]^q$ as per Equation (5)). This ultimately reduces the concentration of hydrogen defect complexes as well. The $[\text{Si}_{\text{Al}} - H_i]^x$ complex concentration is also not significant, due to the low concentration of $H_i$.

The low concentrations of defect complexes in this case shows that not only does the binding energy of the defect complex matter, but the concentrations of the isolated individual defects also must be significant to ensure complex formation. It is worthwhile to point out here that this reduced concentration of free $H_i$ and hydrogen defect complexes ultimately result in the overall hydrogen solubility to be lowered in Si-doped $\alpha$-$\text{Al}_2\text{O}_3$ to nearly $1/3$rd that of the undoped case. Additionally, it must be noted that diffusivity is also rapidly reduced, as hydrogen is present primarily in bound form in the lattice as $[V^{+}_{\text{Al}} - H_i]^+$ and $[V^{0}_{\text{Al}} - H_i]^-$. These have high binding energies of 3.24 eV and 2.71 eV respectively. Thus, ultimately the overall permeability is likely to be lowered. Similar trend is seen in Ti-doped $\alpha$-$\text{Al}_2\text{O}_3$. Thus, with Si- and Ti-doping of $\alpha$-$\text{Al}_2\text{O}_3$, not only

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**Fig. 4.** Defect concentrations as a function of $p_{\text{H}_2}$ in (a) undoped (b) Mg-doped (c) Si-doped (d) Ti-doped (e) Fe-doped (f) Cr-doped $\alpha$-$\text{Al}_2\text{O}_3$ at 300 K. Only those defects with concentration $[D] > 10^{-18}$ per f.u. $\alpha$-$\text{Al}_2\text{O}_3$ are shown. Point defects are represented by Kröger-Vink notation.
does the total hydrogen concentration decrease to 1/3 of that of the undoped case, the drastically lowered concentration of diffuse hydrogen ($H^i$) and the presence of hydrogen predominantly in bound form will likely reduce the hydrogen diffusivity, resulting in an overall lowered permeability. This will improve the hydrogen permeation resistance properties of $\alpha$-Al$_2$O$_3$. If the cyto-violence form of $\alpha$-Al$_2$O$_3$ is a deep donor, its defect level at 3.93 eV. In all cases, the equilibrium Fermi level established is deep in the band gap (see SI Fig. S7), away from the conduction and valence band edges. The deep defect level is also seen in the density of states plot, shown in SI Fig. S8. Thus, interstitial hydrogen is compensated by other defects present in the oxide, without contributing to the electronic conductivity. It must be emphasized here that the trends and predominant hydrogen defects revealed are also important while considering the effect of hydrogen in oxides used in other applications. One example as mentioned in Section 1 is that of resistive switching, in which the oxide electrolytes have exhibited sensitivity to humidity [24]. Knowledge of the defect state of hydrogen in the oxide by utilizing the methods employed in this study can help unambiguously establish the defect compensation mechanisms during device operation and elucidate the effect of hydrogen on device performance. Therefore, in SI Fig. 4, the x-axis can be replaced by voltage, which can be used in such devices to control the hydrogen concentration. 

Fig. 5 summarizes the roles of the concentrations of $V_{Al}^a$, $H_i$ and $[V_{Al}^\bullet - nH]^a$ ([Mg$_{Al}$ - $H]^a$ for Mg-doped case) in determining the total concentration of H-defects, $H_{tot}$. Although other charge states of $V_{Al}$ and $H_i$ do contribute to the H-defect complexes and overall H concentration, we focus mainly on $V_{Al}$ and $H_i$ as they are involved in the maximum number of H-complexes and overall H concentration. The clear effect of donor dopants in eliminating $H_i$ is seen in SI Fig. 5b. In Fig. 5c, it is shown that with Si and Ti doping, the low $H_i$ offsets the increase in $V_{Al}^a$ seen in SI Fig. 5a, leading to a decrease in the concentration of hydrogen-defect complexes relative to the undoped case. The trend in SI Fig. 5d matches exactly that in SI Fig. 5c, showing that in all cases, total hydrogen solubility is dictated by largely H-defect complex concentration due to binding energy effects at low temperature. This figure summarizes the main results of our work.

3.5. Effect of varying dopant concentrations on hydrogen permeability

In the results presented above, the dopant concentration was fixed at 1 ppm. As mentioned in Section 2.2, this is simply a representative doping concentration, chosen to be within the solubility limit of the various dopants in $\alpha$-Al$_2$O$_3$ and to ensure that the dilute defect approximation is still valid. However, a reasonable question arises, i.e., whether changing the dopant concentration will impact the relative trends in concentrations of free $H_i$ (impacting hydrogen diffusivity) and the total concentration of hydrogen.

To answer this, two additional dopant concentrations were studied: 10 and 100 ppm. As seen in SI Fig. 59, at 1100 K, increase in dopant concentration leads to (i) increase in compensating defect concentrations and (ii) increase in transition $p_{D} \rightarrow p_{NT}$ for p-type to n-type conductivity in the Si and Ti donor doping cases. However, the relative trends in defect concentrations do not change with dopant concentration for the most part, except for the Cr-doped case. In the 1 ppm Cr-doped case, holes are the 2nd predominant defect, whereas in the 10 and 100 ppm Cr-doped cases, $Cr_{Al}$ is the 2nd predominant defect. 

Fig. 6 shows the effect of doping levels on the concentrations of $V_{Al}^a$, $H_i$ and $[V_{Al}^\bullet - nH]^a$ ([Mg$_{Al}$ - $H]^a$ for Mg-doped case), which ultimately determine hydrogen diffusivity and solubility (similar to Fig. 5). From SI Fig. 6a it is clear that increase in Si and Ti concentration by 10x and 100x leads to a corresponding increase in $V_{Al}^a$ concentration, due to the exact compensation of $Si_{Al}$ and $Ti_{Al}$ by $V_{Al}^\bullet$. Increase in $V_{Al}^a$ in the Fe and Cr doped cases is not as high, as Fe and Cr are predominantly isovalent in $\alpha$-Al$_2$O$_3$. SI Fig. 6b shows that despite an increase in Mg, Si, and Ti concentration, the free $H_i$
concentration does not change significantly for these dopants. This is because the Fermi level in these cases is effectively pinned by the charged ionic defect with the largest (fixed) concentration (therefore the pinning effect is strongest for Mg, Si and Ti), and changing dopant concentration by 10x or 100x is not enough to change the Fermi level significantly. The pinning effect isn’t as strong for Fe and is the least strong for Cr as these are primarily isovalent in $\alpha$-Al$_2$O$_3$. Although a large drop in $H^i$ concentration is seen in the Cr doped case as the concentration is changed from 1 to 10 ppm, this drop is still negligible at the scale considered (the absolute $H^i$ defect concentrations in donor doped cases are of the order 10$^{-5}$ ppm). Thus, increase in dopant concentration does not have a significant effect on concentration of diffusible $H^i$. From Fig. 6c we see that an increase in Mg concentration to 10 and 100 ppm leads to an increase in $[\text{MgAl} - H]^n$ concentration, as expected. However, increasing Si and Ti doping to 10 and 100 ppm leads to an increase in hydrogen complex concentration by 2x and 12x respectively, compared to the undoped case. This increase (as opposed to the decrease in hydrogen complex concentration when Si and Ti doping was 1 ppm) is due to the increase in $V^\text{Al}$ concentration, which increases the complex concentration via Equation (5). The decrease in $H^i$ upon increased Si and Ti doping is not enough to offset the effect of increased $V^\text{Al}$ concentration. At 10 ppm Fe, the H-complex concentration remains nearly the same as that in the undoped case. However, at 100 ppm, Fe reduces the hydrogen complex solubility by 2x, compared to the undoped case. This is because at this concentration, the decrease in $H^i$ concentration is enough to offset the slight increase in $V^\text{Al}$, leading to a net decrease in overall concentration of H-complexes. Cr doping at 10 ppm slightly increases the H-complex concentration by 2x, and at 100 ppm the H-complex concentration remains nearly equal to that in the undoped case. Finally, the total hydrogen concentration follows the trend of the hydrogen complex concentrations, due to the significant binding energy effects at low temperature. Thus, an increase in the Si and Ti concentrations increases the total hydrogen concentration by ~2x (10 ppm doping) and ~12x (100 ppm doping). At 100 ppm, only Fe reduces the overall solubility by 2x.

This effect of varying dopant concentrations brings us back to the point mentioned in Section 3.4 that it is the concentration of isolated vacancies $N^\text{vac}$, isolated hydrogen interstitials $(N^H)^n$ and the binding energy $E_b$ of the defect complex $[\text{V}^\text{Al} - nH]^n$ that together decide the effect of the dopant on the overall hydrogen solubility of the system. Increasing the dopant concentration does not change the free $H^i$ concentration (and in turn, the hydrogen diffusivity) significantly. Thus, donor dopants like Si, Ti, Fe and Cr, with their near negligible $H^i$ concentrations and hydrogen predominantly present in complex form, will aid in reducing hydrogen permeability. Moreover, doping Si and Ti at 1 ppm gave the best results, as in addition to lowered diffusivity due to near negligible concentration of $H^i$, the total hydrogen concentration is reduced by 3x at this lower doping level, compared to the 2x decrease exhibited by doping Fe at 100 ppm. Thus, doping $\alpha$-Al$_2$O$_3$ with Si and Ti at 1 ppm appears to be the overall best strategy to reduce its hydrogen permeability.

4. Conclusions

To create better hydrogen permeation barriers, experimental and theoretical work related to the behavior of hydrogen in $\alpha$-Al$_2$O$_3$, a popular coating material, and ways to improve its adherence onto
steel, already exist. This study systematically assessed the effect of dopants to further reduce hydrogen permeation in \( \alpha\)-Al\(_2\)O\(_3\). Our analysis took into account the equilibrium high temperature defect concentrations during \( \alpha\)-Al\(_2\)O\(_3\) formation, and the binding energy effects at low temperature to find the defect concentrations during hydrogen-rich operating conditions. In the undoped case, the predominant hydrogen defect is \( \frac{1}{2}\)V\(_{\text{Al}}\)/C\(_{\text{O}}\), a defect complex with high binding energy. This gives insights into the reason behind the superior performance of \( \alpha\)-Al\(_2\)O\(_3\) as a hydrogen permeation barrier: hydrogen predominantly exists as clusters trapped on the V\(_{\text{Al}}\) site, which presumably increases the activation energy for migration in the lattice. Analyzing the effect of dopants, with the doping level fixed at 1 ppm, it was found that the impact on the total hydrogen concentration \( \text{H}_{\text{tot}} \) (with consequences on solubility) and the concentration of free hydrogen interstitials \( \text{H}_i \) (with consequences on diffusivity) is distinct and substantial. In Mg-doped \( \alpha\)-Al\(_2\)O\(_3\), both \( \text{H}_{\text{tot}} \) and isolated \( \text{H}_i \) concentrations are higher than in the undoped case by 10\(^4\) and 10\(^7\) times respectively. This makes Mg an unsuitable choice for a dopant for improving the properties of \( \alpha\)-Al\(_2\)O\(_3\) as a hydrogen permeation barrier. In both Fe- and Cr-doped \( \alpha\)-Al\(_2\)O\(_3\), \( \text{H}_{\text{tot}} \) increases slightly by 1.5 times and 1.25 times respectively, however, the more mobile species \( \text{H}_i \) is found to be literally eliminated. Thus, although Fe and Cr doping increase the hydrogen solubility slightly, the hydrogen is trapped on the V\(_{\text{Al}}\) sites with high binding energies in the form of \( [V_{\text{Al}} - 3H]^n \). This implies that doping with Fe and Cr could help reduce the overall permeability by reducing the diffusivity of hydrogen in the lattice. The best dopants were found to be Si and Ti. These reduce \( \text{H}_{\text{tot}} \) to nearly 1/3 that of the undoped case. Additionally, the more mobile species \( \text{H}_i \) is eliminated. Thus, upon Si and Ti doping, the overall permeability will be reduced because the solubility is lower and most of the hydrogen present is trapped on the V\(_{\text{Al}}\) sites with high binding energy (~3 eV) in the form of \( [V_{\text{Al}} - H]^n \) and \( [V_{\text{Al}} - H]^+ \), thus reducing diffusivity. Comparing these results obtained at 1 ppm doping level with those obtained at doping levels of 10 and 100 ppm, doping \( \alpha\)-Al\(_2\)O\(_3\) with Si and Ti at 1 ppm was found to be the overall best strategy that reduced both hydrogen diffusivity and solubility by the greatest amount. The established effects of the dopants on electrical properties of \( \alpha\)-Al\(_2\)O\(_3\) and on the predominant hydrogen defects in the lattice can also be extended to other applications such as resistive switching, where electrical conductivity, hydrogen defects, and defect compensation mechanisms play a significant role in determining device performance.

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